Gravity: How does it Arise from Gauge Theory?

Herman Verlinde

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The search for a unified theory of natural forces is in a critical phase. The micro- and macroscopic world are successfully described by QFT and GR, but there are deep tensions between different fundamental principles:

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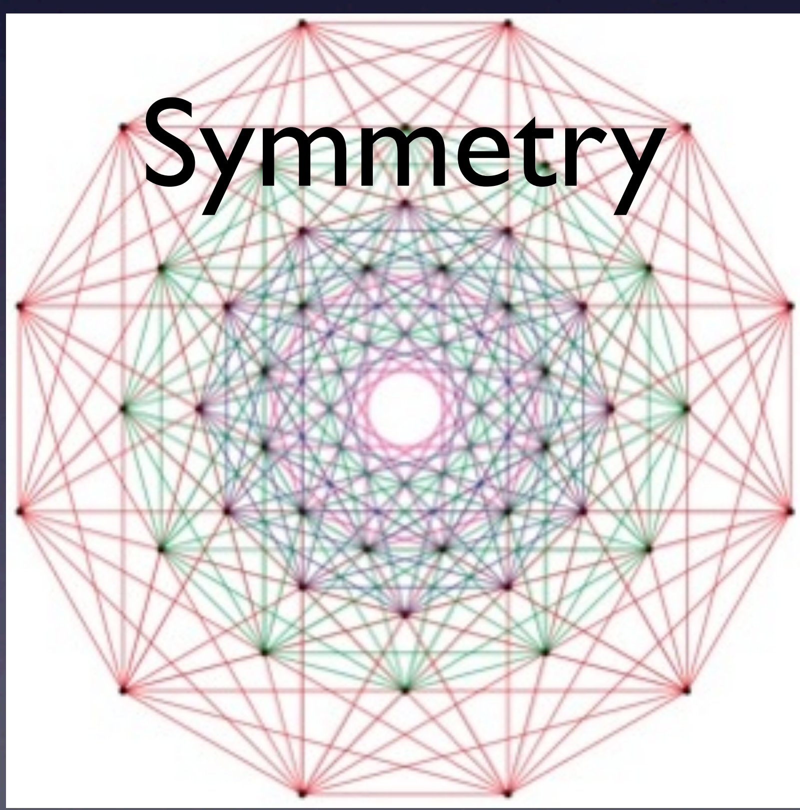

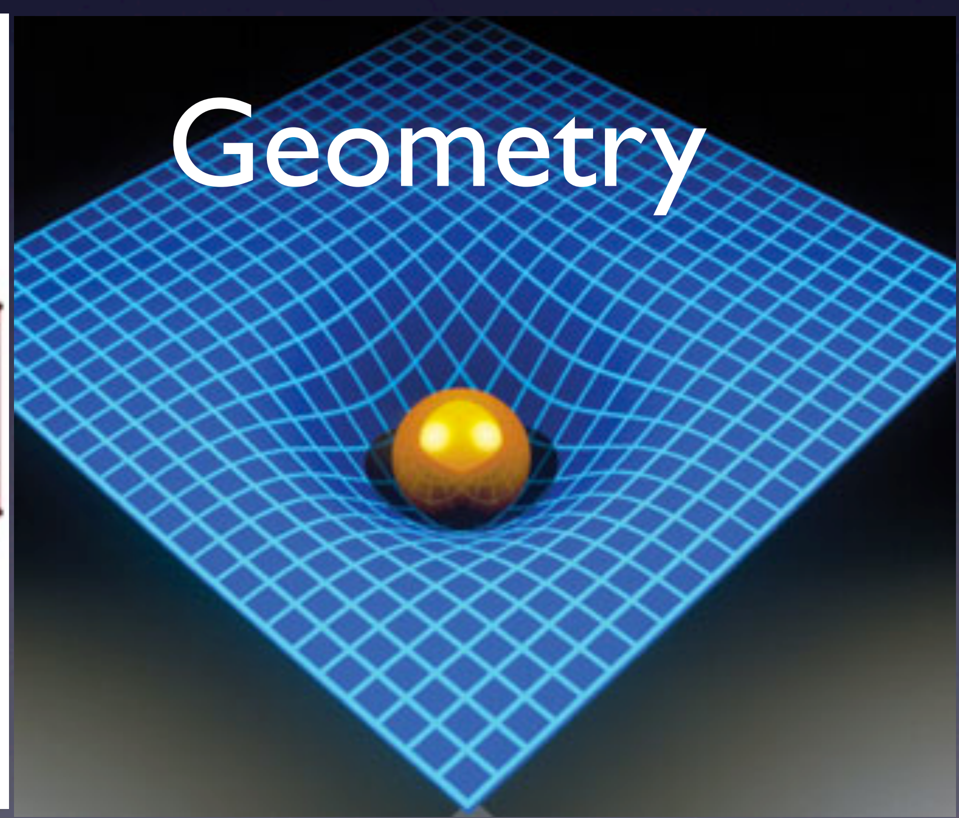

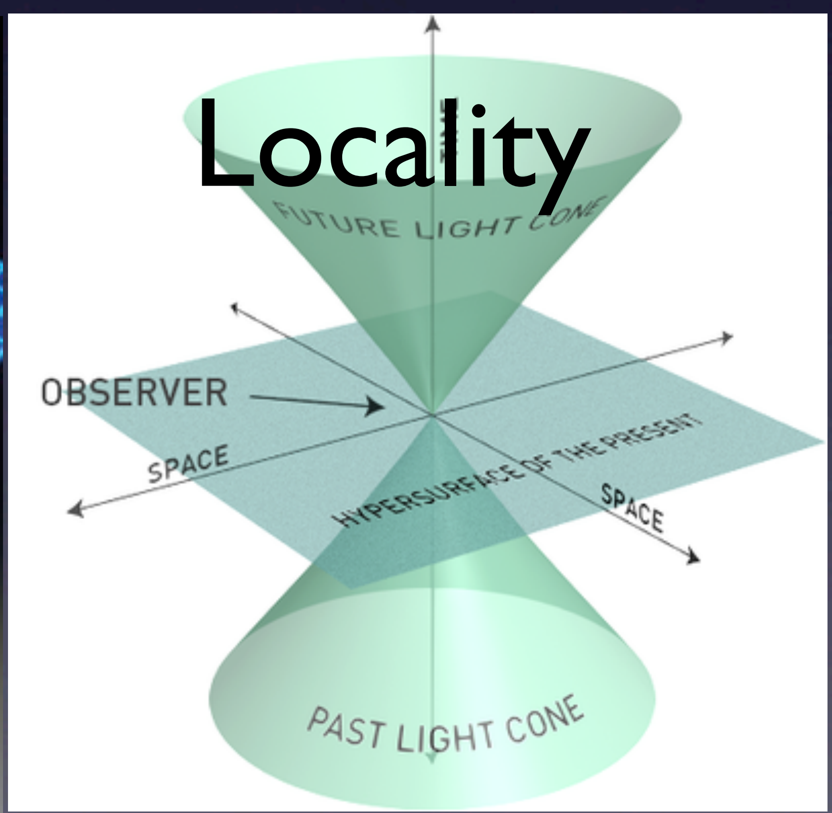

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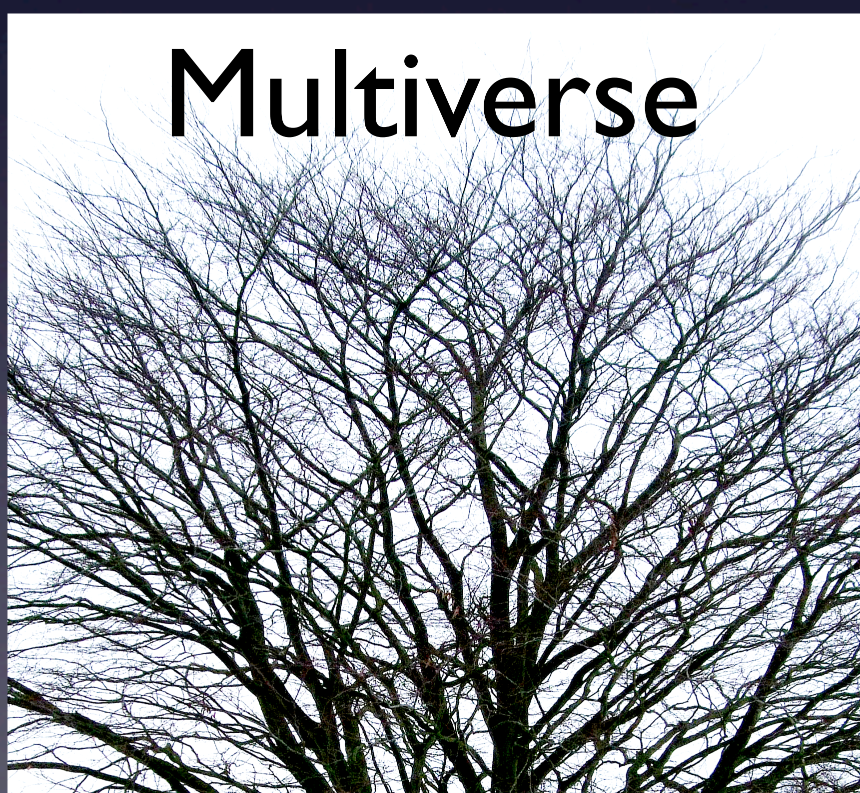

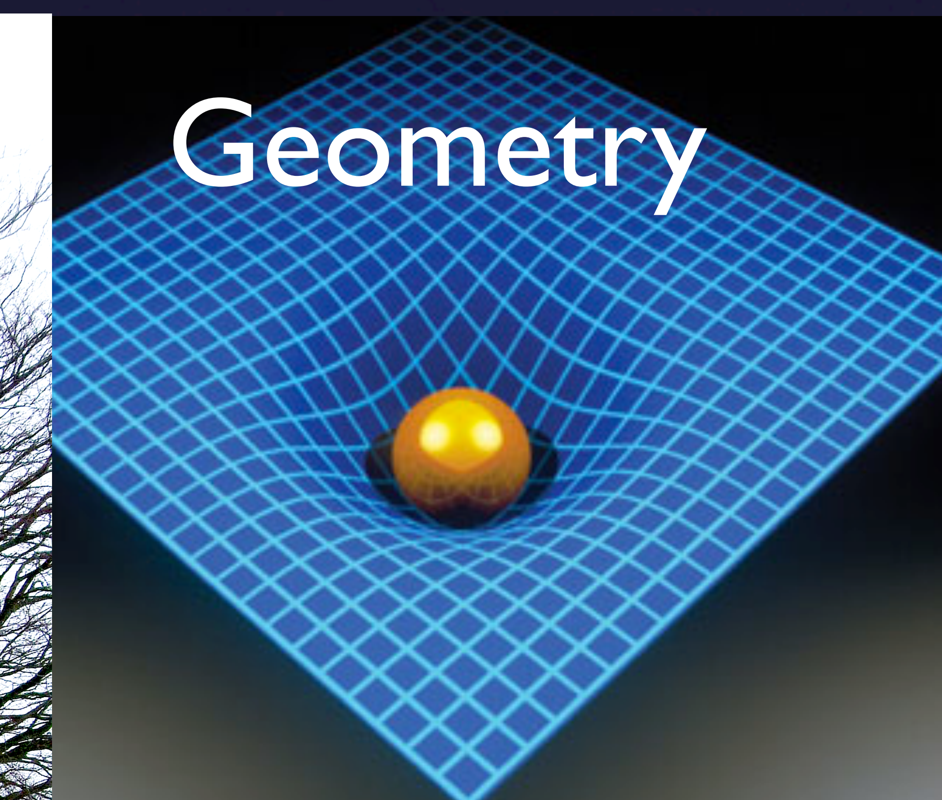

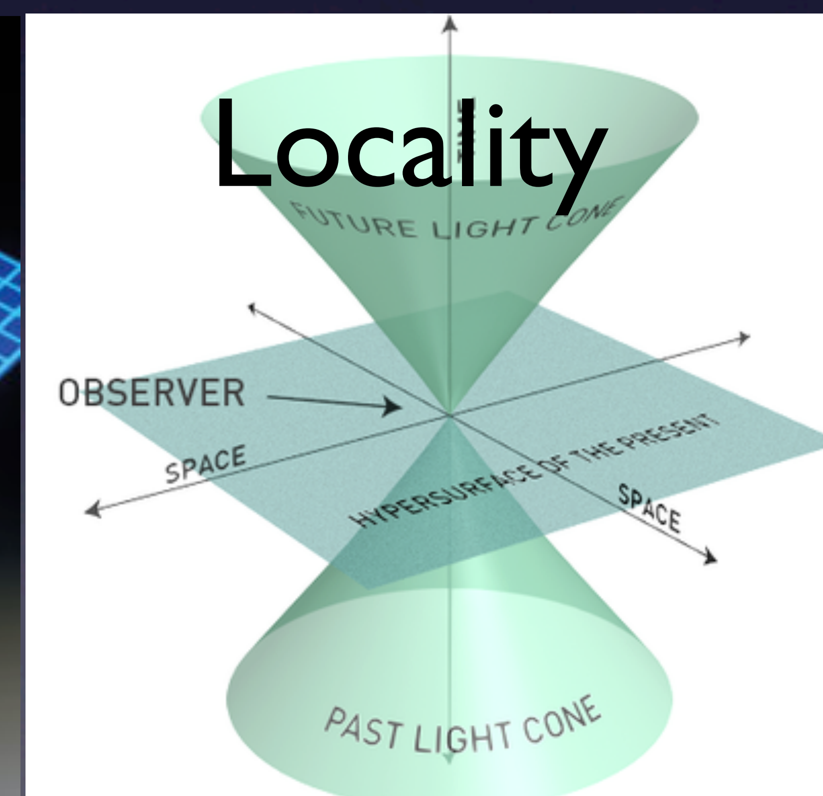

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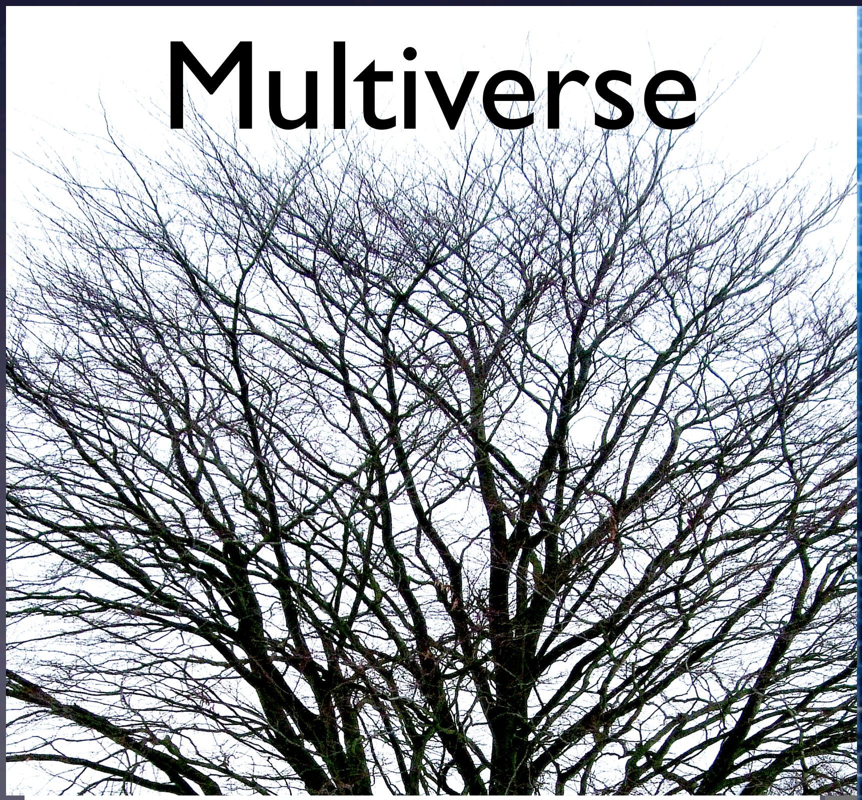

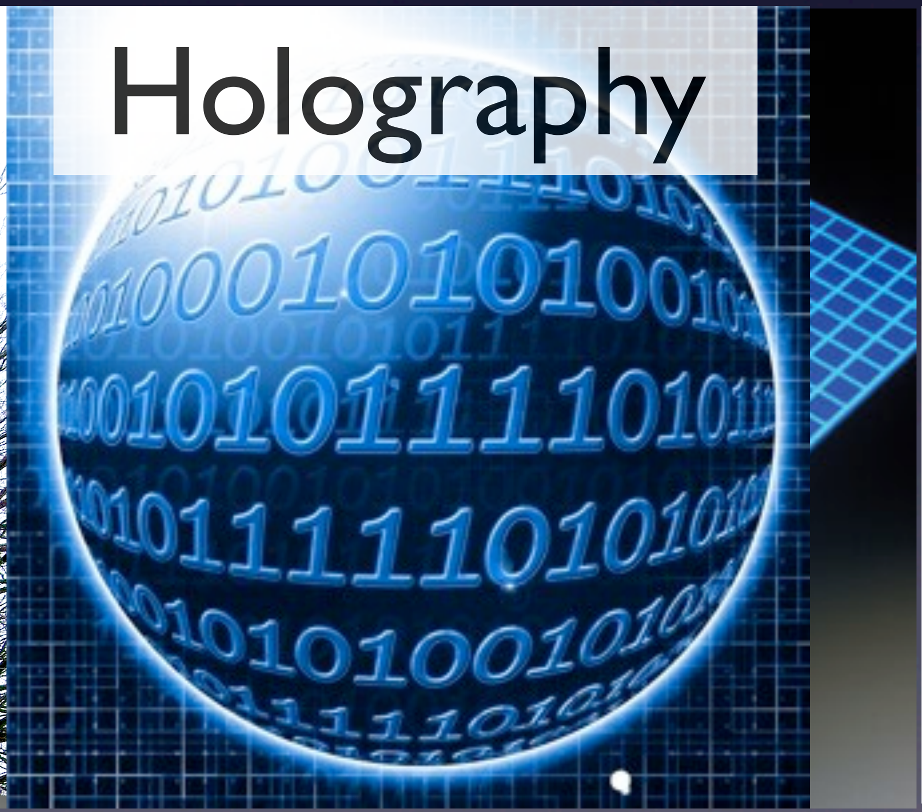

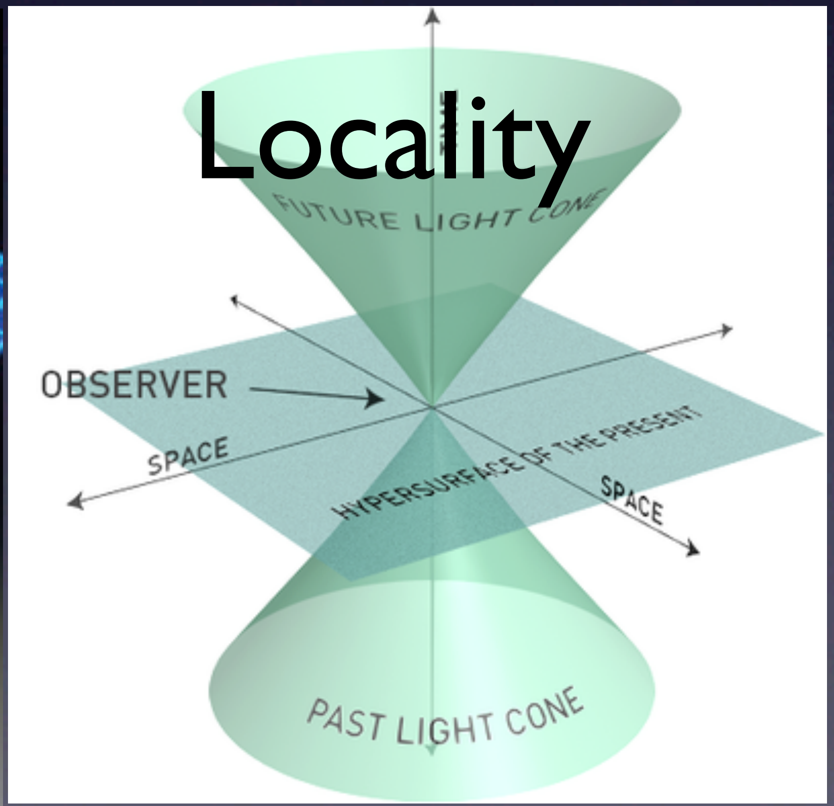

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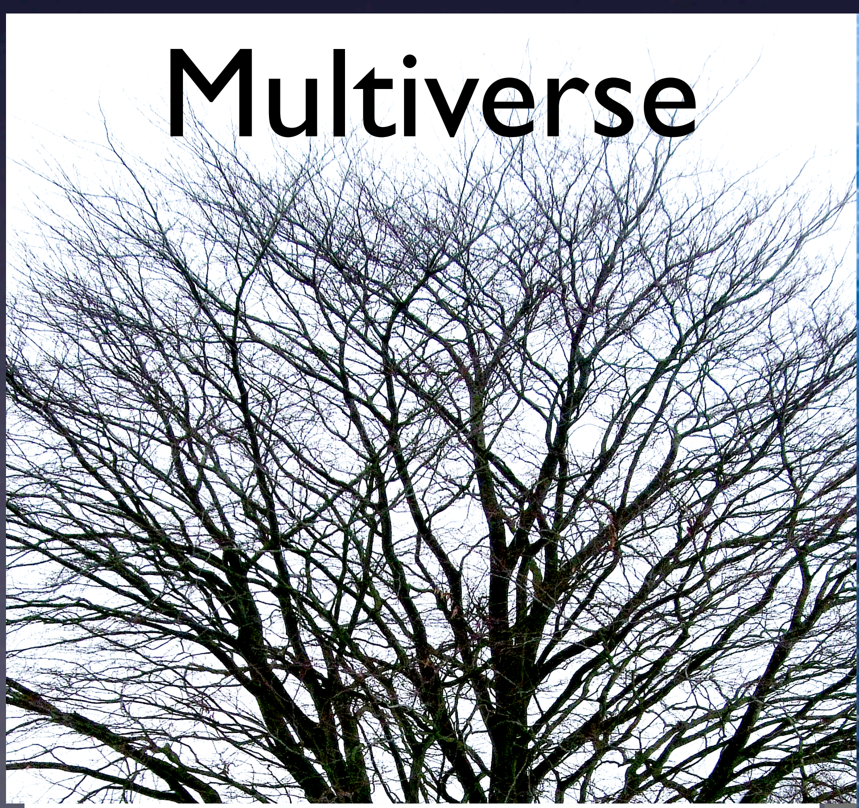
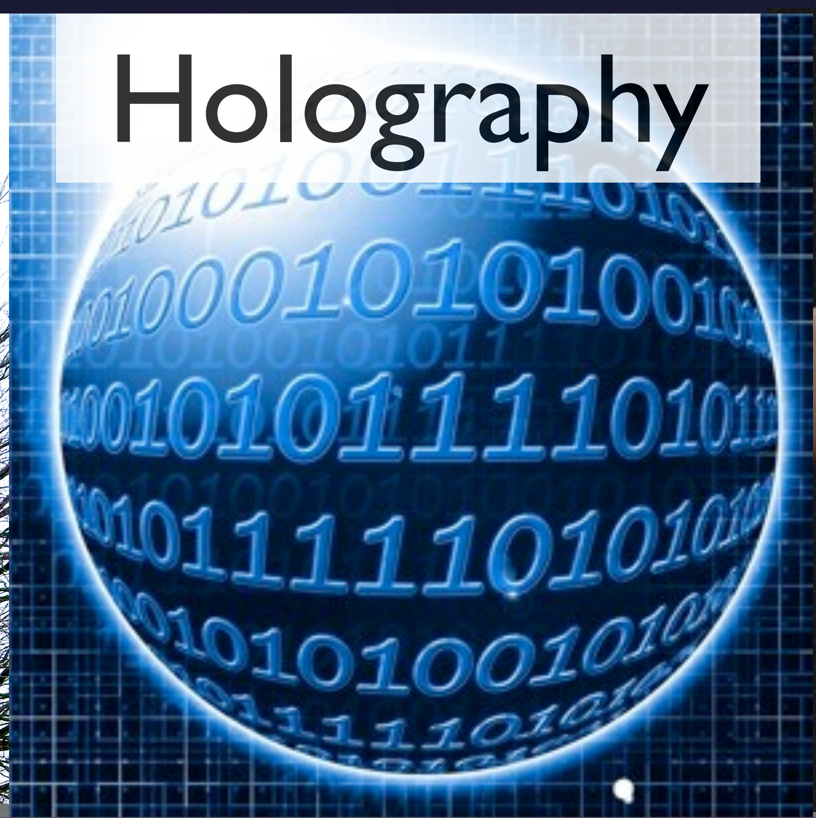
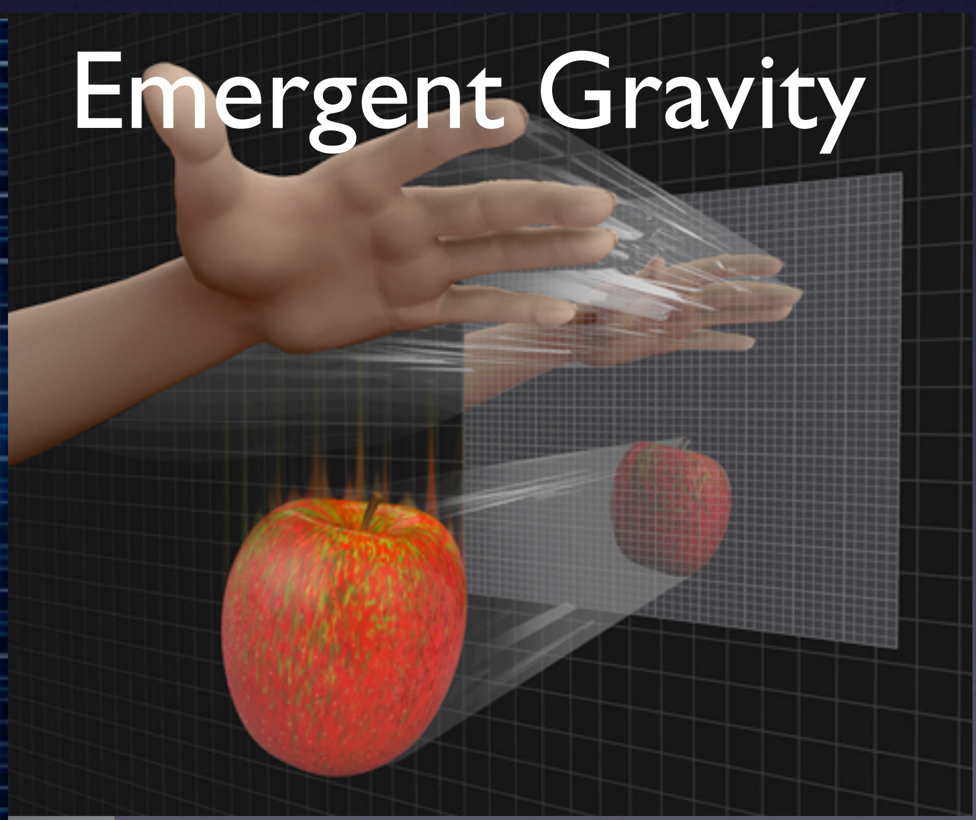
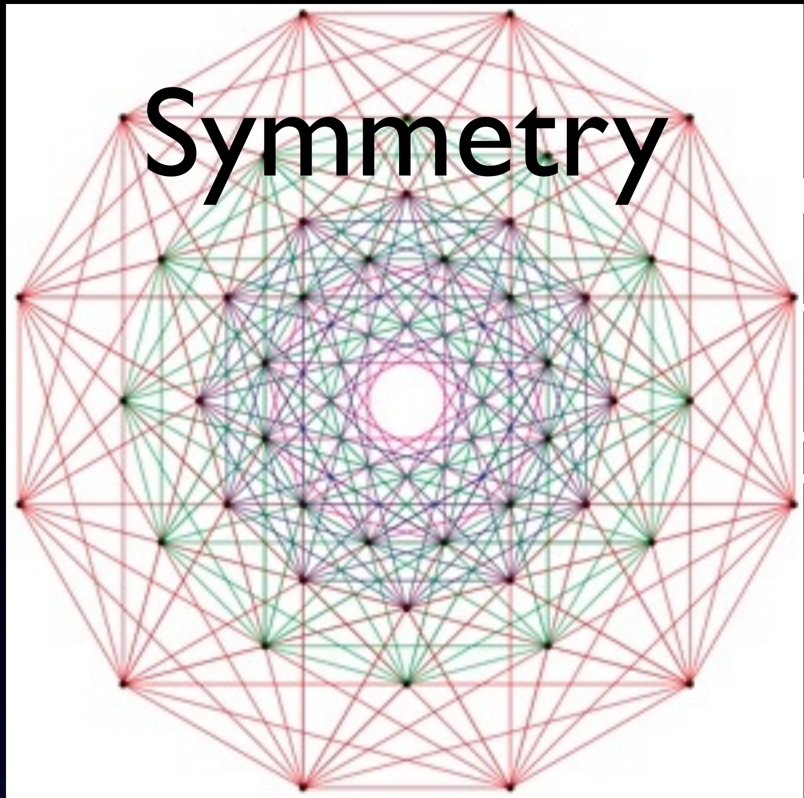
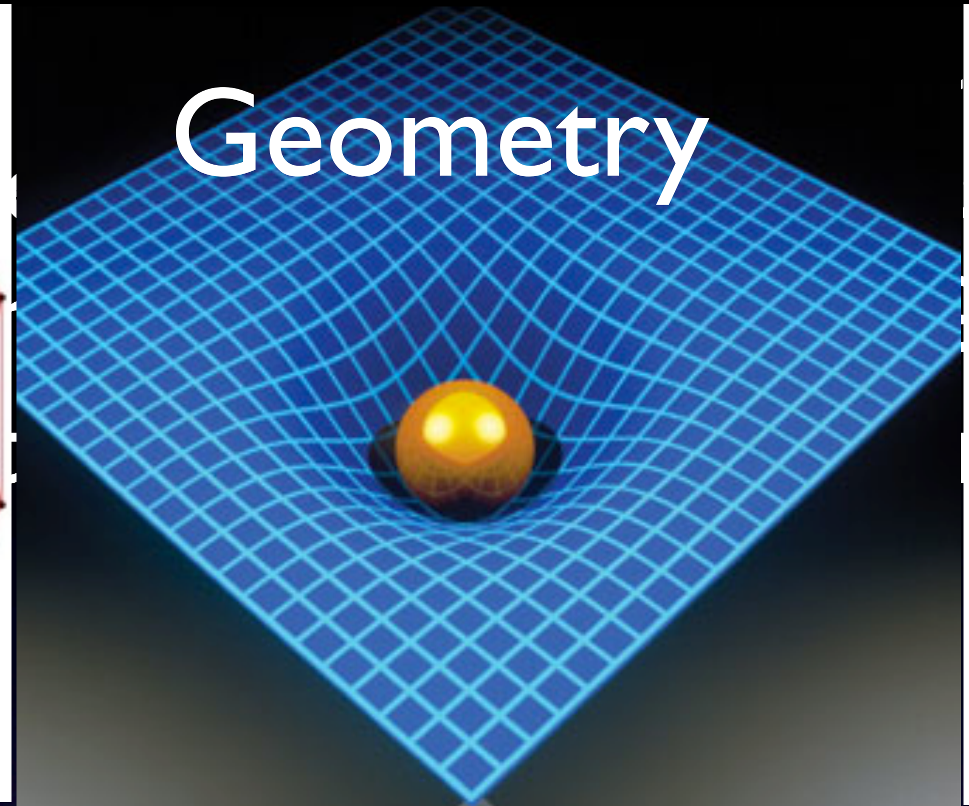
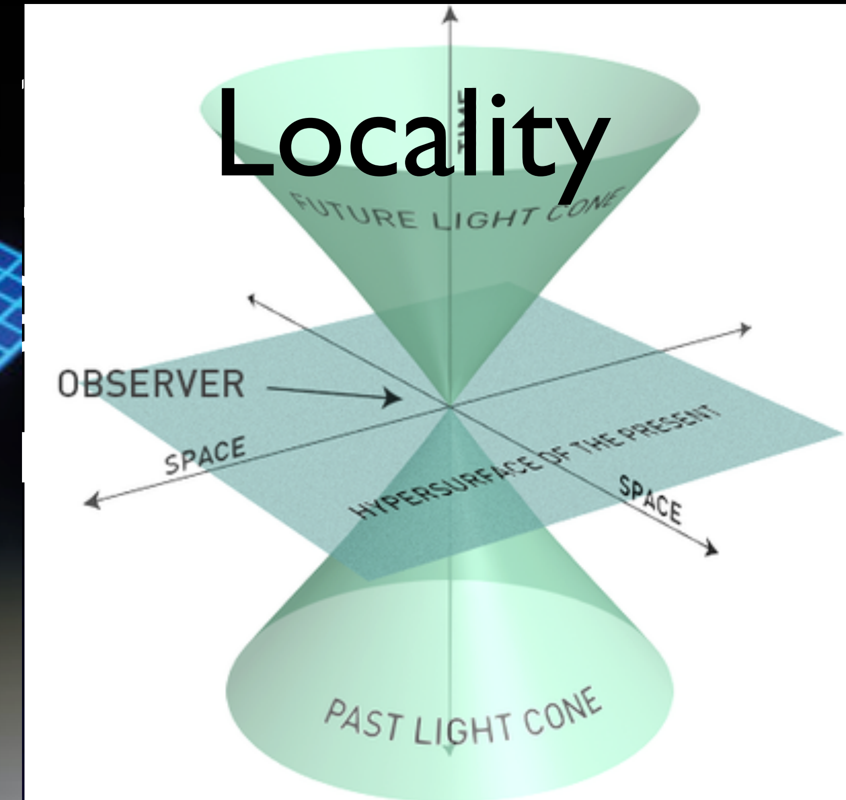


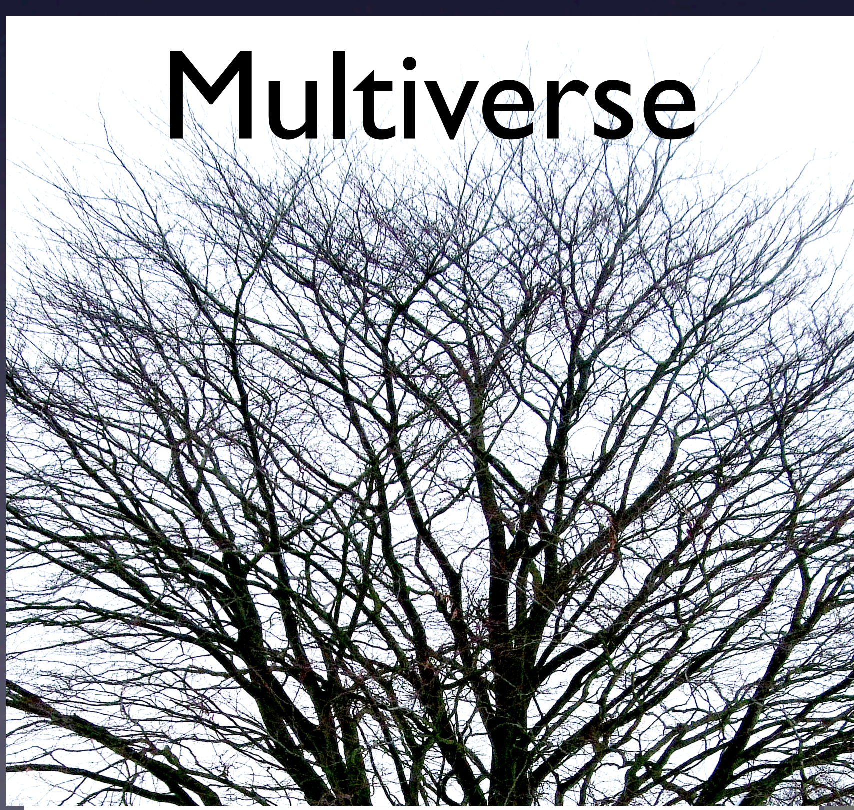


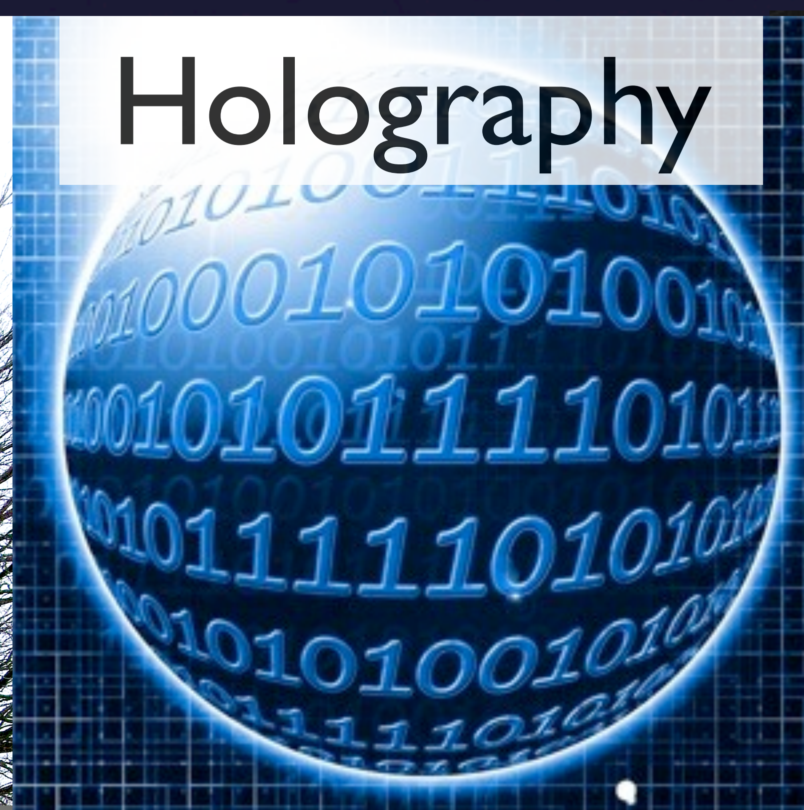


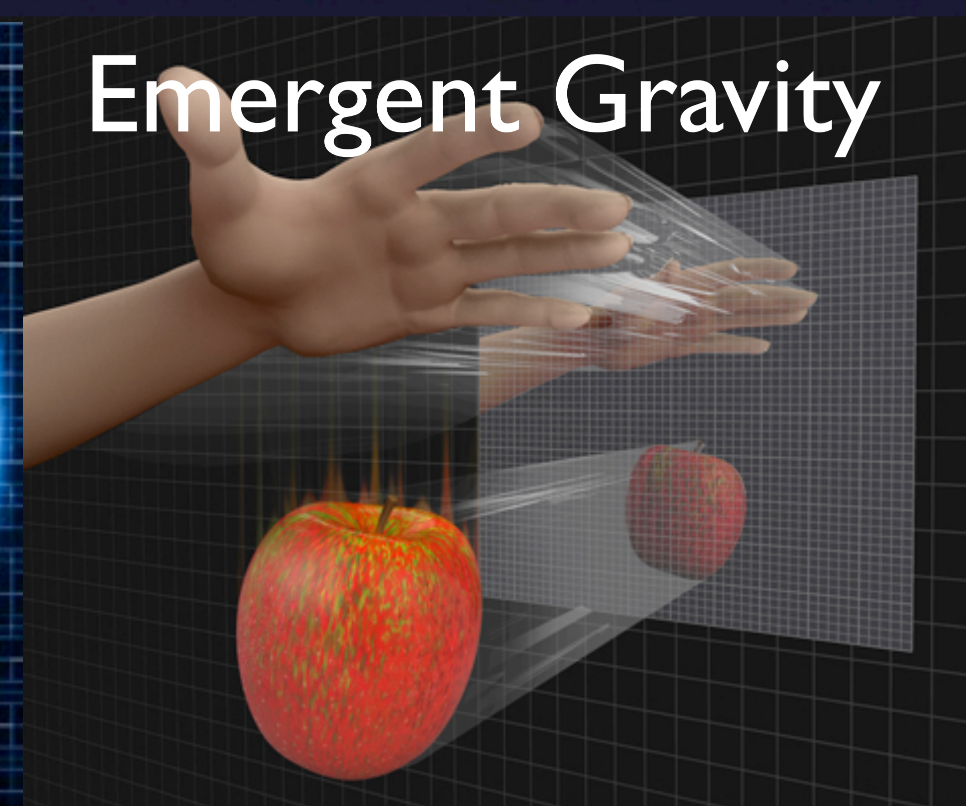

Symmetry


Geometry


Locality
FUTURE LIGHT CONE
OBSERVER
SPACE
HYPERSURFACE OF THE PRESENT
SPACE
PAST LIGHT CONE


Multiverse


Holography


Emergent Gravity

Matrix Theory

Early example of gravity arising from gauge theory:

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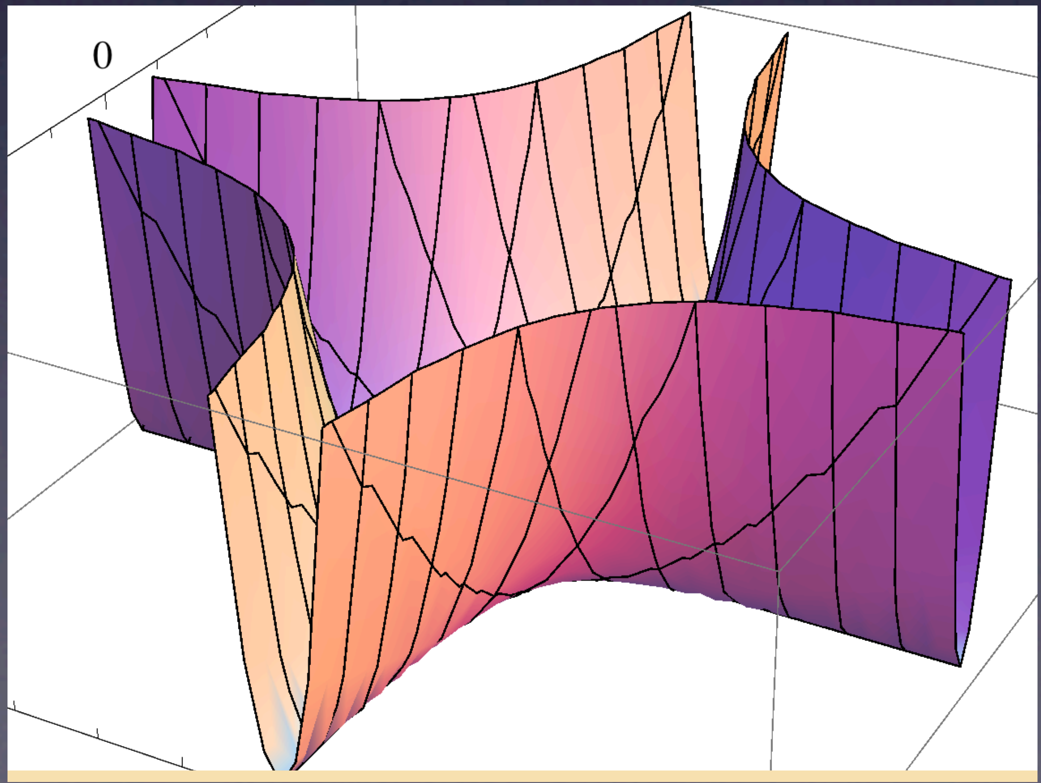
Early example of gravity arising from gauge theory:

* Large N matrix quantum mechanics
* Eigenvalues are positions of particles

$$\mathrm{H} = \mathrm{tr}\left(P_I^{\,2} + [X_I, X_J]^2 + \Psi^* X_I \Gamma^I \Psi\right)$$



Matrix Theory

Early example of gravity arising from gauge theory:

* Large N matrix quantum mechanics
* Eigenvalues are positions of particles
* Off-diagonal modes are open strings
*Quantum fluctuations induce gravity:

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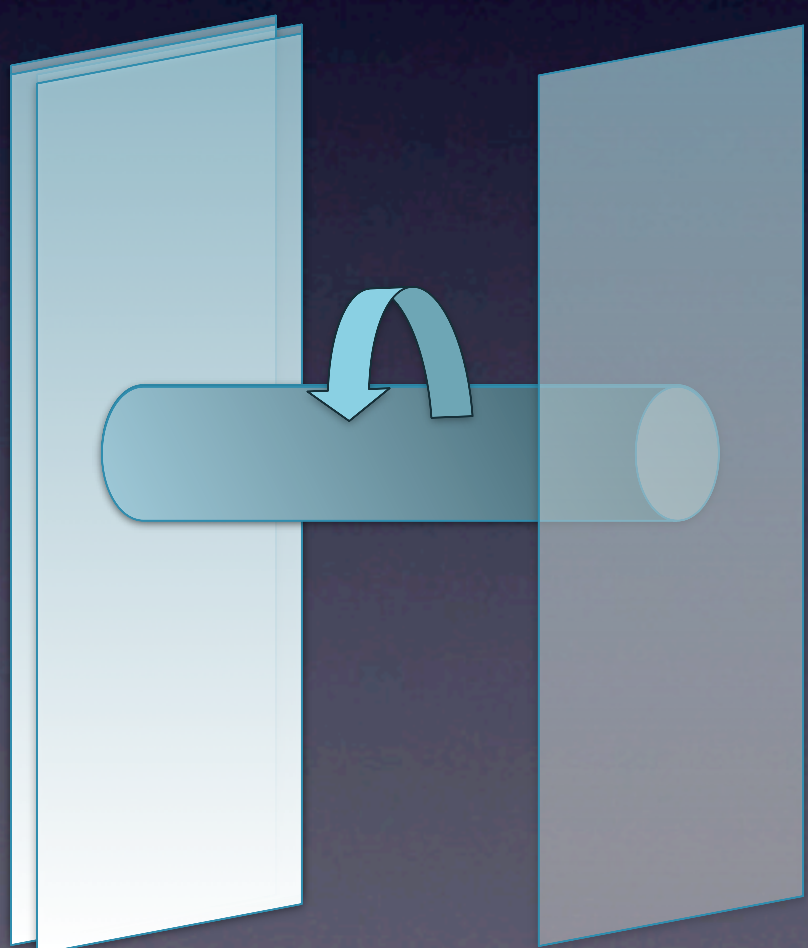
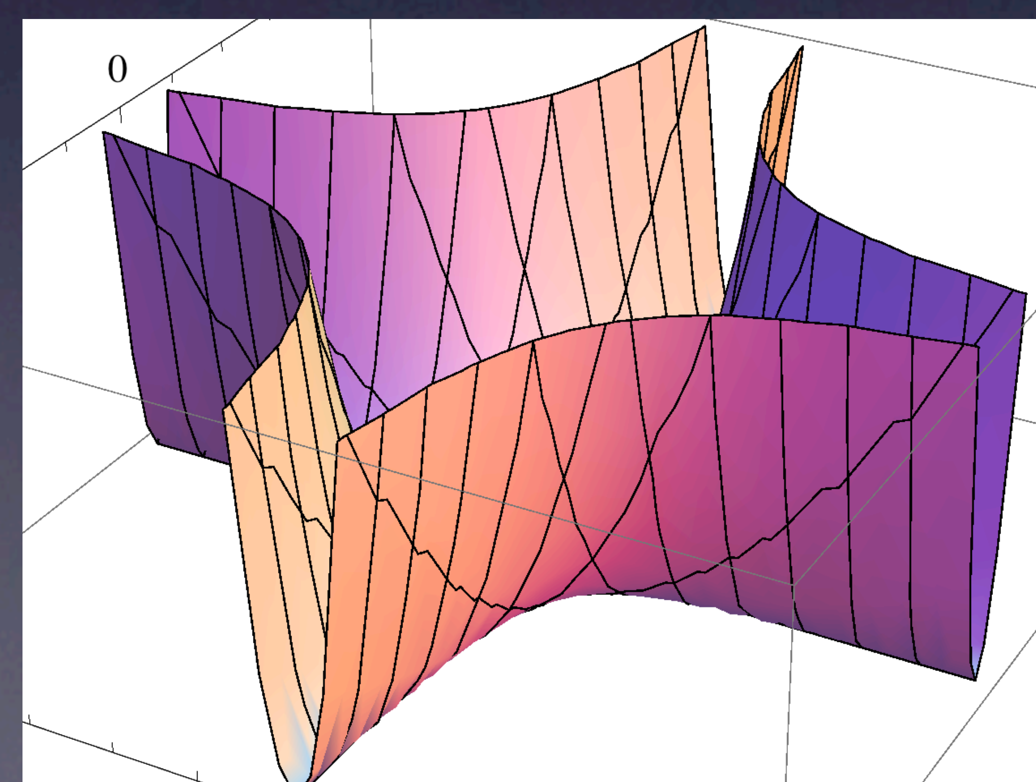
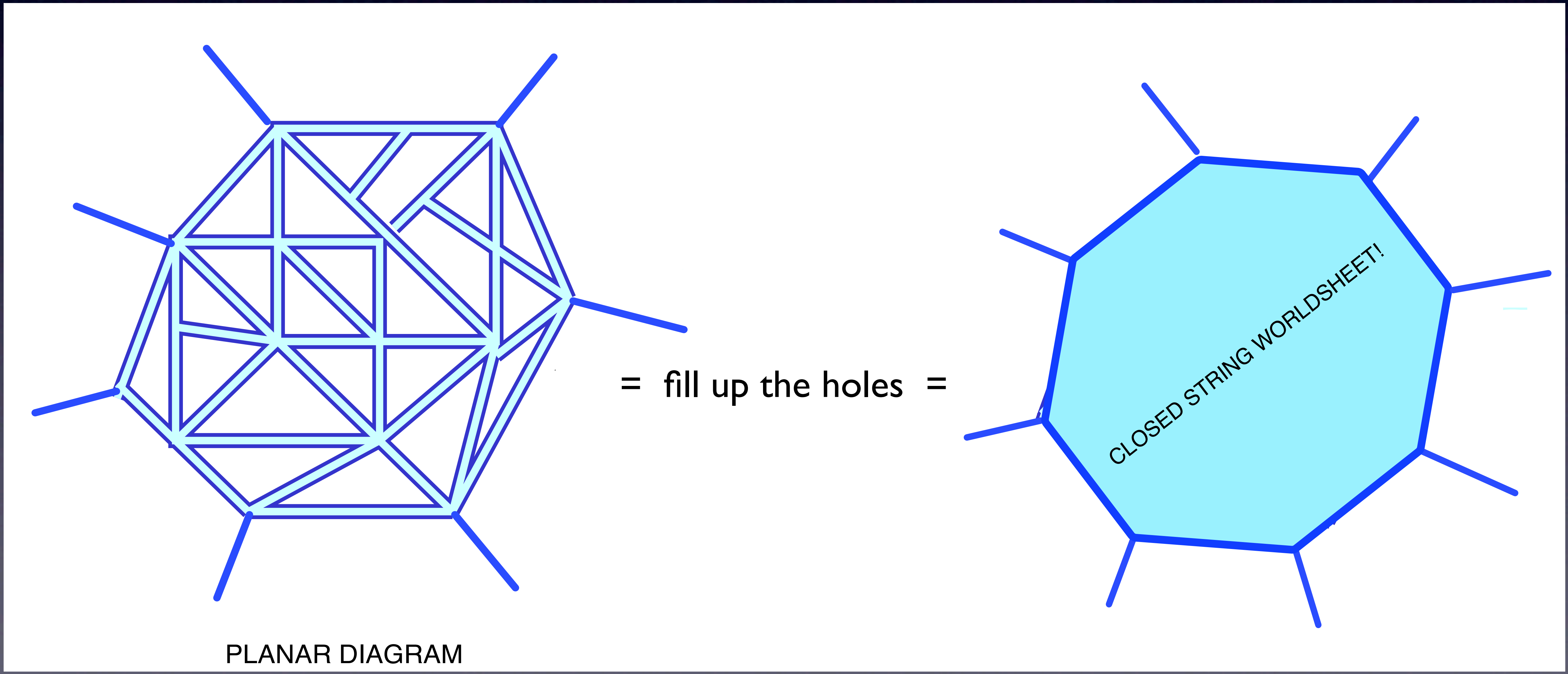




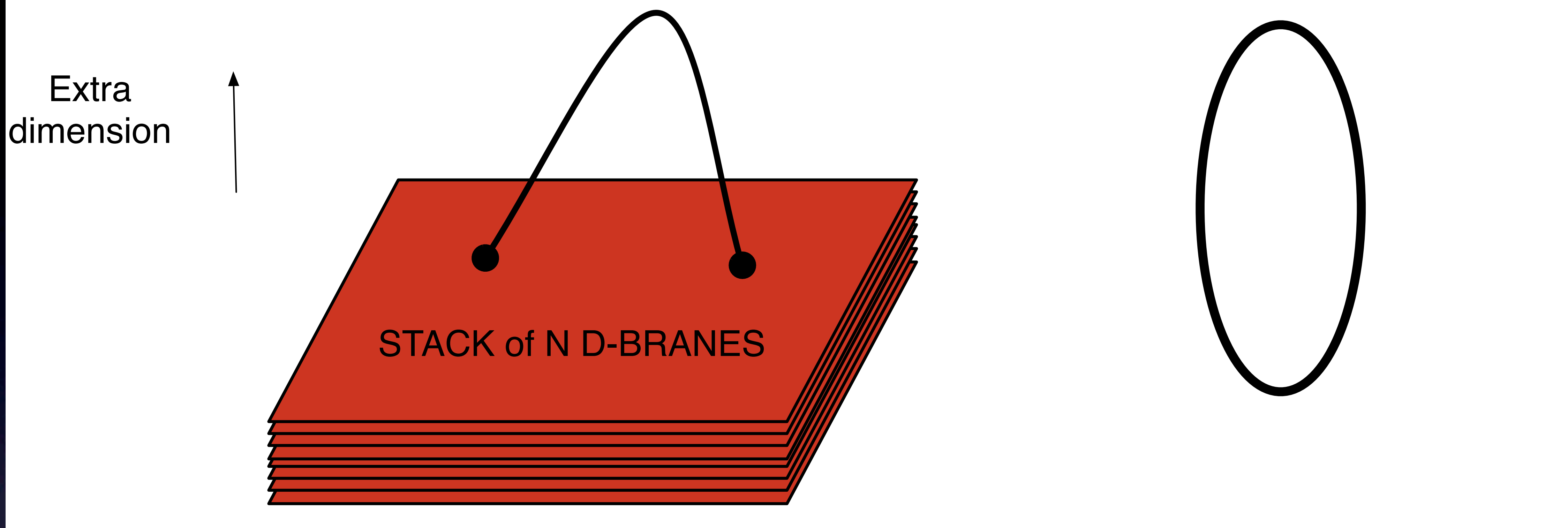
' t Hooft's argument

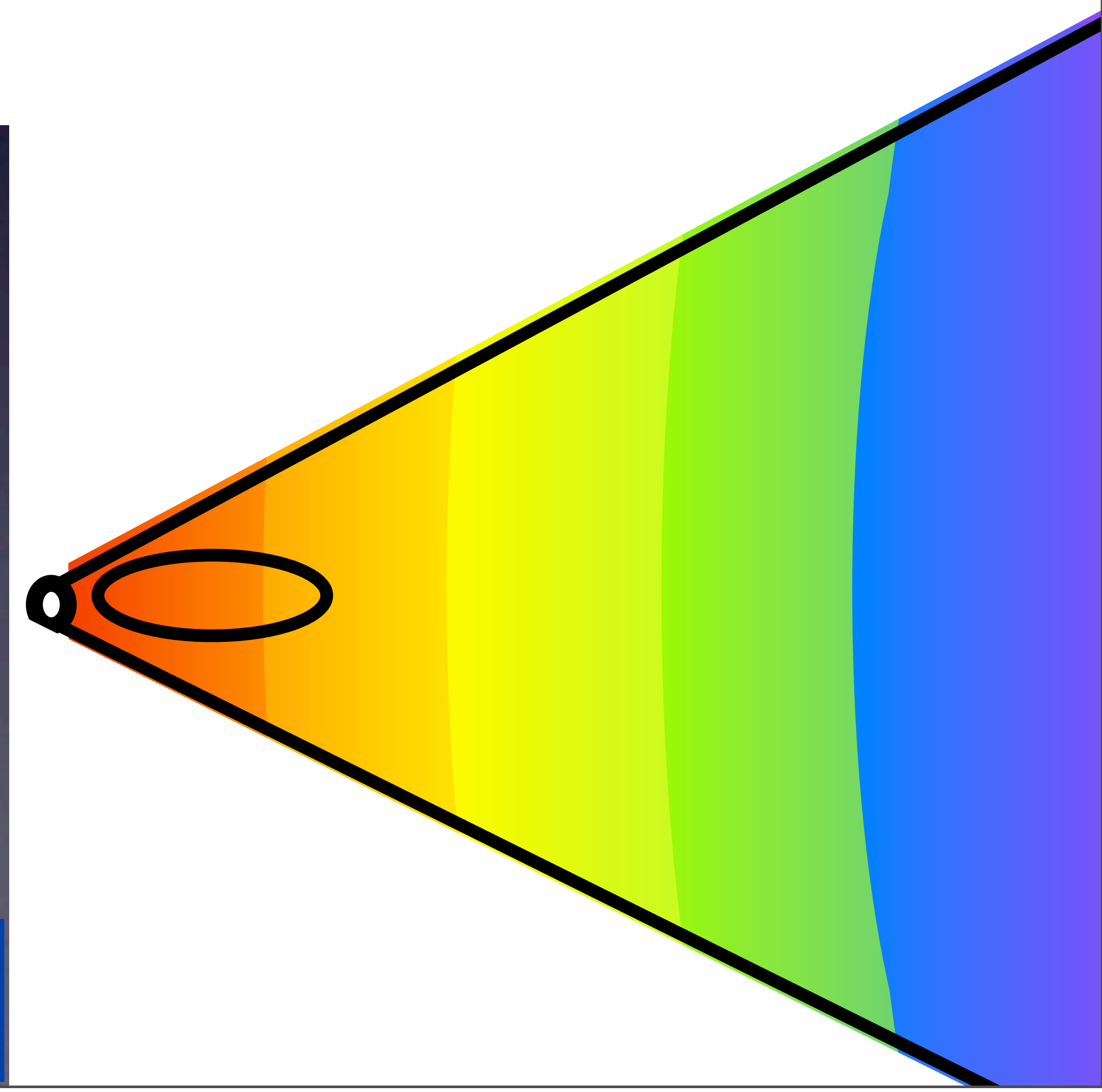
* 1/N expansion = string perturbation theory

* planar diagram = triangulation of world sheet



Maldacena's argument





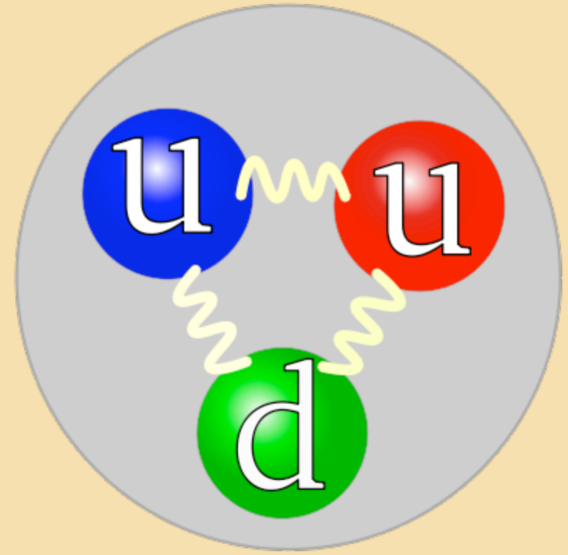
Gravitational back reaction creates a warped geometry

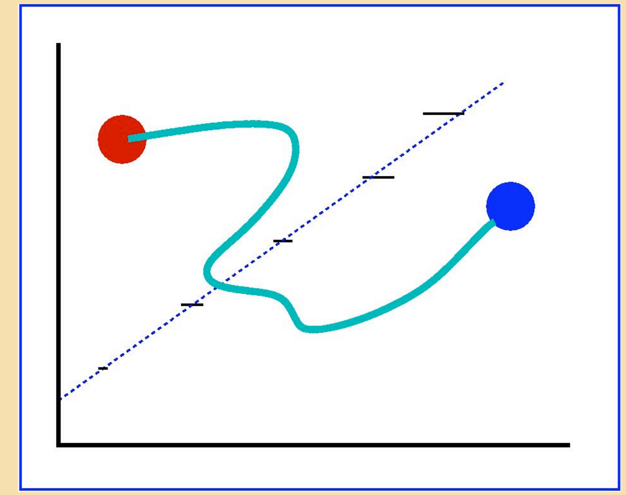
$$ds^2 = a^2(r)\, \eta_{\mu\nu} dx^\mu dx^\nu + dr^2$$

Extra dimension = RG scale

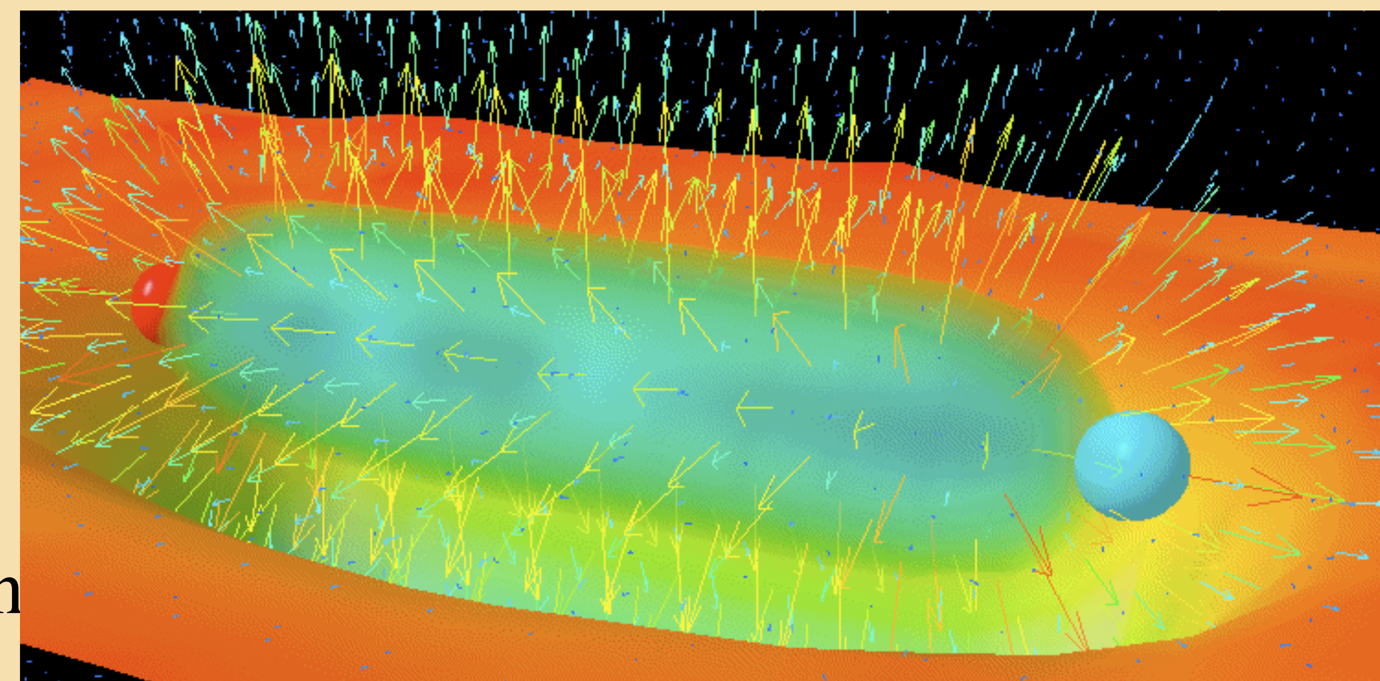
Large N QCD

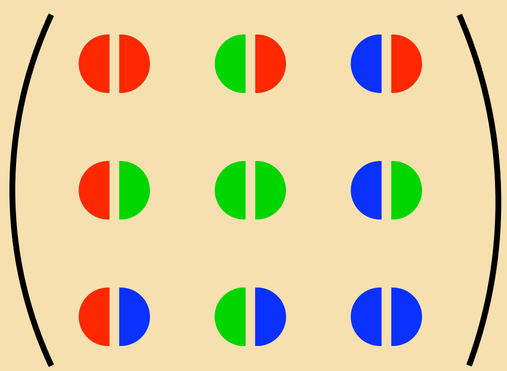
<u>defines</u> a particular string theory, in a highly curved target space. In QCD, this string is strongly coupled.





QCD String



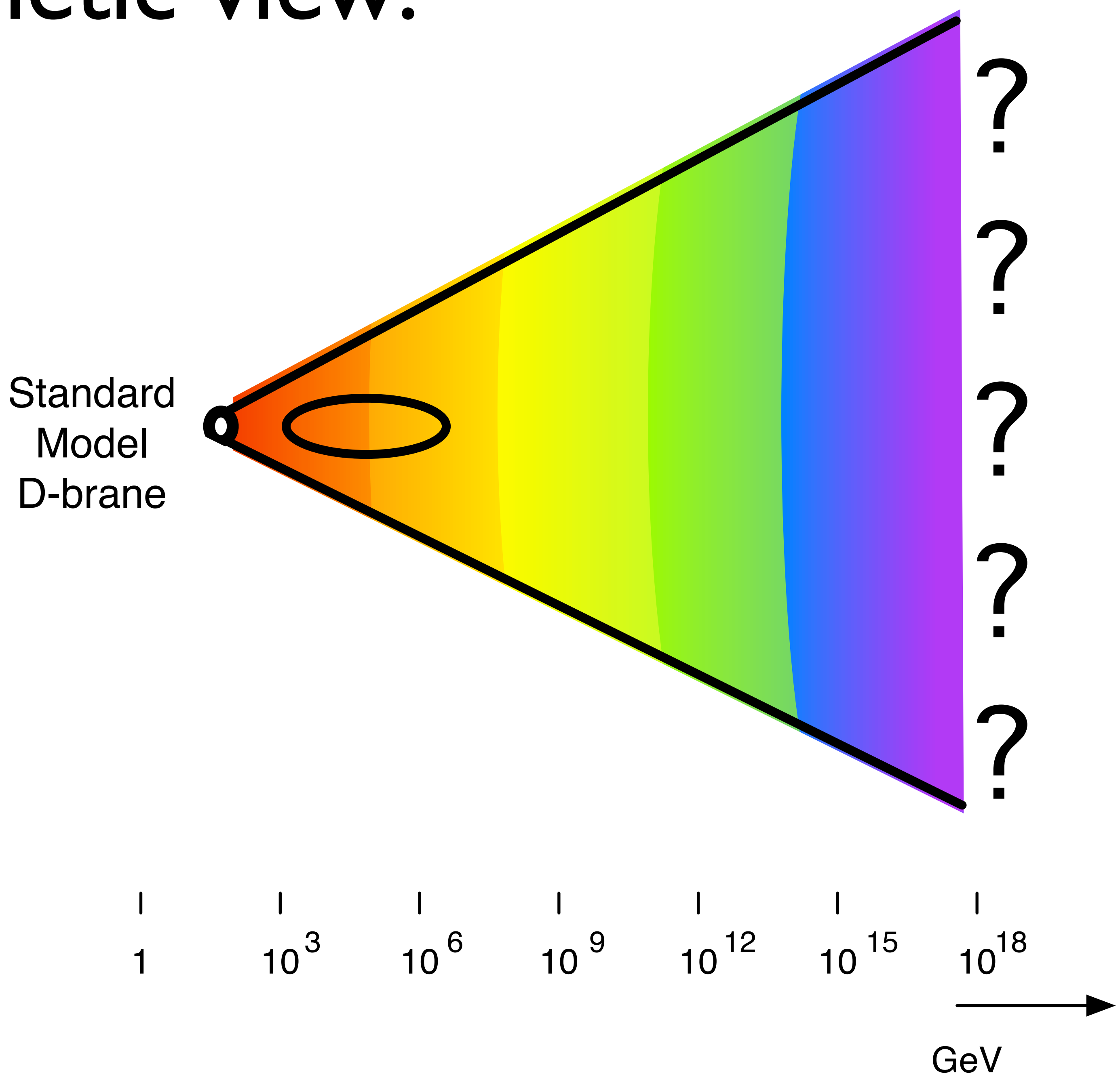
Gluon =  = Matrix

String =  = $(\text{Matrix})^n$

Large N spin chain = World sheet theory

What is the UV completion of Standard Model?

A geometic view:



There are many possibilities:

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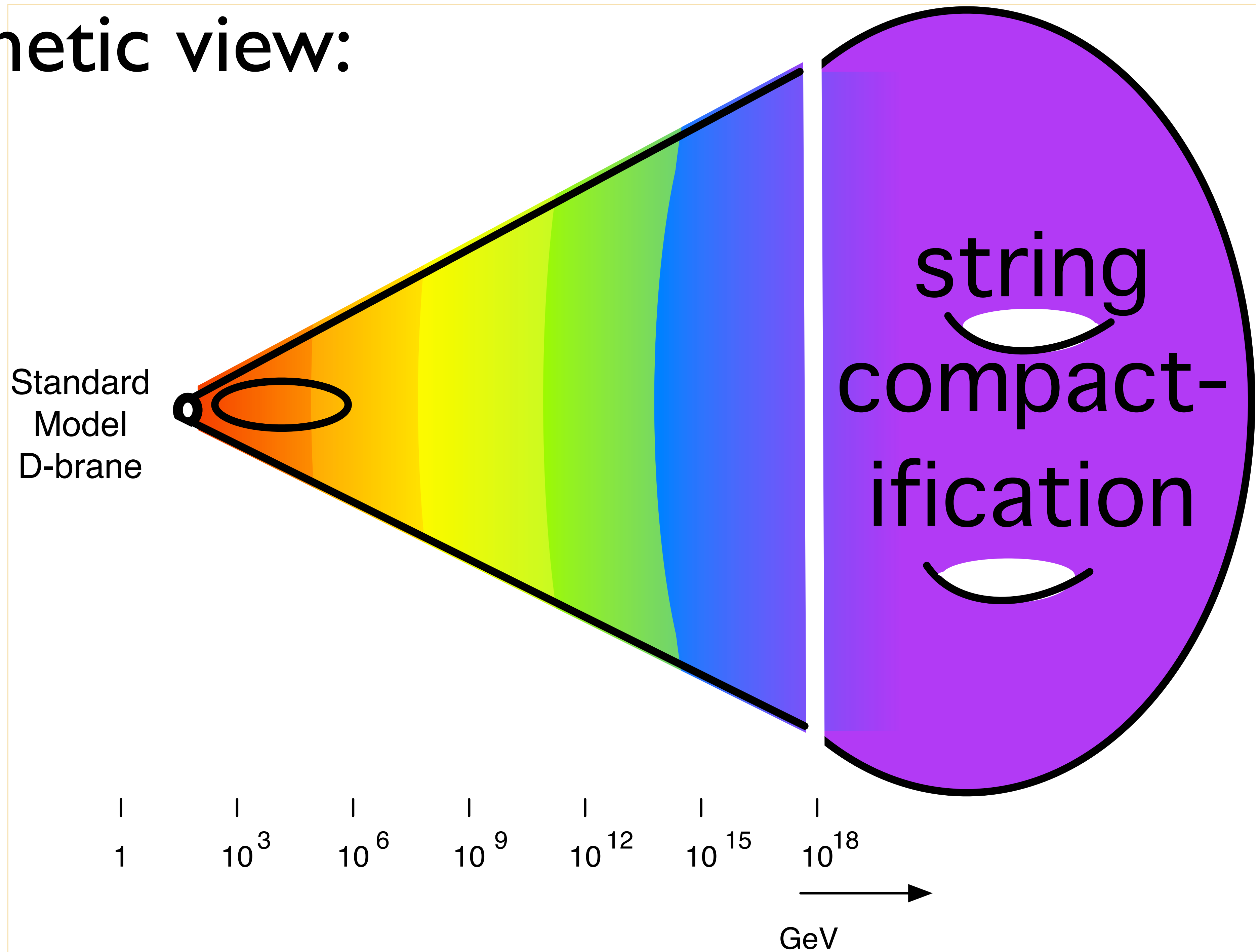
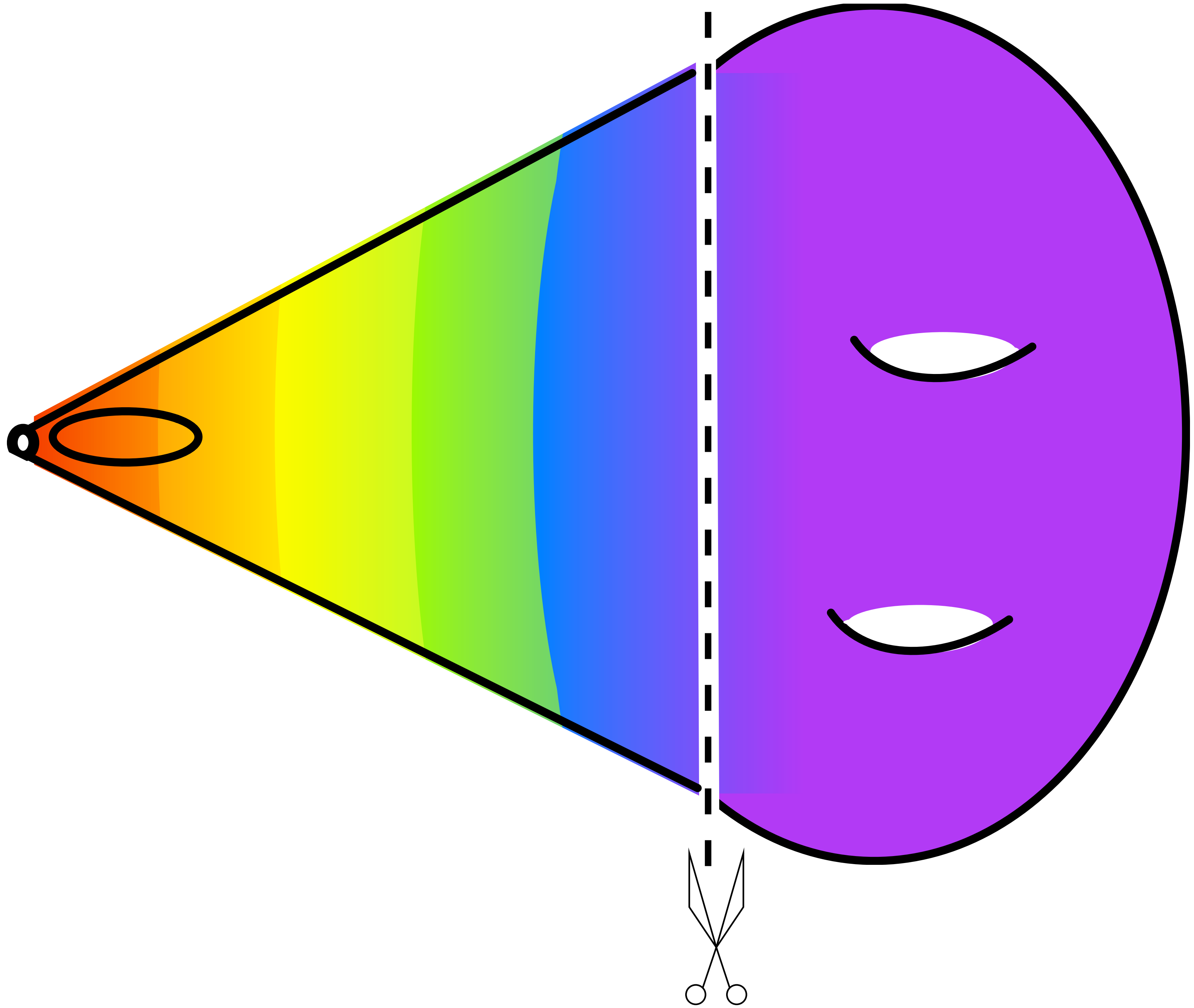


There are many possibilities: String Landscape

Holographic Renormalization Group



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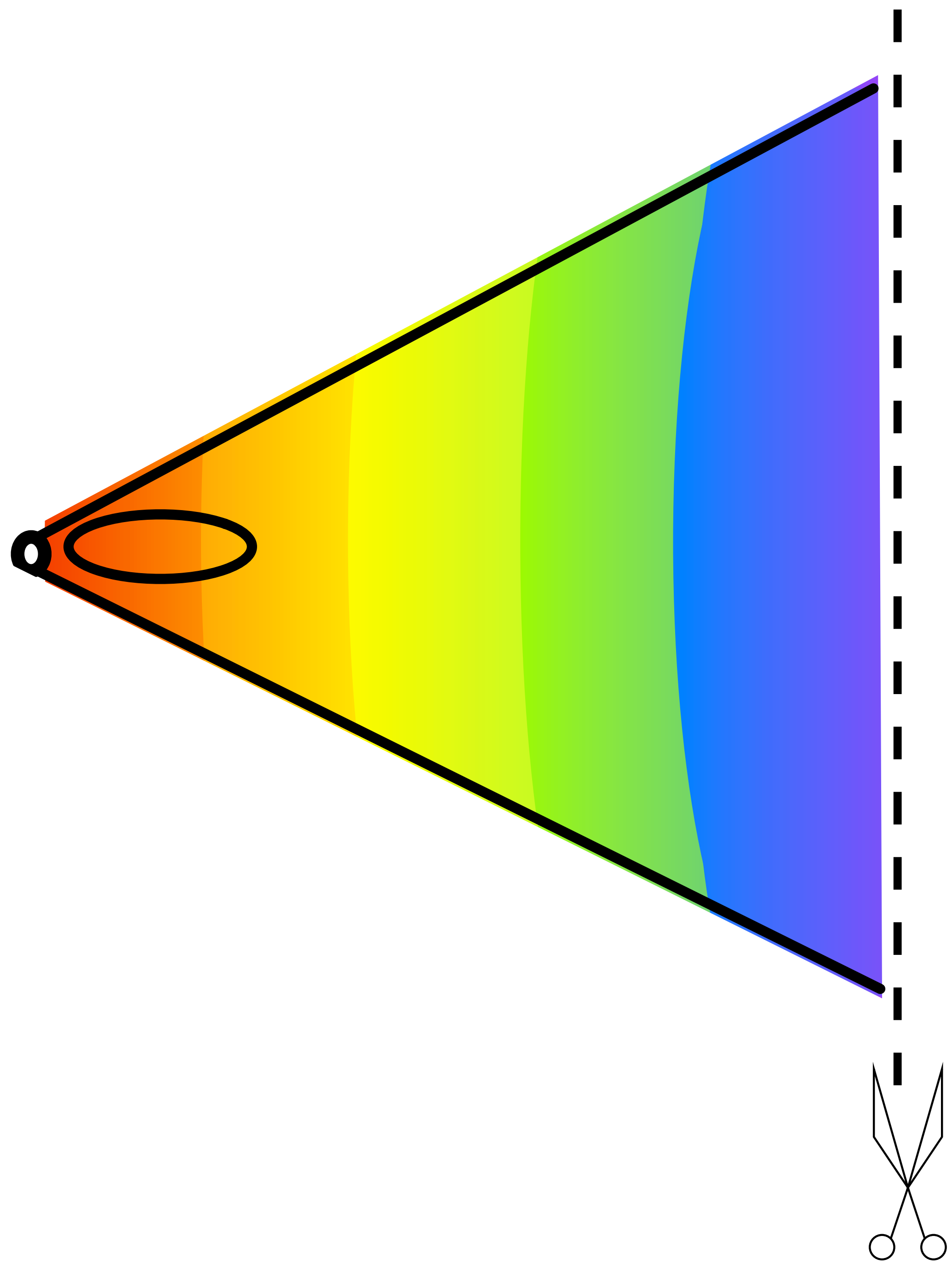

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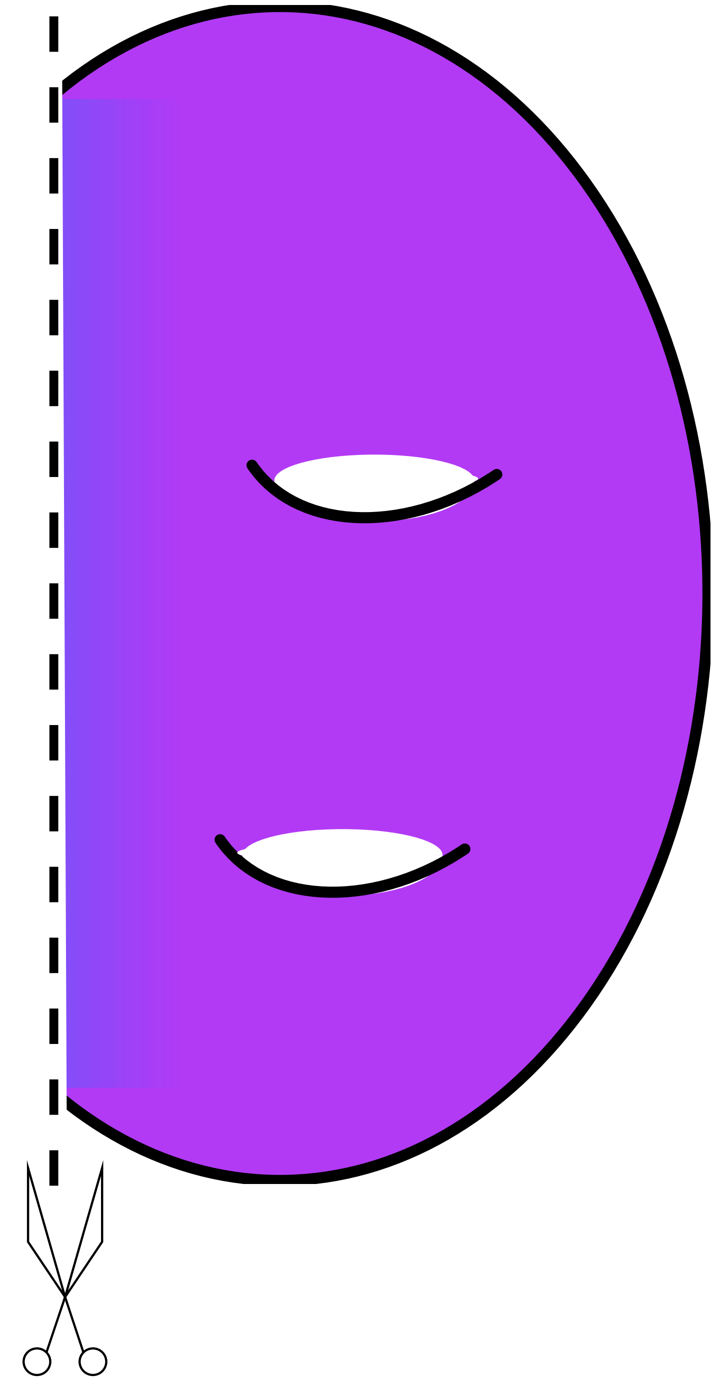
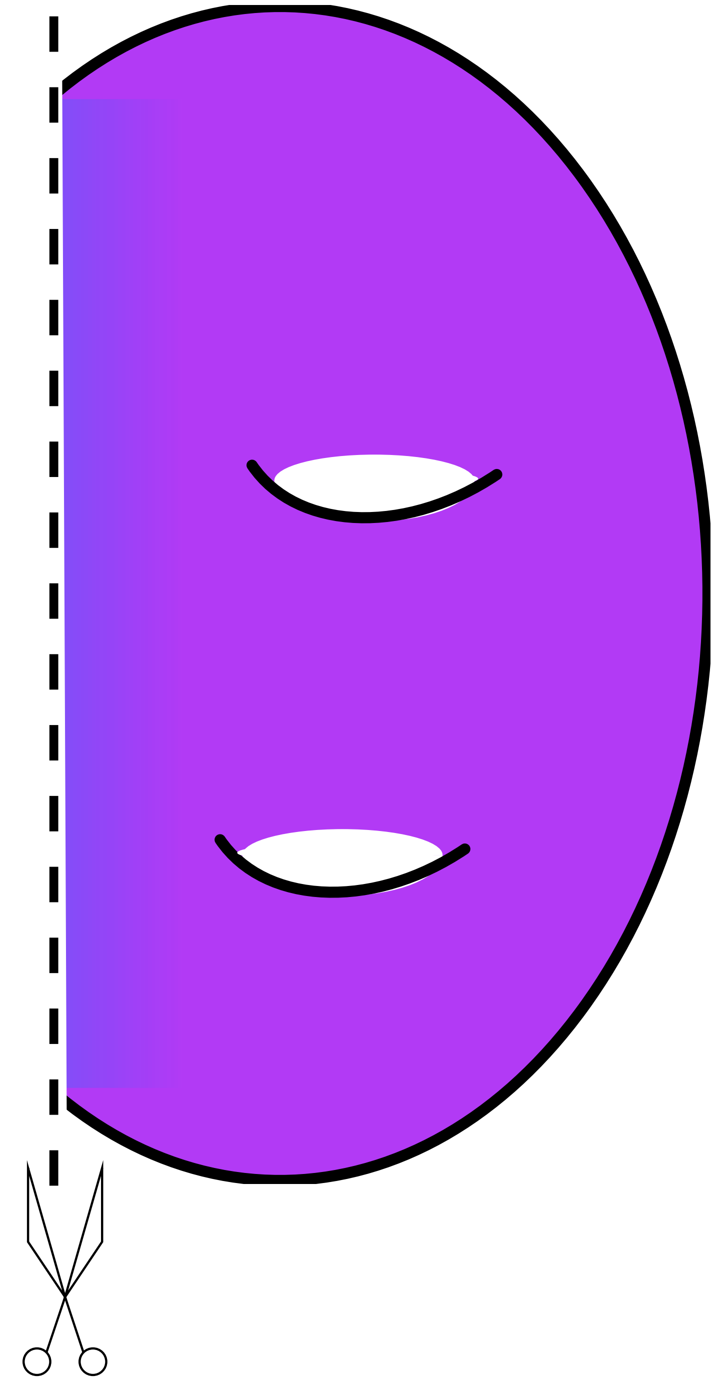


Holographic Renormalization Group

* cut away the AdS space

* path integral over exterior defines a wave function = Wilsonian effective action

* radial Schrodinger eqn = Wilsonian RG evolution



$$\hbar \partial_\Lambda e^{-\frac{1}{\hbar} S_{grav}(\phi, \Lambda)} = \hat{H}_{\text{grav}}\, e^{-\frac{1}{\hbar} S_{grav}(\phi, \Lambda)}$$

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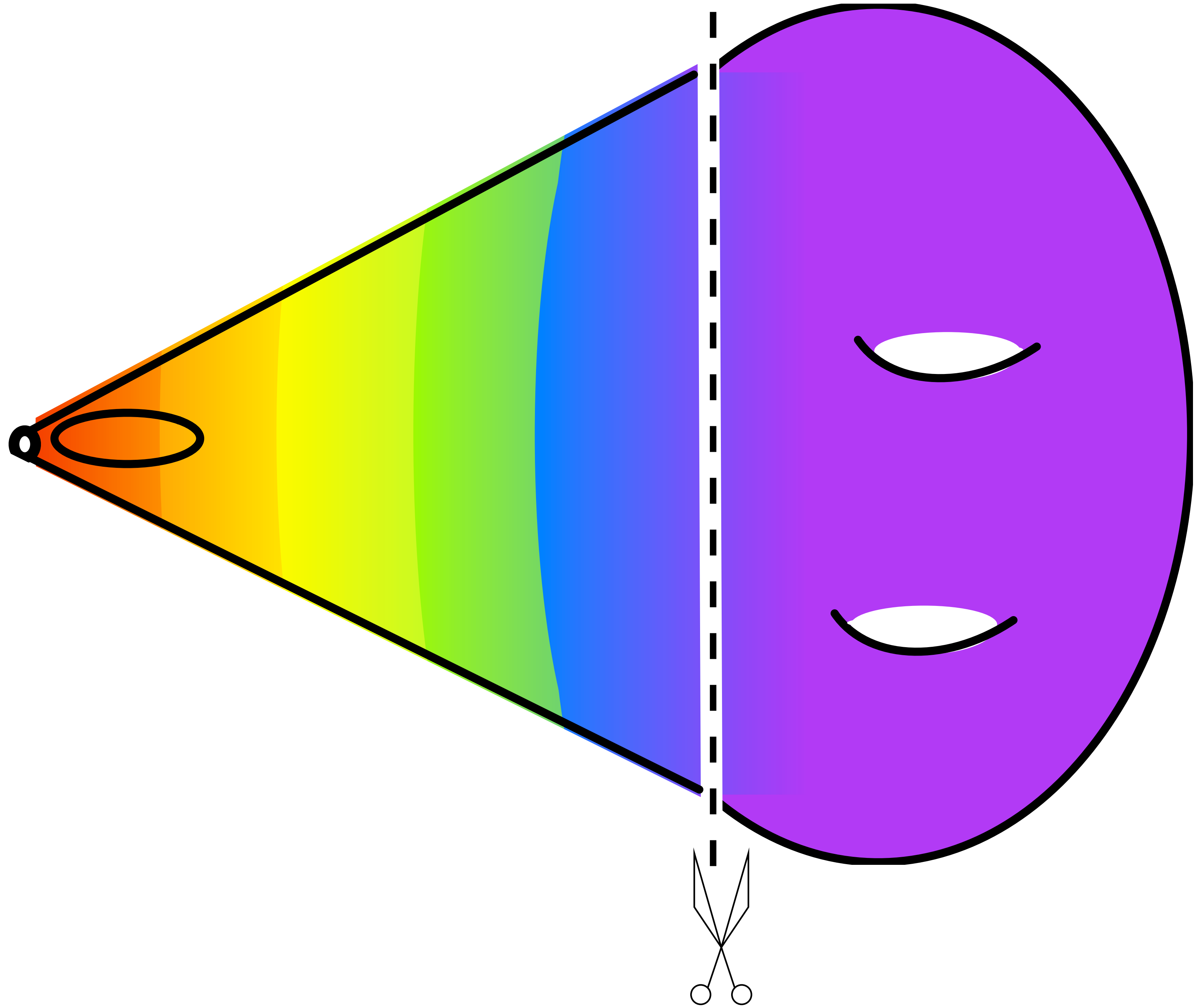
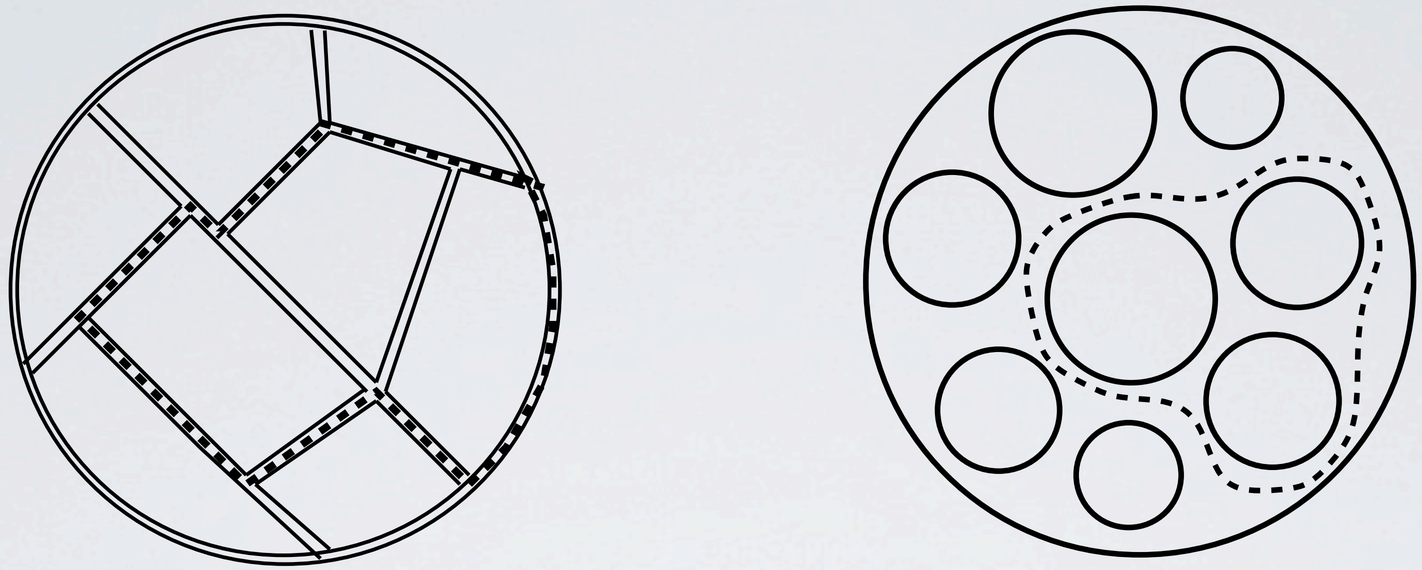


Reconstruct extra dimension from exact RG

Open/Closed String Duality



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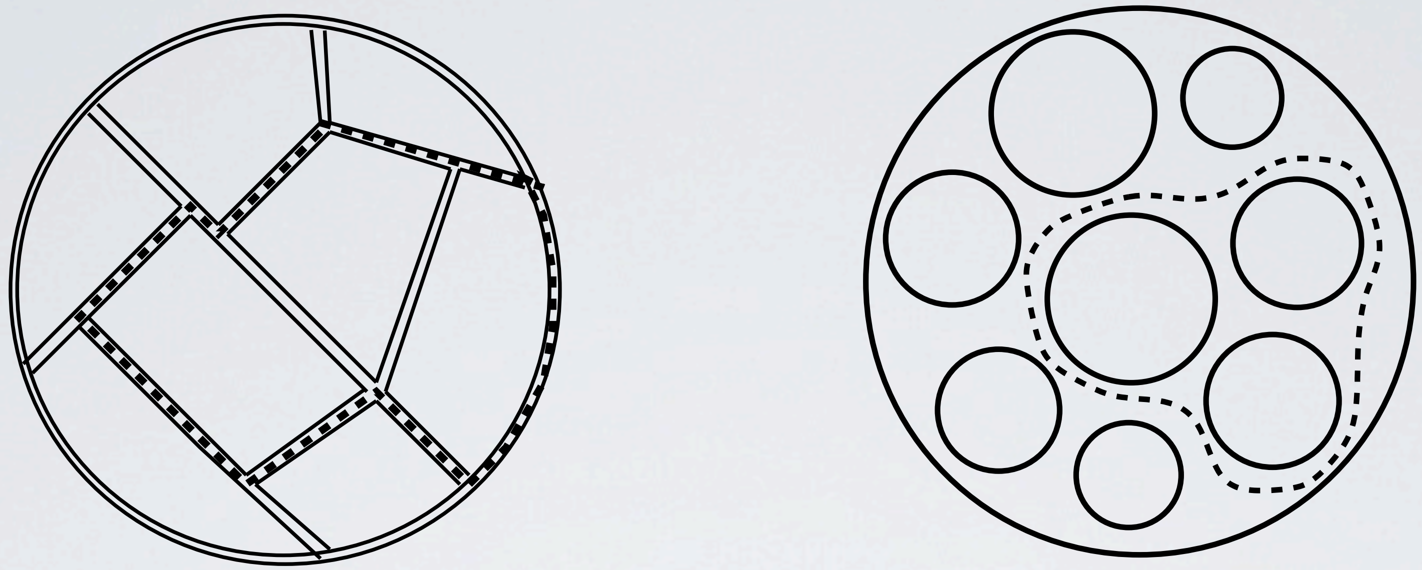
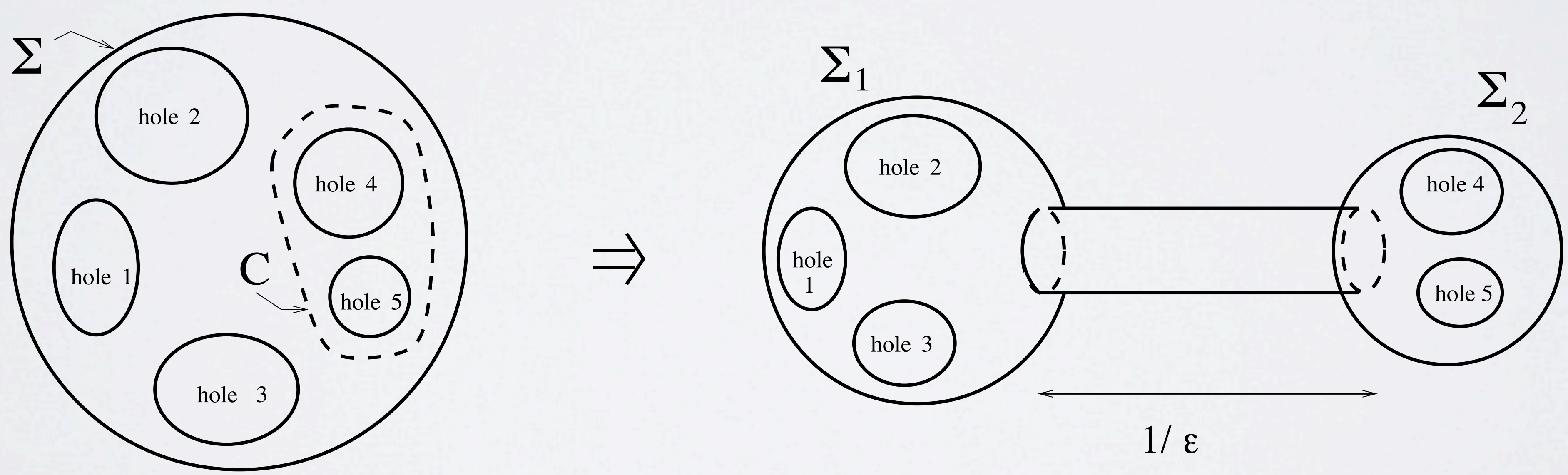


UV divergences in a QFT can be thought of as due to on-shell closed strings that propagate in a dual channel



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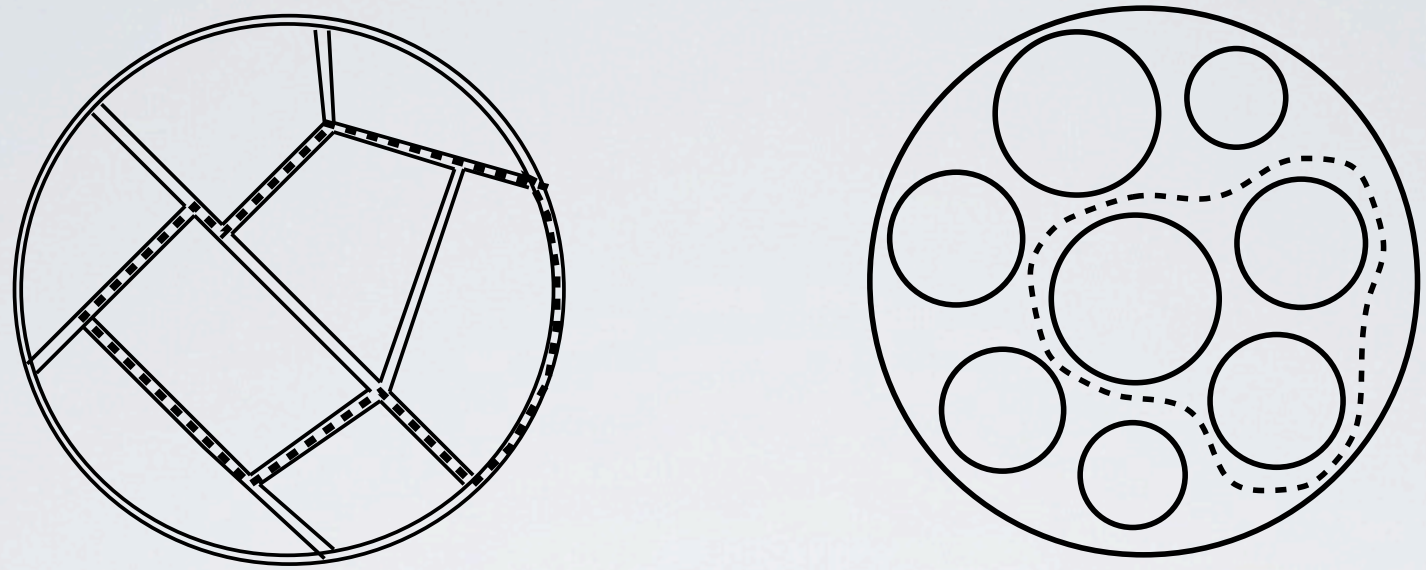

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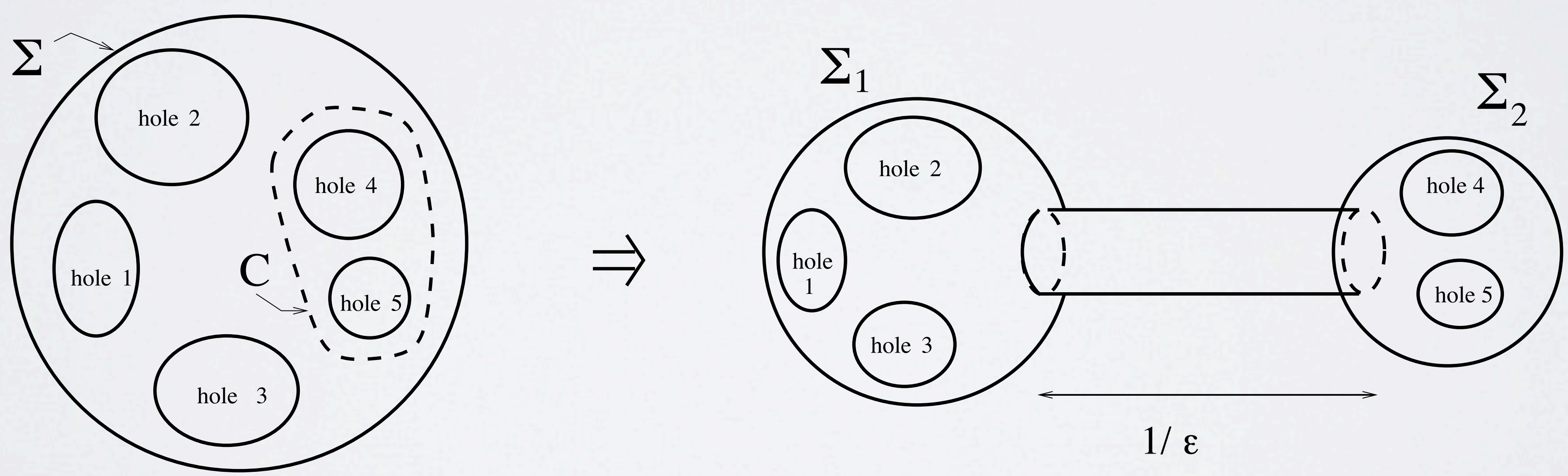

Counter terms in a (large N) QFT satisfy the eqns of motion of a (classical) closed string field theory.

A schematic derivation, using Polchinski's RG eqn:

$$\delta_\Lambda e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)} = \int \mathcal{D}a\; e^{-\frac{1}{\hbar}\left(\frac{\alpha_{ij}}{2\delta\Lambda}{\rm tr}(a^i a^j)\; +\; S_{\rm int}(A+a;\Lambda)\right)}$$

$$\hbar\partial_\Lambda e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)} = \hat{H}_{\rm gauge}\; e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)}$$

$$\hat{H}_{\rm gauge} = \hbar^2\alpha^{ij}\,{\rm tr}\left(\frac{\partial^2}{\partial A^i\partial A^j}\right)$$

Perform a generalized Hubbard-Stratonovic transform:

$$e^{-\frac{1}{\hbar} S_{\text{int}}(A,\Lambda)} = \int \mathcal{D}\phi \; e^{-\frac{1}{\hbar}\left(S_0(A;\phi) + S_{\text{grav}}(\phi;\Lambda)\right)}$$

$$S_0(A;\phi) = \sum_{(i_1 \ldots i_n)} \phi_{(i_1 \ldots i_n)} \operatorname{tr}\left(A^{i_1} \ldots A^{i_n}\right)$$

Single trace couplings = closed string fields

Open/closed string duality follows from the identity:

$$\hat{H}_{\text{gauge}}\, e^{-\frac{1}{\hbar} S_0(A,\phi)} = \check{H}_{\text{grav}}\, e^{-\frac{1}{\hbar} S_0(A,\phi)}$$

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exact RG evolution = Schrodinger equation

$$\hbar\partial_\Lambda e^{-\frac{1}{\hbar}S_{grav}(\phi,\Lambda)} = \hat{H}_{\rm grav}\, e^{-\frac{1}{\hbar}S_{grav}(\phi,\Lambda)}$$

RG evolution at large N = Hamilton-Jacobi equation

Some general comments:

* The 5-d graviton is one of the closed string modes
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* RG involves dividing the modes into momentum shells
* There is an associated entanglement entropy: UV <=> IR

* Entanglement entropy is proportional to area: `holography'
* 5-d Gravity can be understood as a consequence of the thermodynamic laws that relate entropy and energy.

T. Jacobson
E. Verlinde

Jacobson's argument: QFT + Thermo = Einstein Eqn

A) In a Rindler wedge, the vacuum has temperature and entropy:

$$T = \frac{\hbar}{2\pi}, \qquad S = \alpha A$$

Entropy scales with area because entanglement is dominated by short wavelength modes that cross the Rindler horizon.

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$$dS = \frac{dQ}{T}$$

Clausius relation

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A + B + general geometric facts imply Einstein eqns, with

$$\alpha = \frac{1}{4\hbar G_N}$$

Can we use these ideas to define a quantum gravity in space-time with positive cosmological constant?

Hint from AdS/CFT:

*Gravity in AdS(d+1) is dual to QFT(d)

* AdS with cut-off = warped compactification

= QFT(d)+ Gravity(d)

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Idea

Look for a consistent, covariant cut-off of gauge theory with a holographic dual.

But how do we find it?

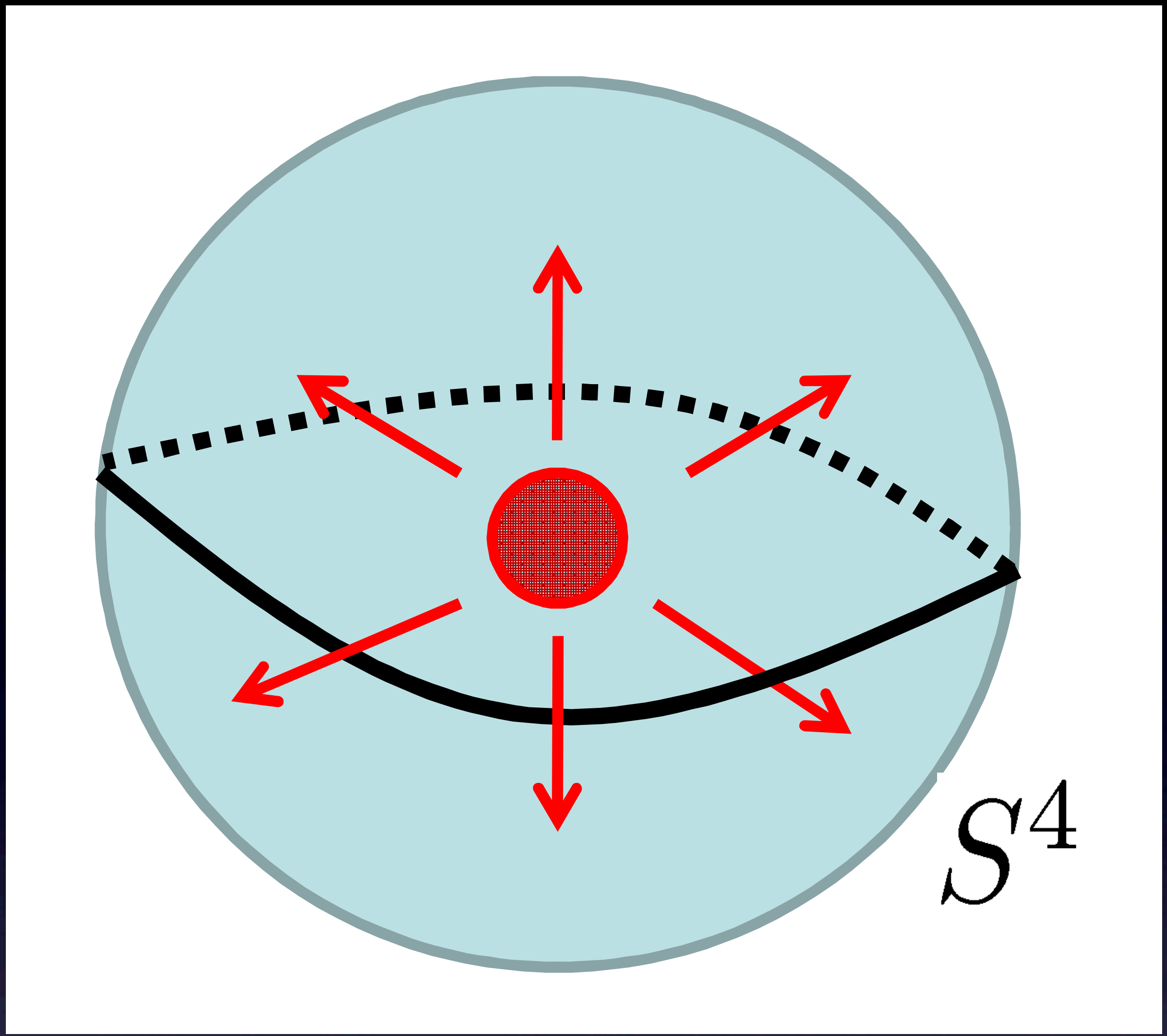
Hint from Matrix Theory: BFSS

* Start from a UV complete string theory
* Identify the most basic constituent d.o.f.
* Saturate the number N of constituent d.o.f.
    * Take a large N decoupling limit

New idea:

Consider $U(N N_c)$ gauge theory on a four sphere Set $g_{ym}=0$ and turn on a maximal homogeneous U(N) instanton flux. So:

$U(N N_c) \to U(N_c)$



c.f. Zhang & Hu (2002)
Taylor et al (1998)

Yang monopole:
homogeneous instanton
with maximal charge k

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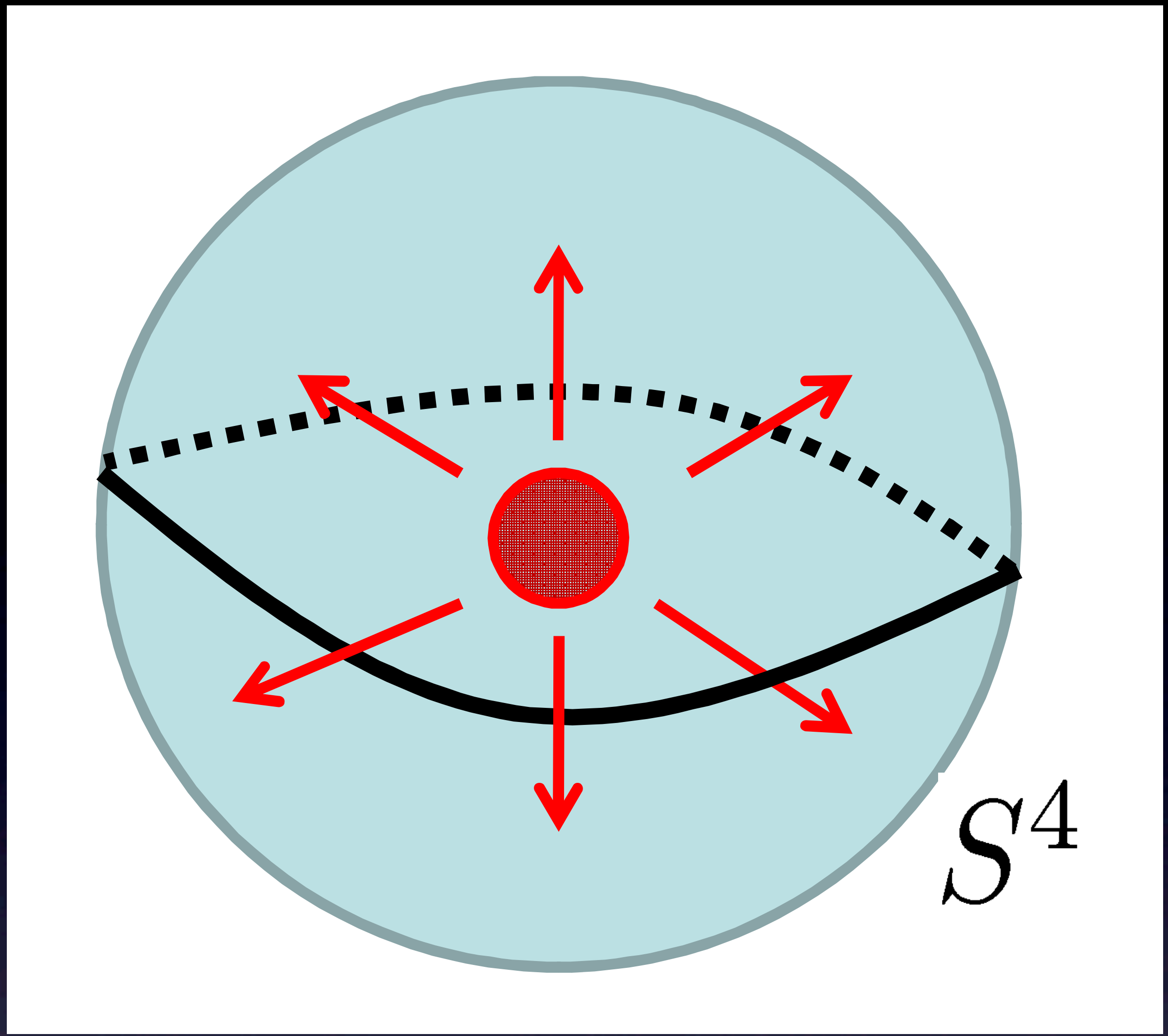

LLL dynamics defines a matrix model (ADHM)
Take a large N and flat space limit, so that the Planck cells stay finite.

c.f. Zhang & Hu (2002)
Taylor et al (1998)

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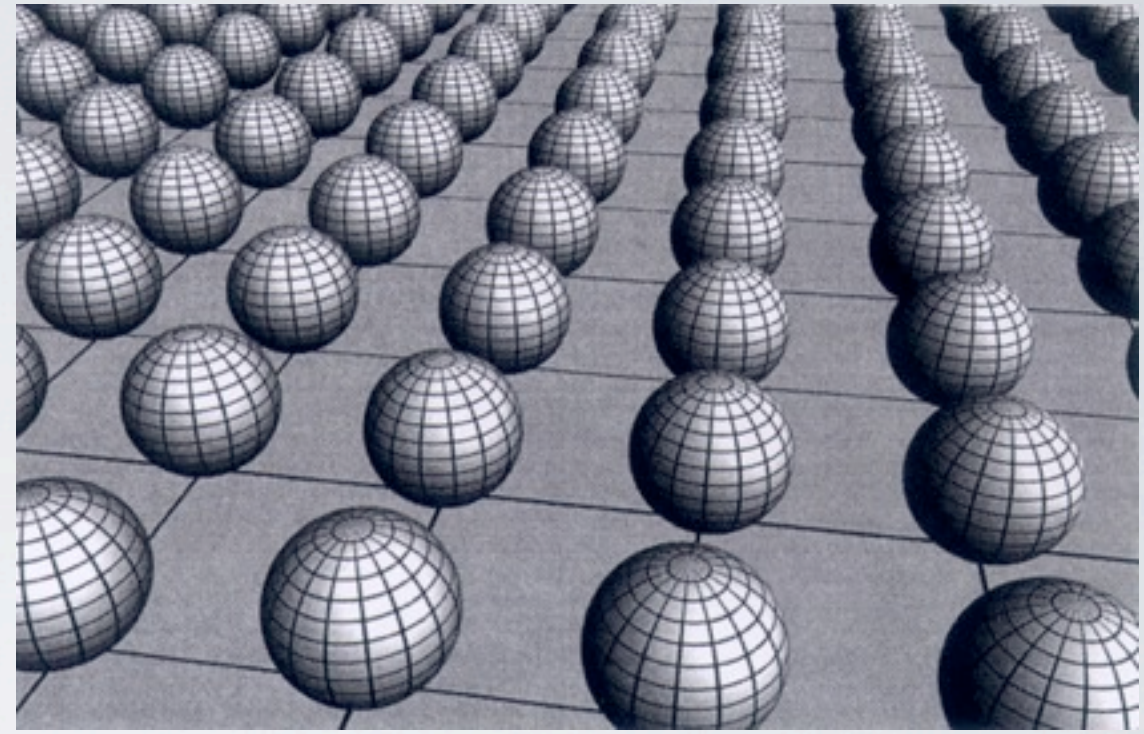
Geometric tool:

Twistor Space

$\mathbb{CP}^1 \longrightarrow \mathbb{CP}^3$ ⟵ twistor space

$\downarrow$

S^4 ⟵ spacetime



Given a light-like momentum p and space-time coordinate x

$$p_{AA'} = \widetilde{\pi}_A \pi_{A'} \qquad \omega^A = i x^{AA'} \pi_{A'}, \qquad \tilde{\omega}^{A'} = -i \tilde{\pi}_A x^{AA'}.$$

Penrose (1967)

$\mathbf{PT}_*$ $$Z^\alpha = (\omega^A, \pi_{A'}), \qquad \widetilde{Z}_\beta = (\widetilde{\pi}_A, \widetilde{\omega}^{A'})$$ **$\mathbf{PT}^*$**

The canonical commutation relations [p,x] = i imply that

Penrose (1967)

$$[Z^\alpha, \widetilde{Z}_\beta] = \hbar\delta^\alpha_\beta \quad \rightarrow \quad \begin{array}{l} [\omega^A, \widetilde{\pi}_B] = \hbar\delta^A_B \\ [\pi_{A'}, \widetilde{\omega}^{B'}] = \hbar\delta^{B'}_{A'} \end{array}$$

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$$Z^\dagger_\alpha Z^\alpha = \hbar N$$

What does the low energy theory look like?

Lowest Landau Level = Fuzzy Twistor Space!

$$\mathfrak{D}\,|\Psi\rangle = \hbar N\,|\Psi\rangle \qquad\qquad \mathfrak{D} = \widetilde{Z}_\beta Z^\beta$$

This gives a finite Hilbert space of dimension:

$$k = (N+1)(N+2)(N+3)/6.$$

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The twistor lines:

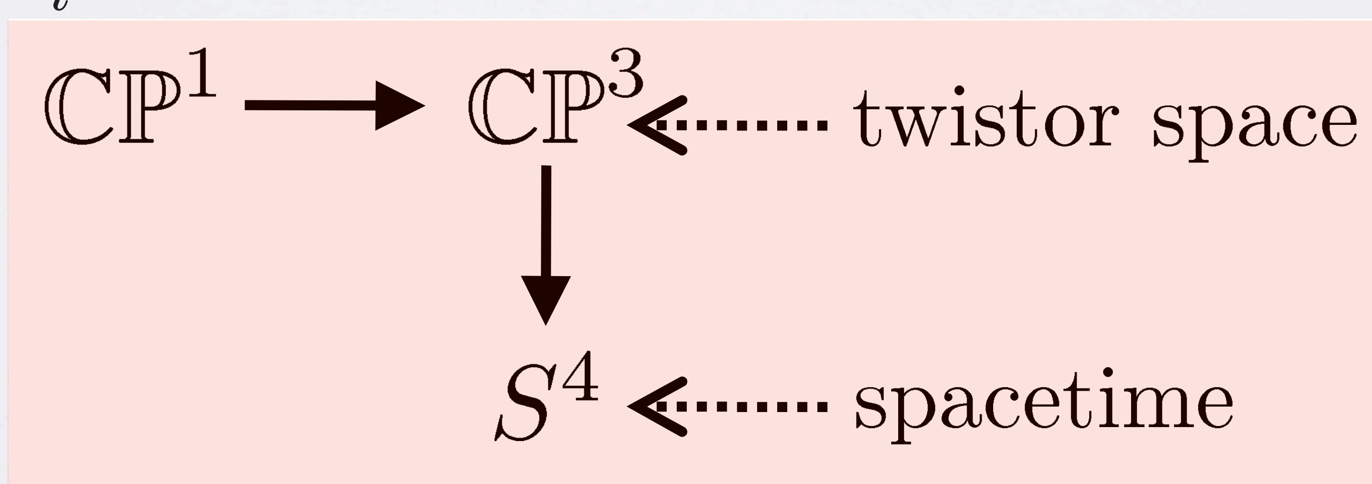
$$(\omega^A - i x^{AA'} \pi_{A'})|p\rangle = 0$$

are fuzzy spheres with n = N+1 points

Covariant space-time non-commutativity:

$$[n_i, n_j] = \frac{2}{N}\epsilon_{ijk}n_k\,, \qquad [y_\mu, y_\nu] = \frac{\ell^2}{2N}\eta^i_{\mu\nu}n_i, \qquad [n_i, y_\mu] = \frac{2}{N}\eta_{i\mu\nu}y^\nu.$$

$$n_i n^i = 1$$



$$y^A y_A = \ell^2.$$

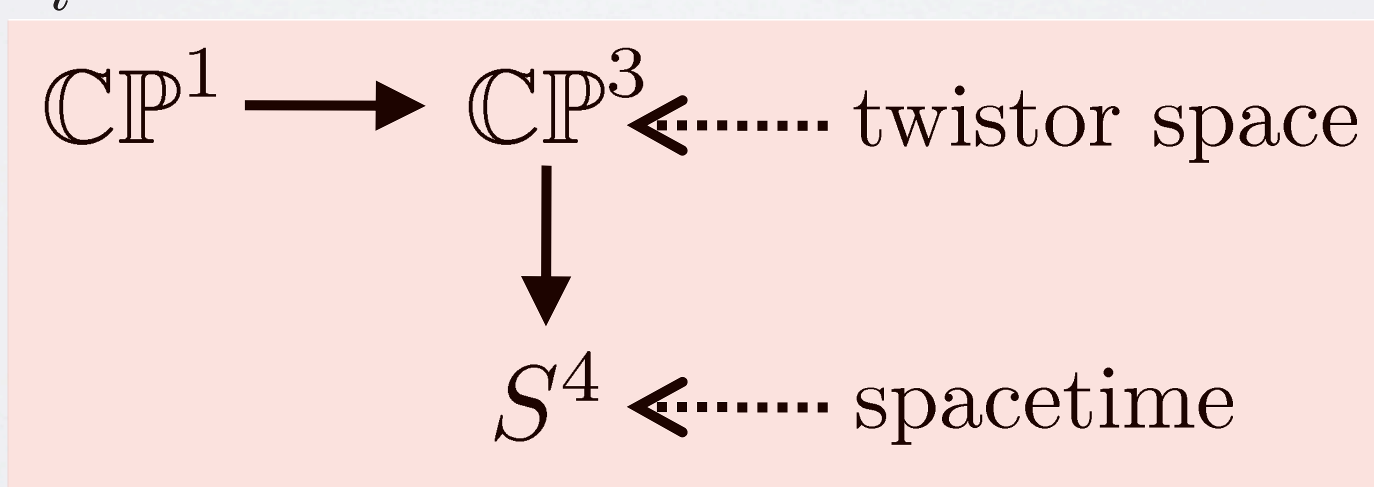
Covariant space-time non-commutativity:

$$[n_i, n_j] = \frac{2}{N}\epsilon_{ijk}n_k\,, \qquad [y_\mu, y_\nu] = \frac{\ell^2}{2N}\eta^i_{\mu\nu}n_i, \qquad [n_i, y_\mu] = \frac{2}{N}\eta_{i\mu\nu}y^\nu.$$

CP^1 = space of complex structures on R^4

superposition of all possible
unimodular self-dual non-commutativities

$$n_i n^i = 1$$



$$y^A y_A = \ell^2.$$

Our Proposal:

UV theory: N=4 BF Theory with Flux:

decoupling ↓ limit

IR theory: ADHM Matrix Model

large N ↓ limit

Emergent theory: N=4 SYM Theory

Our **Bold** Proposal:

UV theory: N=4 BF Theory with Flux:

decoupling ↓ limit

IR theory: ADHM Matrix Model

large N ↓ limit

Emergent theory: N=4 SYM Theory

1/N Corrections = Einstein Gravity

Matrix Action = Fuzzy Chern-Simons + Matter

$$\mathcal{A} = \mathcal{A}_I(Z^\dagger, Z, \psi^\dagger, \psi)d\overline{\mathcal{Z}}^I$$

$$S(\mathcal{A}) = \mathrm{Tr}_{\mathfrak{D}\leq N}\left(\epsilon^{\alpha\beta\gamma\delta}\mathcal{F}_{\alpha\beta}\mathcal{F}_{\gamma\delta}\right)$$

$$\mathcal{F}_{\alpha\beta} = \left[D_\alpha, D_\beta\right], \qquad D_\alpha = \tfrac{1}{\hbar}Z_\alpha - \mathcal{A}_\alpha$$

$$S_{\mathrm{defect}}(Q, \tilde{Q}, \mathcal{A}) = \mathrm{Tr}\left(\mathcal{I}_{IJ}\widetilde{Q}\mathcal{D}^I Q \mathcal{Z}^J\right)$$

Why do the 1/N corrections give rise to 4D Einstein gravity?

Hints:

* Jacobson's argument

* Gauge/gravity correspondence

* Twistor string theory contains conformal gravity

* Penrose's `non-linear graviton'

* Gravity MHV amplitudes