IOFFE PHYSICAL-TECHNICAL INSTITUTE

# Nanostructures as the Instrument for Materialization of the Problems from the Textbook on Quantum Mechanics

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with

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# Presentation consists of two Sections:

1. Coulomb States in Nanostructures, Accidental Degeneracy, and the Laplace-Runge-Lenz Operator

2. Electron in Periodic Potential under Action of Constant Electric Field and Bloch Oscillations

# Section 1

# Coulomb States in Nanostructures, Accidental Degeneracy, and the Laplace-Runge-Lenz Operator

*M. A. Semina and R. A. Suris*

- Introduction: Accidental Degeneracy for Coulomb potential and harmonic oscillator potential
- How to get transition between two of these cases using Quantum Well nanostructure
- Looking after the transition
- Conclusions to the Section

JETP Letters , Volume 94, Number 7 (2011), 574-578

# 3D Hydrogen Atom

$$\widehat{H} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} - \frac{2}{r}$$

$$\Psi = Const\cdot\left(\frac{2r}{n}\right)^l \exp(-\frac{r}{n})Y_{lm}(\theta,\varphi)L_{n+l}^{2l+1}\left(\frac{2r}{n}\right) \qquad Ry = \frac{m_e e^4}{2\varepsilon^2\hbar^2}$$

$$E_n = -\frac{1}{(n+1)^2},\ \ n = 0, 1, .. \qquad a_B = \frac{\hbar^2\varepsilon}{m_e e^2}$$

**Degeneracy of the State with Main Quantum Number *n*:**

$$l = 0, 1, ..n, \quad m = -l, -l+1, .., 0, .., l-1, l$$

**Total:** $(n+1)^2$

## Accidental degeneracy

## Laplace – Runge –Lentz Operator:

$$\widehat{\mathbf{A}} = (\widehat{\mathbf{p}}\times\widehat{\mathbf{L}} - \widehat{\mathbf{L}}\times\widehat{\mathbf{p}}) - \frac{2}{r}\mathbf{r}$$

$\mathbf{A} = \mathbf{p}\times\mathbf{L} - \frac{2}{r}\mathbf{r}$ **- in classical mechanics – constant of motion**

$$[\widehat{A}_i, \widehat{A}_k] = -2\mathrm{i}e_{ikl}\widehat{H}\widehat{L}_l \qquad [\widehat{L}_i, \widehat{A}_k] = \mathrm{i}e_{ikl}\widehat{A}_l$$

*V.A. Fock, Z. Phys. **98**, 145 (1935), W. Pauli, Z. Phys. **36**, 336 (1926)*

# 3D ISOTROPIC HARMONIC OSCILLATOR

$$\widehat{H} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + Kr^2$$

$$\Psi = Const\cdot r^l \exp\left(-\frac{\sqrt{K}r^2}{2}\right)Y_{lm}(\theta,\varphi)F\left(-\frac{n-l}{2}, l+\frac{3}{2}, \sqrt{K}r^2\right)$$

$$E_n = 2\sqrt{K}\left(n+\frac{3}{2}\right),\ \ n = 0,1,2,..$$

**Degeneracy:**

**even *n*** $l = 0, 2, ..n, \quad m = -l, -l+1, .., 0, .., l-1, l$ **Всего:** $\frac{n+2}{2}$

**odd *n*** $l = 1, 3, ..n, \quad m = -l, -l+1, .., 0, .., l-1, l$ **Всего:** $\frac{n+1}{2}$

**Operators commuting with Hamiltonian:**

$$\widehat{A}_{xx} - \widehat{A}_{yy},\ \widehat{A}_{yy} - \widehat{A}_{zz},\ \widehat{A}_{xz} + \widehat{A}_{zx},\ \widehat{A}_{yz} + \widehat{A}_{zy},\ \widehat{A}_{xy} + \widehat{A}_{yx}$$

$$\widehat{A}_{ij} = \widehat{p}_i\widehat{p}_j + Kx_ix_j,\ \ i,j = x,y,z$$

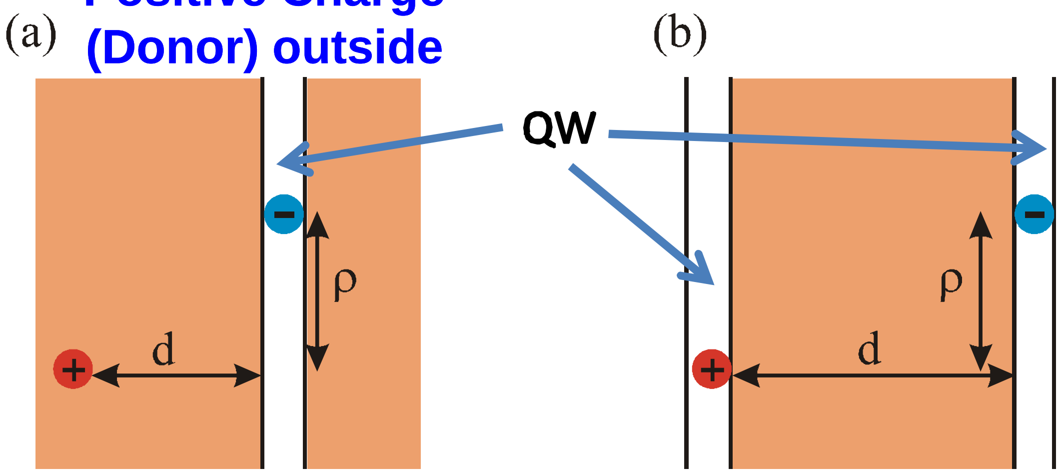
$$\widehat{H} = \widehat{A}_{xx} + \widehat{A}_{yy} + \widehat{A}_{zz}$$

*J.M. Jauch and E.L. Hill, Phys. Rev.* ***57****, 641 (1940)*

# 2D Electron in Quantum Well Heterostructures

**Electron inside QW and Positive Charge (Donor) outside**

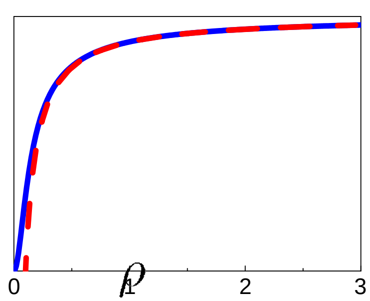
**Spatially indirect exciton**




$$d \to 0$$

## 2D Hydrogen Atom

$$-\frac{2}{\sqrt{\rho^2+d^2}} \approx -\frac{2}{\rho}$$




**Hamiltonian:**

$$\hat{H} = -\Delta - \frac{2}{\sqrt{\rho^2+d^2}}$$

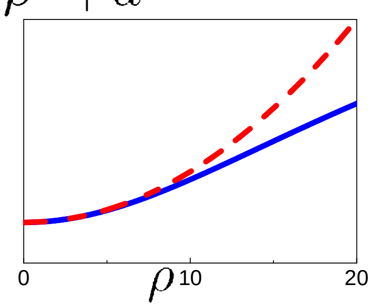
$$Ry = \frac{m_e e^4}{2\varepsilon^2 \hbar^2}$$

$$a_B = \frac{\hbar^2 \varepsilon}{m_e e^2}$$

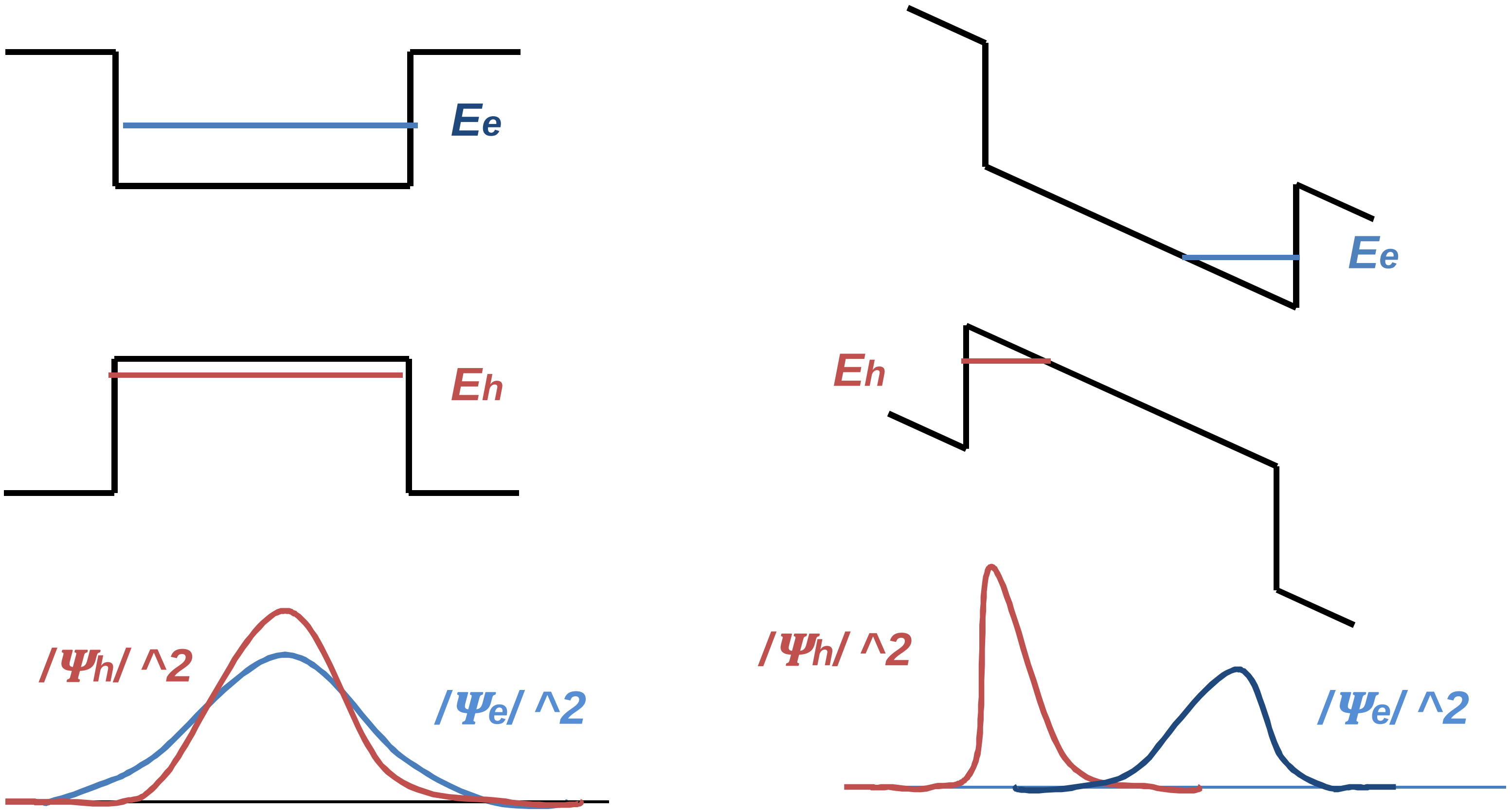
$$d \to \infty$$

## 2D Isotropic Harmonic Oscillator

$$-\frac{2}{\sqrt{\rho^2+d^2}} \approx -\frac{2}{d} + \frac{\rho^2}{d^3}$$



# Field Control of Electron – Hole Interaction in Single Quantum Well



# 2D Hydrogen Atom

**Hamiltonian:**

$$\widehat{H} = -\Delta - \frac{2}{\rho}$$

**Energy Levels:**

$$E_n = -\frac{1}{(n+1/2)^2}$$

**Degeneracy:**

$$m = -n, -n+1, .., n-1, n$$

**total:** $2n+1$

**Wave Functions:**

$$\psi_{nm}(\rho,\phi) = C_{n,m} \exp\left(-\frac{2\rho}{2n+1} - \mathrm{i}m\phi\right)\left(\frac{4\rho}{2n+1}\right)^{|m|} L^{2|m|}_{n_1-|m|}\left(\frac{4\rho}{2n+1}\right)$$

## 2D Laplace – Runge –Lentz Operator:

$$\widehat{\mathbf{A}} = (\widehat{\mathbf{p}} \times \widehat{\mathbf{L}}_z - \widehat{\mathbf{L}}_z \times \widehat{\mathbf{p}}) - \frac{2}{\rho}\boldsymbol{\rho}$$

$$\widehat{\mathbf{L}}_z = [\boldsymbol{\rho} \times \mathbf{p}]_z \boldsymbol{e}_z$$

*D.G.W. Parfitt and M.E. Portnoi, J. Math. Phys. 43, 4681 (2002)*

# 2D Isotropic Harmonic Oscillator

**Hamiltonian:**

$$\widehat{H} = -\Delta - \frac{2}{d} + \frac{\rho^2}{d^3}$$

**Eigenenergies:**

$$E_p = -\frac{2}{d} + \frac{2}{d^{3/2}}(p+1)$$

**Degeneracy:**

$$m = -p, -p+2, .., p-2, p$$

**total:** $p+1$

**Eigenfunctions:**

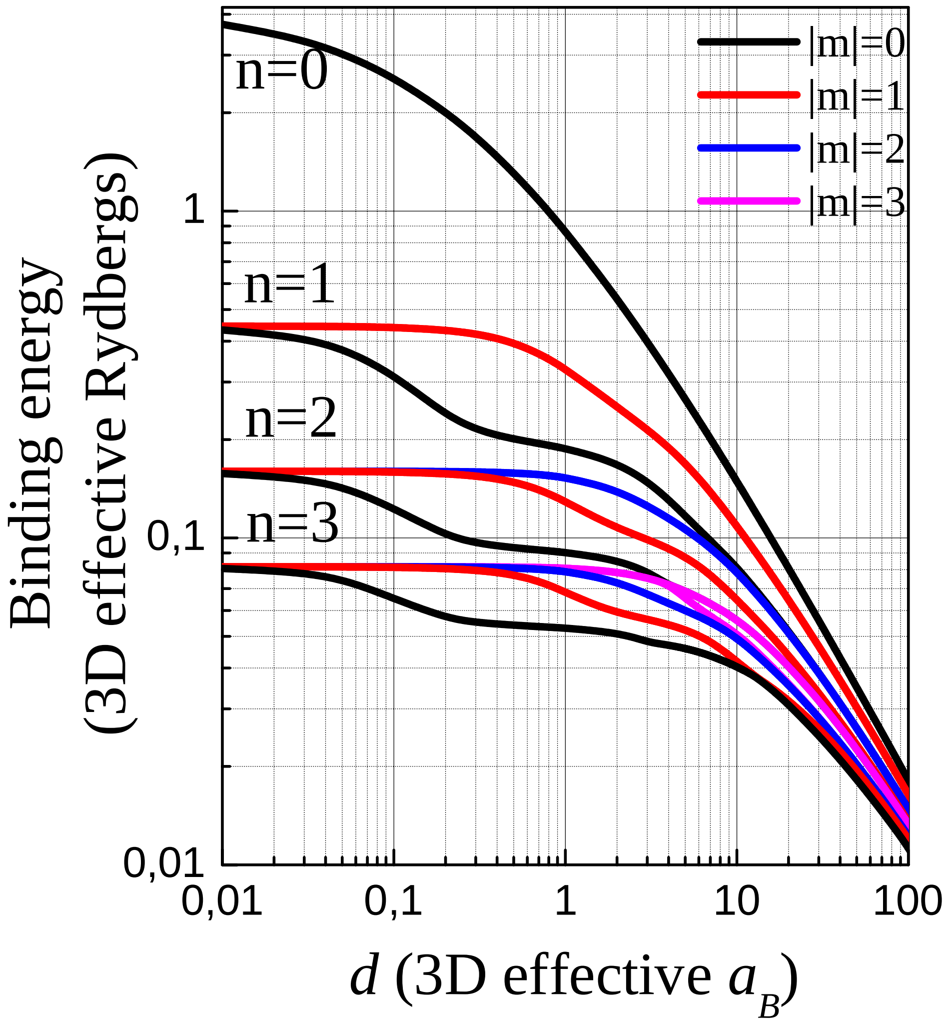
$$\psi_{pm}(\rho,\phi) = B_{p,m} \exp\left(-\frac{\rho^2}{2d^{3/2}} - im\phi\right)\left(\frac{\rho}{d^{3/2}}\right)^{|m|} L^{|m|}_{p/2-|m|/2}\left(\frac{\rho^2}{d^{3/2}}\right)$$

**2 Component Operator commuting with Hamiltonian**

$$S_x = p_x p_y + \frac{xy}{d^3}, \quad S_y = \frac{p_y^2 - p_x^2}{2} + \frac{y^2 - x^2}{2d^3}$$

*R.D.Mota, V.D. Granados, A. Queijeiro and J. Garcia, J. Phys. A: Math. Gen.* ***35****, 2979 (2002)*

# Eigenenergies between Coulomb Potential and Harmonic Oscillator




## Direct diagonalization

**Basis@*d<20***

Hydrogen Atom Eigenfunctions Set

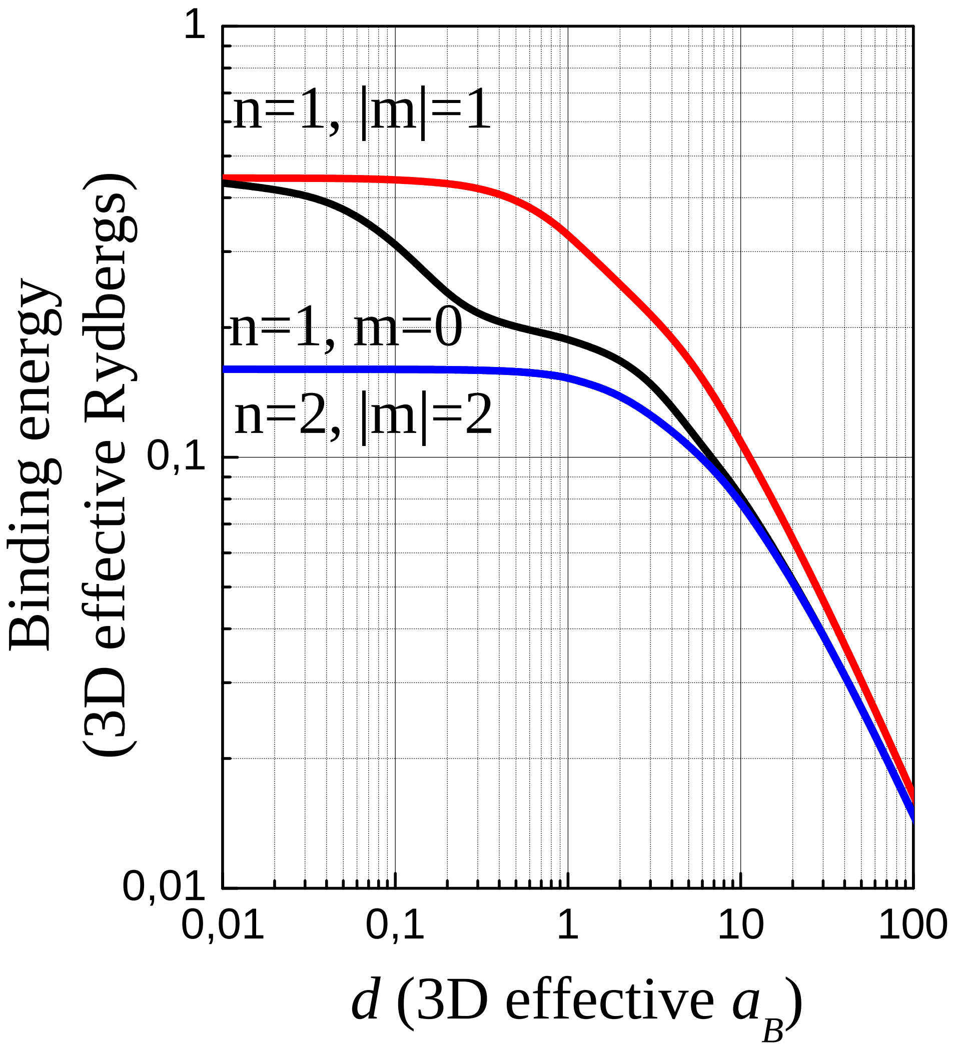
**Basis@*d>100***

Oscillator Eigenfunctions Set

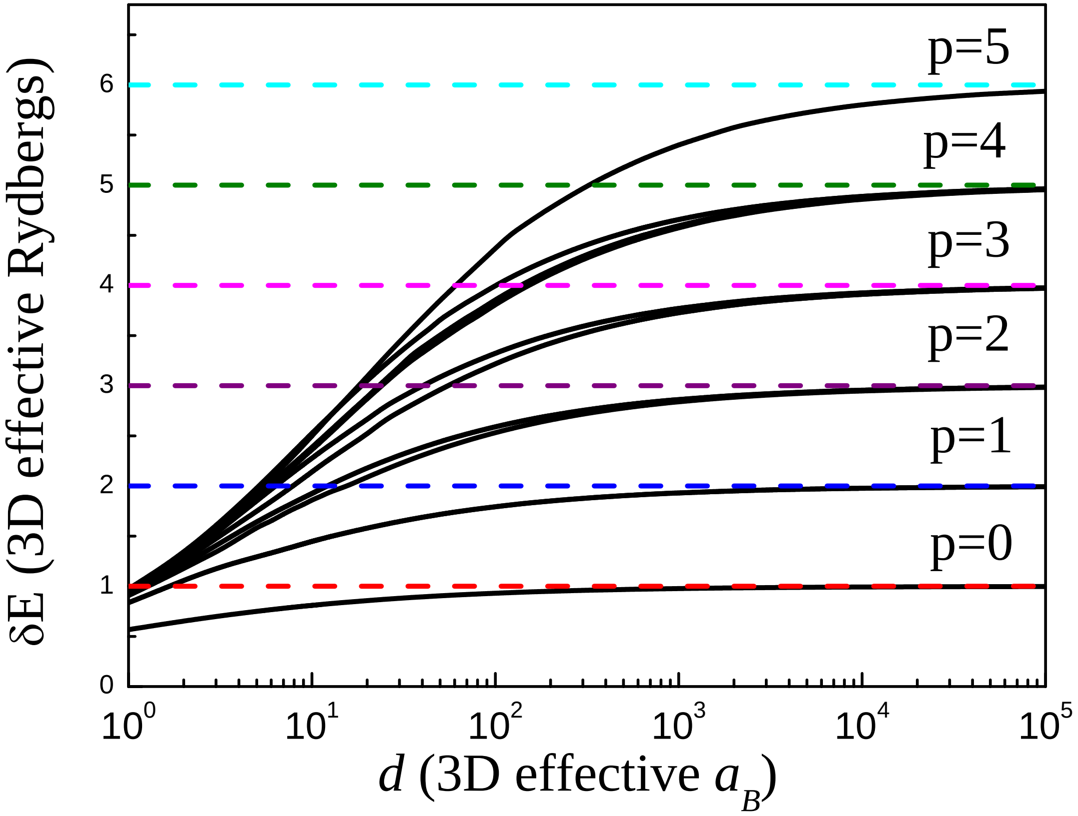
**Basis@*20<d<100***

**Both Sets**

# Rearrangement of Energy Levels



# Asymptotics of Energy Levels at *d >> 1*




$$E_p = -\frac{2}{d} + \frac{2}{d^{3/2}}(p+1)$$

$$\delta E = \frac{1}{2}\left(E_p + \frac{2}{d}\right)d^{3/2}$$

# Conclusions to the Section 1

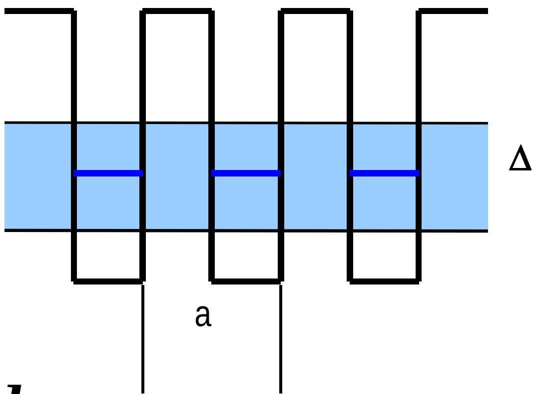
❖ Semiconductor Quantum Well Nanostructures allow us to look after transition of eigenstates and eigenenergies between two systems with accidental degeneration

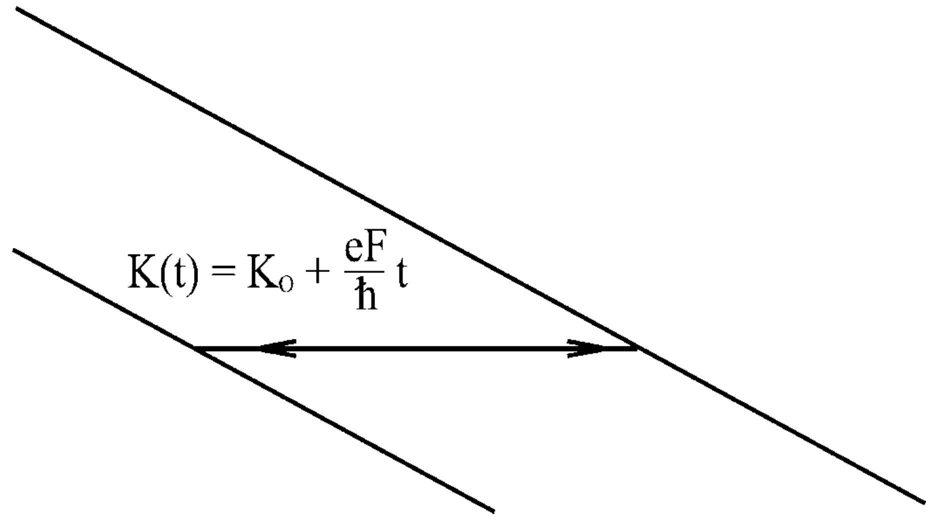
❖ In intermediate region, mixing of states with different principal quantum numbers becomes important in addition to the level splitting

❖ The region of applicability of both limiting case approximations depends on the eigenenergy, i.e., on the typical size of the corresponding eigenfunction

# Section 2.
# Electron in Periodic Potential under Action of Constant Electric Field and Bloch Oscillations

## Bloch oscillations: semi-classical description




$$\frac{d}{dt}\hbar K = eF$$

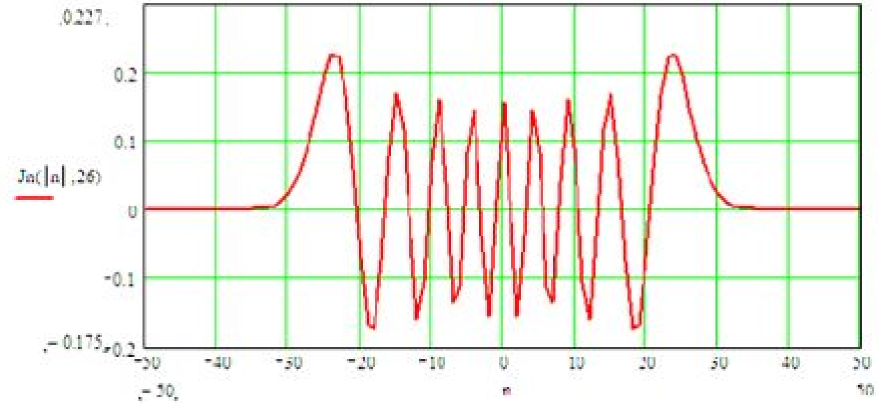
$$= K(0) + \frac{e}{\hbar} F t$$

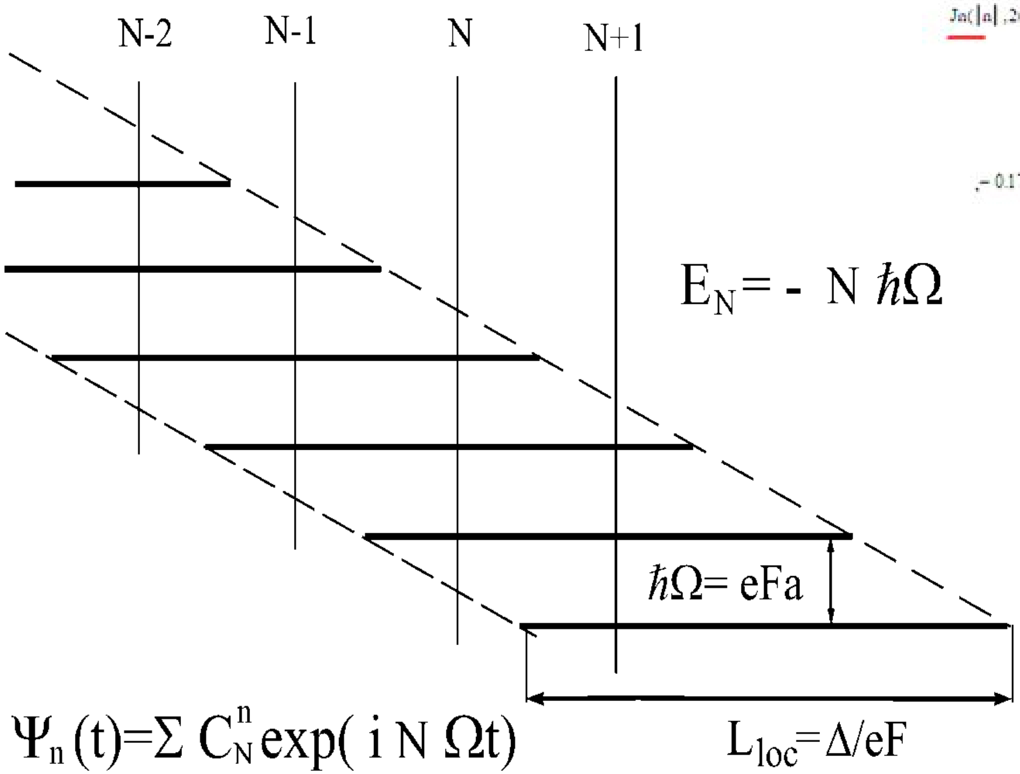
$$v(t) = \frac{\partial}{\partial K} E(K)\Big|_{K(t)}$$

***E(K)* – periodic function on *K***
***It is why an electron oscillates relative some spatial position with Bloch frequency***

**Bloch Frequency**
**$\Omega \equiv eFa/\hbar$**

# Wannier-Stark Ladder




BO – Quantum Beating

# Where Bloch Oscillations can be observed?

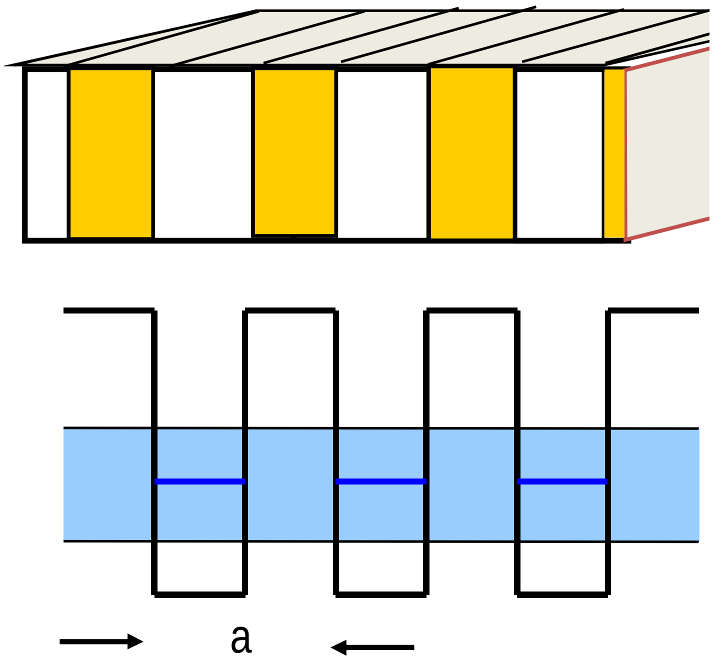
**In natural crystals it is impossible to realize BO. It would require too strong electric field at which electric breakdown should occur.**

**However, it is possible in artificial  crystals: semiconductor superlattices with lattice period strongly exceeding period of natural crystals (Man-made  in contrast to God –made crystal: L. Esaki)**

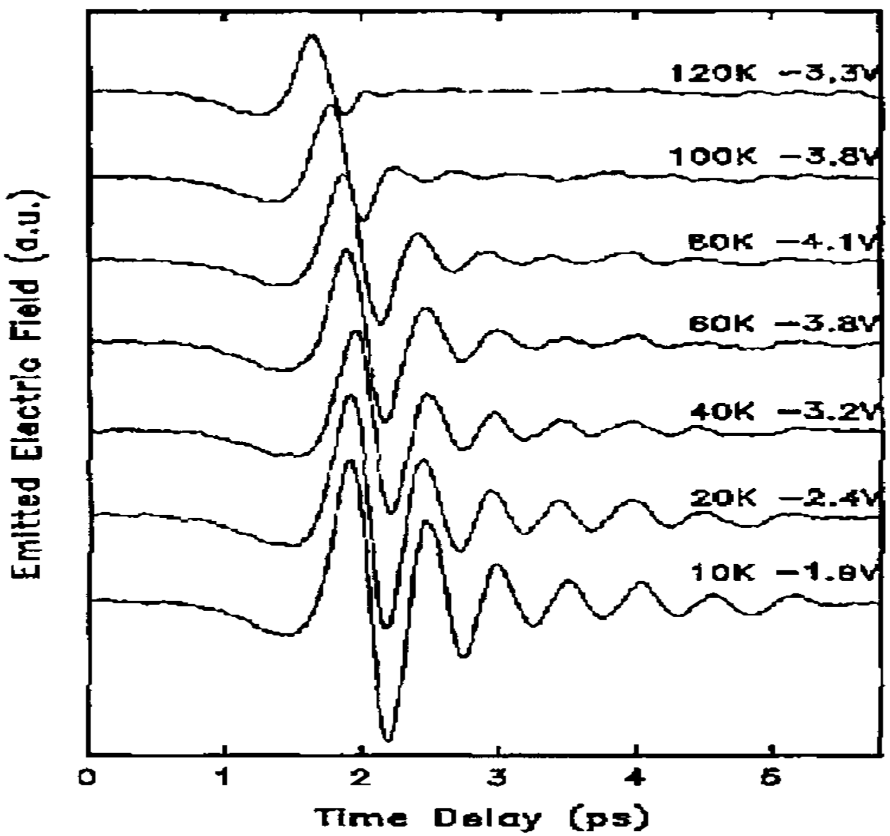
**Very promising properties of the layered superlattices were described by V.A.Yakovlev (1961) and L.V. Keldysh (1962)**

# Artificial Layered Superlattice

The SL structures are fabricated with MBE or MOCVD technologies

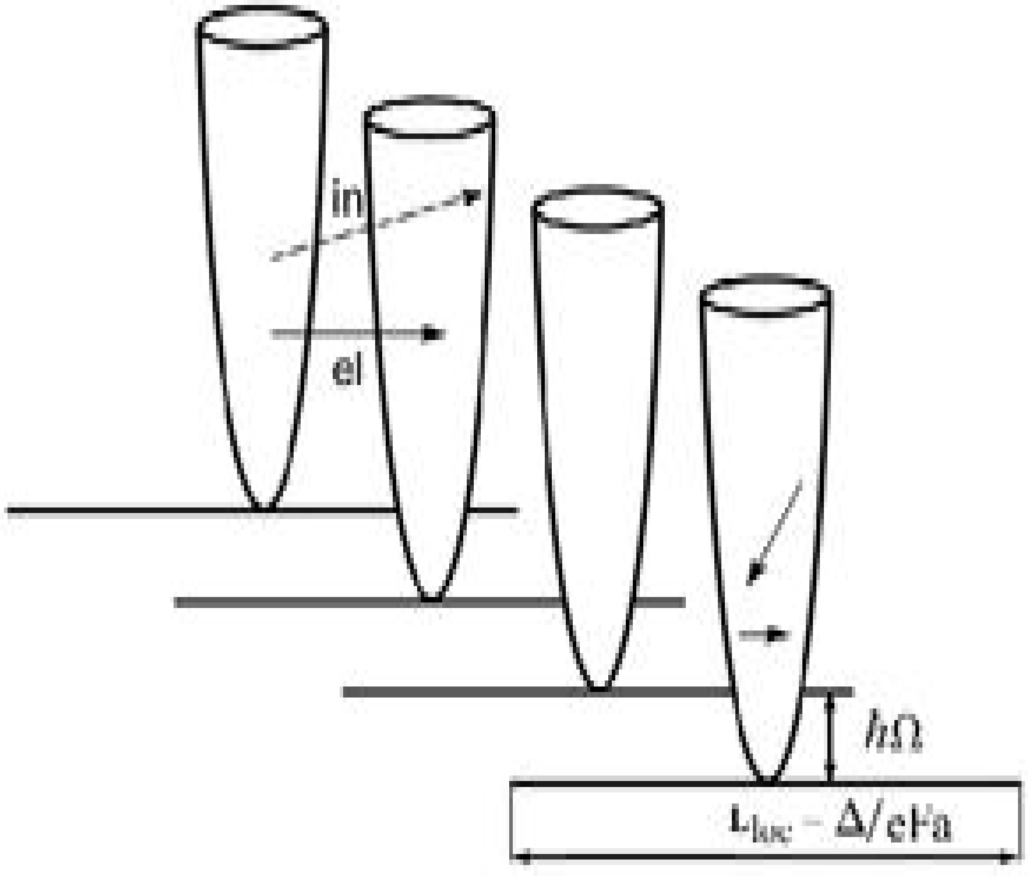



GaAs –$Ga_x Al_{1-x}As$



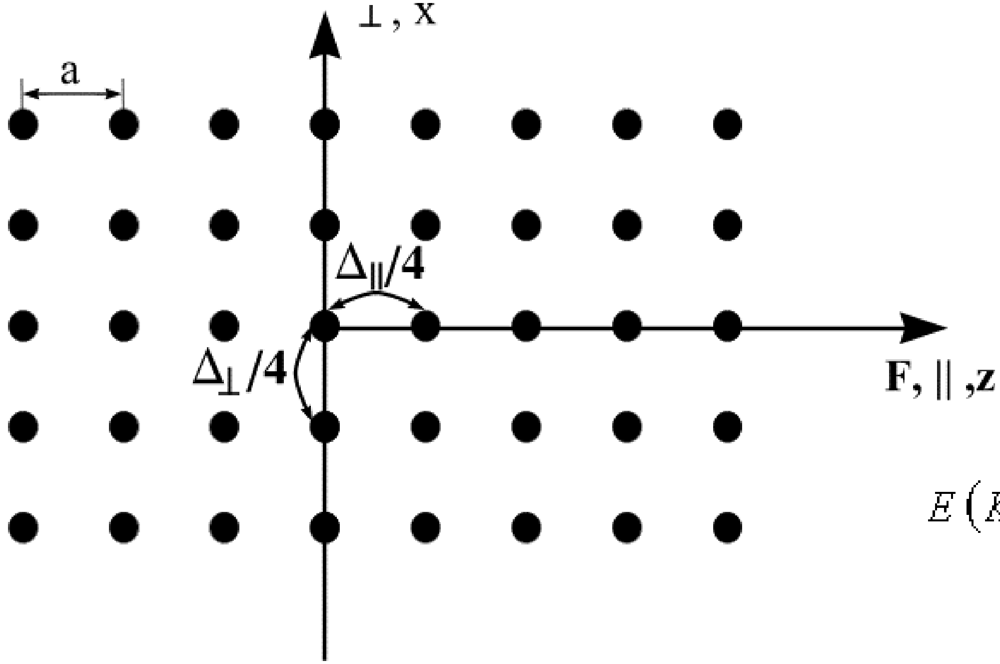

C.Waschke, H.G.Roskos, K Leo, H. Kurz, K.Köhler, Semicond. Sci.Technol., 9, 416, 1994.

# Scattering Processes Layered Superlattice



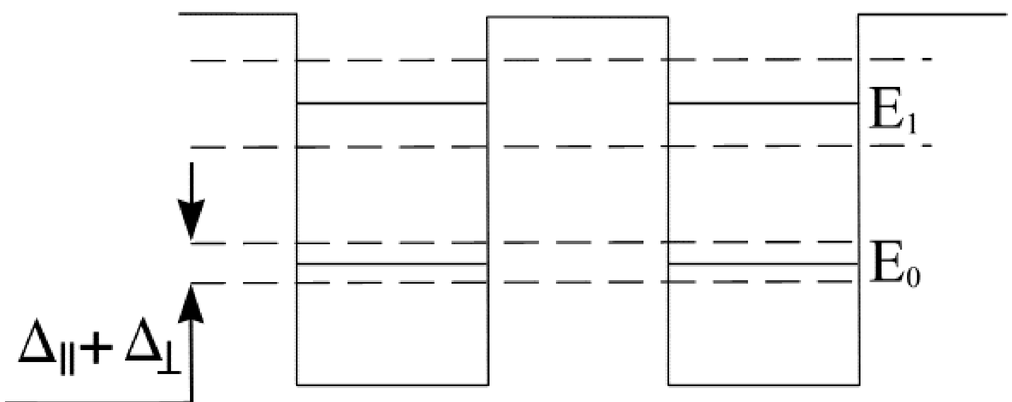

$$E_N(k_\perp) = -eFN + \frac{\hbar^2 k_\perp{}^2}{2m}$$

# Quantum Dot Superlattice




$$E(K) = \frac{\Delta_{\perp}}{2}\cos(k_{\perp}a) + \frac{\Delta_{\|}}{2}\cos(k_{\|}a)$$

$$\Delta_{\perp} + \Delta_{\|} << |E_0 - E_1|$$



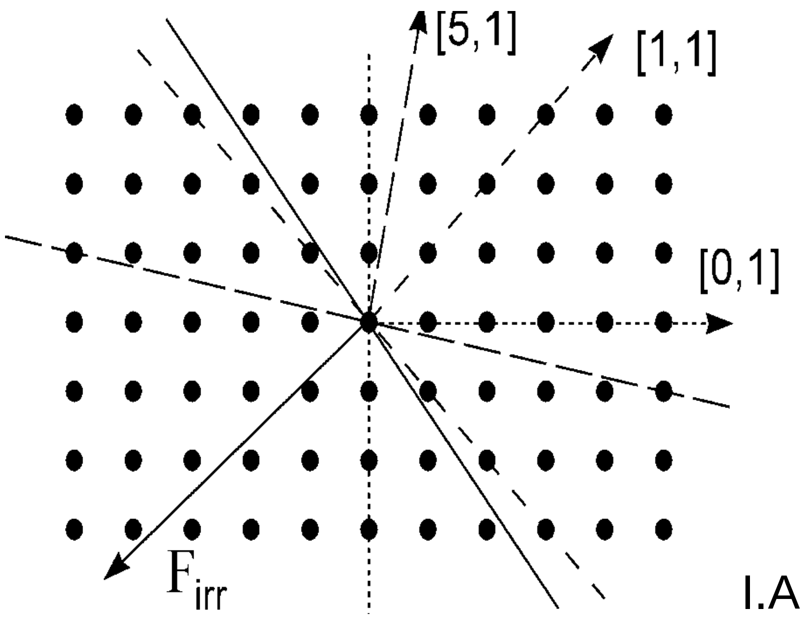
# Exponential dependence of transverse minibands width in QDSL

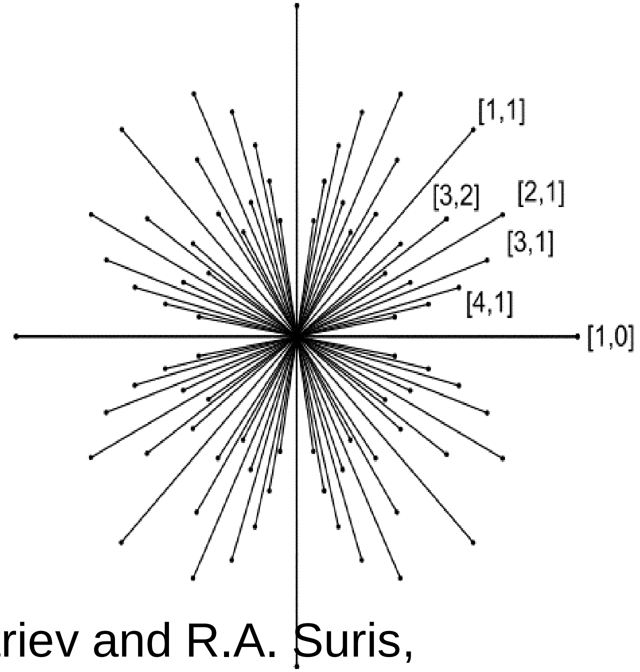
$$E_{\mathrm{R}}(\mathbf{k}) = -\mathrm{e}\mathbf{F}\cdot\mathbf{R} + \sum_{\rho}\frac{\Delta_{\rho}}{4}\left(1-\cos(\mathbf{k}\boldsymbol{\rho})\right)$$

***Rational Electric Field Direction***

$$E_{\mathrm{R}}(\mathbf{k}) = -\mathrm{e}\mathbf{F}\cdot\mathbf{R} = -\sum n_i \hbar\Omega_i$$

***Irrational Electric Field Direction***

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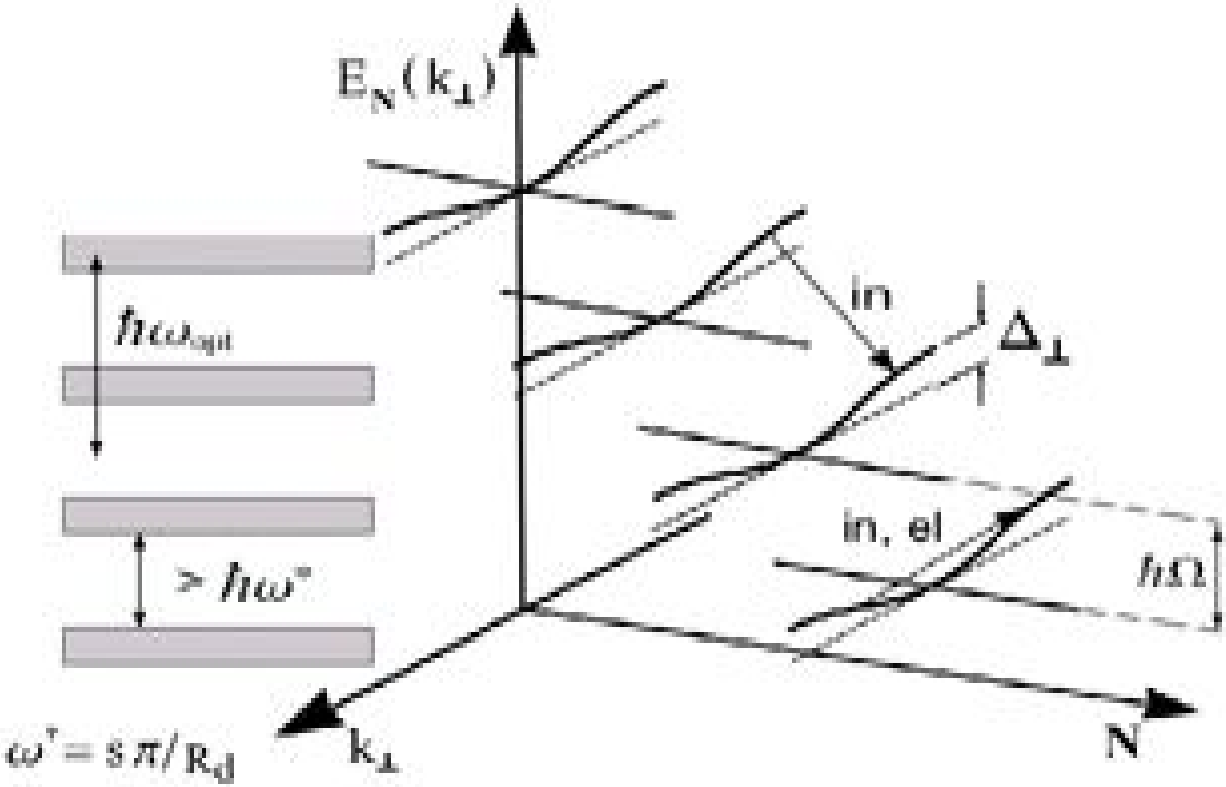
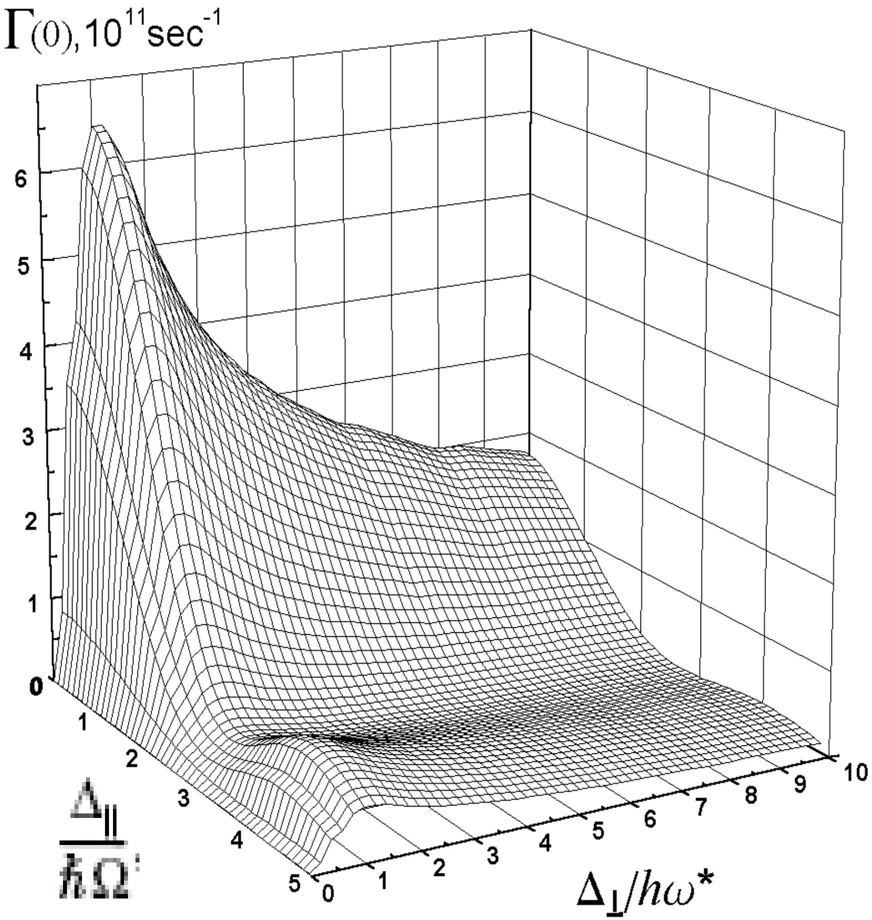
# Scattering Suppression in Quantum Dot Superlattice



# BO damping rate as a function of localization length and transverse miniband width




QDSL with Period a = 100 Å

QD size $R_D$ = 25 Å,

$\hbar\omega^* = \hbar s\,\pi / R_D = 4.3$ meV –

actual acoustic phonon energy.

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# Conclusions to the Section 2

- ♦ QDSL application enables us to control the scattering processes. It results in a profound extent of the BO lifetime.

*Varying the electric field value and orientation in QDSL allows us to:*

- qualitatively change the spectrum of electrons, from the continuous one to the purely discrete one;
- control the transverse miniband width in a wide range since it *exponentially* depends on the field orientation;
- totally suppress the optical phonon scattering which is the main mechanism of scattering in QWSL;
- make negligibly small the acoustic phonon scattering between transverse minibands;
- strongly reduce the intraminiband acoustic phonon scattering by choosing appropriate electric field strength and miniband width.

- ♦ BO damping rate at T=300 K $\gamma \propto 10^{13}$ **sec$^{-1}$ in QWSL** $\gamma \propto 10^{10}$ **sec$^{-1}$ in QDSL**
- ♦ The BO in QDSL are a superposition of 2 (2D SL) or 3 (3D SL) oscillations with frequencies proportional to the field projections on the principal SL axis's.