Non-minimal Higgs Inflation and Frame Dependence in Cosmology

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Outline

1. Non-minimal Higgs inflation
2. Quantum cosmology and initial conditions for inflation
3. One-loop cosmology and frame dependence

Non-minimal Higgs Inflation: Motivation & Model Setup

- A minimally coupled scalar field would lead to Higgs masses far too small:
  $$(\Delta T/T)^2 \simeq 10^{-10} \propto \lambda \quad \Rightarrow M_{\mathrm{H}}^2 \propto \lambda\, v^2 \ll 10^4 \text{ GeV}.$$
- A strong ($\xi \simeq 10^4$) non-minimal coupling $\xi\varphi^2 R$ to gravity leads to:
  $$(\Delta T/T)^2 \simeq 10^{-10} \propto \lambda/\xi^2 \quad \Rightarrow \lambda \text{ compatible with } M_{\mathrm{H}} \simeq 10^2 \text{ GeV}.$$

The Graviton-Higgs Sector:

$$S[g_{\mu\nu},\varphi] = \int d^4x\sqrt{g}\Big(U(\varphi)R(g_{\mu\nu}) - \frac{1}{2}G(\varphi)(\nabla\varphi)^2 - V(\varphi)\Big) + \dots,$$

$$U_{\text{tree}}(\varphi) = \frac{1}{2}\left(M_{\mathrm{P}}^2 + \xi\,\varphi^2\right), \quad V_{\text{tree}}(\varphi) = \frac{\lambda}{4}(\varphi^2 - v^2)^2,$$

$$G_{\text{tree}}(\varphi) = 1, \quad \varphi := |\Phi| = \sqrt{\Phi^a\Phi^b\delta_{ab}}, \quad a = 1,\dots,4\,, \quad v \simeq 246 \text{ GeV}.$$

- Standard Model: Mass generation $m_{\text{part}}(\varphi) \propto \varphi$ via Higgs mechanism.
- Consider only the heaviest particles: top-quark, $W^{\pm}$- and Z boson.

Quantum Corrections & Suppression

- Essential Goldstone contributions are highlighted in blue.

One-Loop Corrections:

$$V_{\text{1-loop}}(\varphi) = \mathbf{A}\frac{\lambda\varphi^4}{128\pi^2}\ \ln\frac{\varphi^2}{\mu^2} + \dots, \qquad U_{\text{1-loop}}(\varphi) = \mathbf{C}\ \frac{\varphi^2}{32\pi^2}\ \ln\frac{\varphi^2}{\mu^2} + \dots$$

$$\mathbf{A} = \frac{3}{8\lambda}\left(2g^4 + (g^2 + g'^2)^2 - 16y_t^4\right) + 6\lambda, \qquad \mathbf{C} = 3\xi\lambda + O(\xi^0)\,.$$

- Each Higgs (but not Goldstone!) propagator is suppressed by:

Suppression Function:

$$s(\varphi) := \frac{U}{GU + 3U'^2} = \frac{M_P^2 + \xi\varphi^2}{M_P^2 + (6\xi+1)\xi\varphi^2} \overset{\varphi \gg \frac{M_P}{\sqrt{\xi}}}{\approx} \frac{1}{6\xi}\,.$$

Inflation in the Einstein Frame & RG Improvement

- Establish connection with standard inflation theory:
  Transformation $S[g_{\mu\nu}, \varphi] \to \hat{S}[\hat{g}_{\mu\nu}, \hat{\varphi}]$ to the Einstein frame.

Effective Potential in the Einstein Frame:

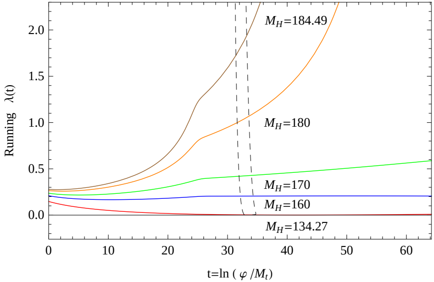
$$\hat{V}_{\text{eff}}(\hat{\varphi}) = \left(\frac{M_P^2}{2}\right)^2 \frac{V_{\text{eff}}(\varphi)}{U_{\text{eff}}^2(\varphi)}\bigg|_{\varphi=\varphi(\hat{\varphi})} \simeq \frac{\lambda M_P^4}{4\xi^2}\left(1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\mathbf{A_I}}{16\pi^2}\ln\frac{\varphi}{\mu}\right).$$

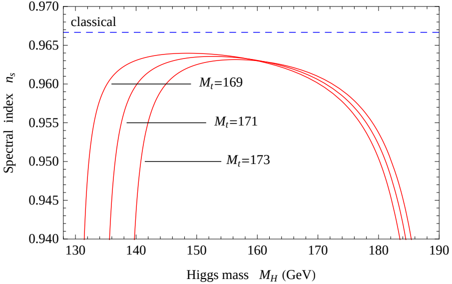
The key quantity $\mathbf{A_I} = \mathbf{A} - 12\lambda$ determines the inflationary dynamics.

- Renormalisation group improvement:[1,2] $g_i \to g_i(t)$ with $t = \ln(\varphi/\mu)$.
- Running from EW scale $t = 0$ ($\varphi \simeq v$) to inflation $t \simeq 35$ ($\varphi \simeq \frac{M_p}{\sqrt{\xi}}$) brings down $\mathbf{A_I}(t)$ to small values compatible with CMB and Higgs mass.

[1] Bezrukov, et al. (2009). Phys. Lett. B, **675**, 88-92.
[2] De Simone, et al. (2009). Phys. Lett. B, **678**, 1-8.

Numerical Results:[3] Running $\lambda(t)$ & Spectral Index





- Instability region: $M_{\rm H} \gtrsim 134.27$ GeV.
- Perturbation theory: $M_{\rm H} \lesssim 190$ GeV.
- $\lambda(t_{\rm end})$ finite $\rightarrow$"asymptotic freedom".

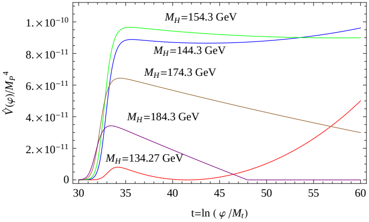
- $n_s = 1 - \frac{2}{N}\frac{x}{e^x - 1}$, $x := \frac{N A_{\rm I}}{48\pi^2}$.
- CMB constraint: $0.94 < n_s < 0.99$.

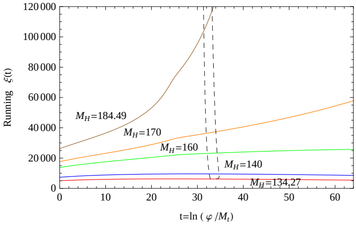
  $\Rightarrow$ $135.6\ \text{GeV} \lesssim M_{\rm H} \lesssim 184.5\ \text{GeV}$.

[3] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer, A. A. Starobinsky and C. S. (2009). JCAP, **12**, 003.

Quantum Cosmology: Initial Conditions for Inflation

- Tunnelling probability distribution: $\rho_{\rm t}(\varphi) := e^{-S_{\rm E}^{\rm eff}(\varphi)} = \exp\left(-\frac{24\,\pi^2\,M_{\rm P}^4}{V_{\rm eff}(\varphi)}\right)$.
- Sharp peak in $\rho_{\rm t}(\varphi)$ $\widehat{=}$ most probable value of φ after tunnelling.



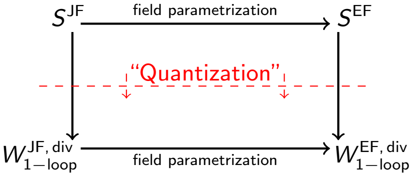


- Peak location $\widehat{=}$ maximum of $\hat{V}_{\rm RG}(\varphi)$: $\varphi_0^2 = -\left.\frac{64\,\pi^2\,M_{\rm P}^2}{\xi\,\mathbf{A}_{\rm I}\,Z^2}\right|_{t=t_0}$.
- Numerically: $t_0 \simeq t_{\rm in} \simeq t_{\rm end} + 2$, $\quad \mathbf{A}_{\rm I\,end} \simeq \mathcal{O}(1) < 0$, $\quad Z_{\rm end} \simeq \mathcal{O}(1)$.
- Initial conditions for inflation: $\varphi_{\rm in} \simeq \varphi_0 \simeq \frac{M_{\rm P}}{\sqrt{\xi_{\rm end}}}$.[8]

[8] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer and C. S. (2010). Phys. Rev. D, **81**, 043530.

One-Loop Corrections & Frame Dependence

- JF: $S^{\mathrm{JF}}[g,\Phi] = \int \mathrm{d}^4x \sqrt{g} \left(U(\varphi)\, R - \frac{1}{2}\, G(\varphi)\, \partial_\mu \Phi^a \partial^\mu \Phi_a - V(\varphi) \right)$.
- Analytic result in closed form: $W^{\mathrm{JF,\,div}}_{1-\mathrm{loop}} = \int \mathrm{d}^4x \sum_i \alpha_i(\varphi)\, O_i[g_{\mu\nu}, \Phi_a]$. [11]



- Transition between JF and EF possible for scalar $O(N)$ multiplet . [12, 13]
- Result: Quantum corrections are frame-dependent. [12]
- Formalism not covariant w.r.t. diffeomorphisms of configuration space. [14]

[11] C. S., A. Y. Kamenshchik (2011). Phys. Rev. D, **84**, 024026.
[12] C. S., A. Yu. Kamenshchik, in preparation.
[13] In contrast to the claim made in: D. I. Kaiser (2010). Phys. Rev. D, **81**, 084044.
[14] G. A. Vilkovisky (1984). Nucl. Phys. B, **234**, 125-137.

Conclusion & Outlook

Main results:

- Higgs-inflation one-loop predictions: $135.6\ \mathrm{GeV} \lesssim M_{\mathrm{H}} \lesssim 184.5\ \mathrm{GeV}$.
- Quantum-cosmological tunnelling sets initial conditions for inflation.
- One-loop effective action for general $O(N)$ field in the Jordan frame.
- Transition between JF and EF for $O(N)$ multiplet possible.
- Cosmological quantum corrections are frame-dependent: JF vs. EF.