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STRING MODELS, STABILITY AND WIDTH OF HADRON STATES

String models of hadrons

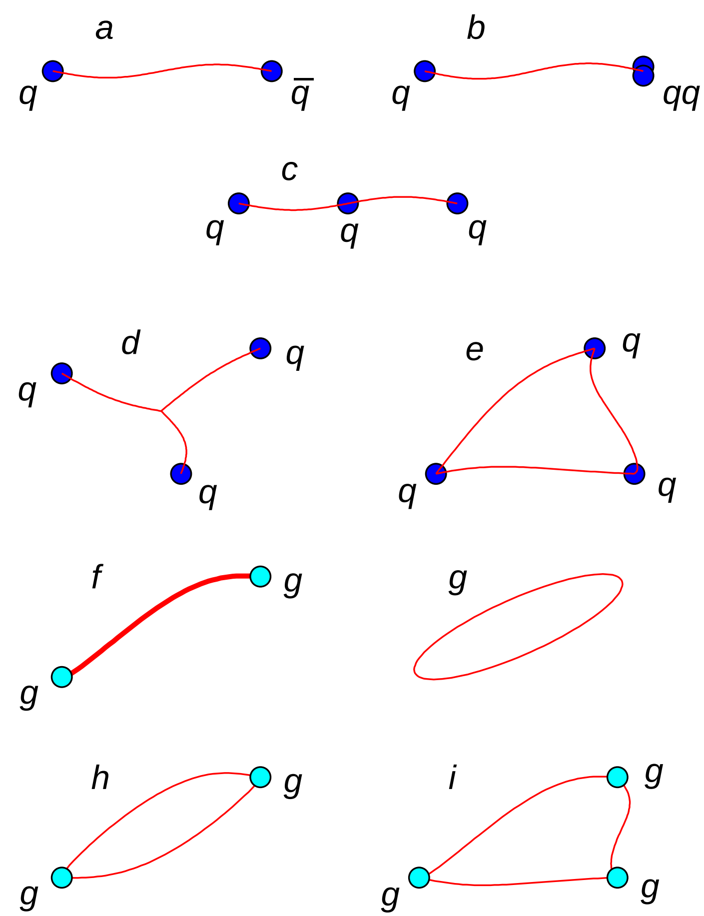
(a) meson string model

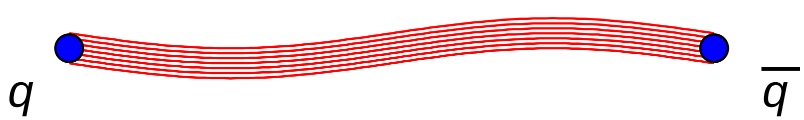
(b) quark-diquark model q-qq

(c) linear configuration q-q-q

(d) Y configuration

(e) “triangle” (or Δ) model

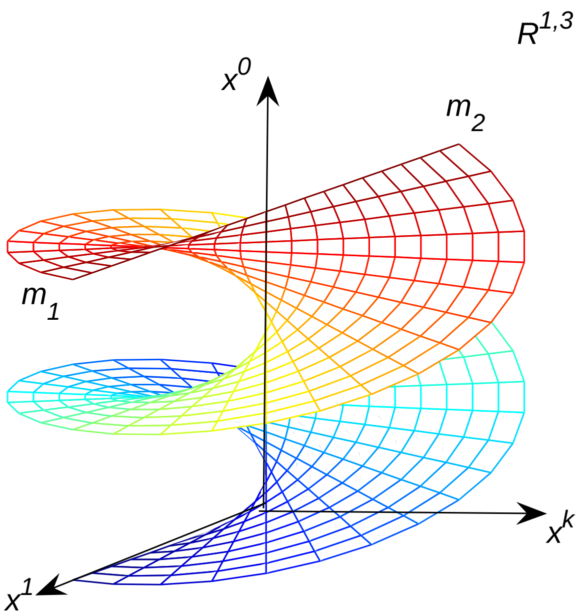




Action

$$S=-\gamma\int\limits_{\Omega}\sqrt{-\det\|g_{ab}\|}\,d\tau\,d\sigma-\sum_{j=1}^{n}m_j\int\sqrt{\dot x_i^2(\tau)}\,d\tau. \qquad (1)$$

Here γ is the string tension, $g_{ab}=\eta_{\mu\nu}\partial_a X^\mu\partial_b X^\nu$, $\eta_{\mu\nu}=\mathrm{diag}\,(1;-1;-1;-1)$ is metric in $R^{1,3}$; $c=1$; m_j are masses and $\sigma=\sigma_j(\tau)$ or $x^\mu=x_j^\mu(\tau)\equiv X^\mu\big(\tau,\sigma_j(\tau)\big)$ are world lines of massive points (quarks), $\dot x_j^\mu(\tau)\equiv\frac{d}{d\tau}x_j^\mu(\tau)$; $(a,b)=\eta_{\mu\nu}a^\mu b^\nu$.



$x^\mu=X^\mu(\tau,\sigma)$ is a string world surface

Dynamical equations result from the action (1) and under orthonormality conditions

$$\dot X^2+X'^2=0,\qquad (\dot X,X')=0 \qquad (2)$$

take the form (here $\dot X^\mu\equiv\partial_\tau X^\mu$, $X'^\mu\equiv\partial_\sigma X^\mu$)

$$\frac{\partial^2X^\mu}{\partial\tau^2}-\frac{\partial^2X^\mu}{\partial\sigma^2}=0 \qquad (3)$$

$$m_j\frac{d}{d\tau}\frac{\dot x_j^\mu(\tau)}{\sqrt{\dot x_j^2(\tau)}}\pm\gamma\Big[X'^\mu+\dot\sigma_j(\tau)\dot X^\mu\Big]\Big|_{\sigma=\sigma_j}=0,\qquad \text{(for an endpoint)} \qquad (4)$$

$$m_j \frac{d}{d\tau} \frac{\dot{x}_j^\mu(\tau)}{\sqrt{\dot{x}_j^2(\tau)}} + \gamma \Big[X'^\mu + \dot\sigma_j(\tau)\dot X^\mu\Big]\Big|_{\sigma=\sigma_j-0} - \gamma \Big[X'^\mu + \dot\sigma_j(\tau)\dot X^\mu\Big]\Big|_{\sigma=\sigma_j+0} = 0, \qquad (5)$$

Rotational states for string models q-$\bar q$ and q-q-q:

$$X^\mu(\tau,\sigma) = \underline{X}^\mu(\tau,\sigma) = \Omega^{-1}\big[\theta\tau e_0^\mu + \cos(\theta\sigma+\phi_1)\cdot e^\mu(\tau)\big], \qquad (6)$$

where $\sigma\in[0,\pi]$, $\Omega=\theta/a_0$ is angular velocity, e_0, e_1, e_2, e_3 is a tetrad in $R^{1,3}$,

$$e^\mu(\tau) = e_1^\mu\cos\theta\tau + e_2^\mu\sin\theta\tau, \qquad \acute{e}^\mu(\tau) = -e_1^\mu\sin\theta\tau + e_2^\mu\cos\theta\tau$$

are unit rotating vectors, $v_1=\cos\phi_1$, $m_j\Omega/\gamma = v_j^{-1} - v_j$.

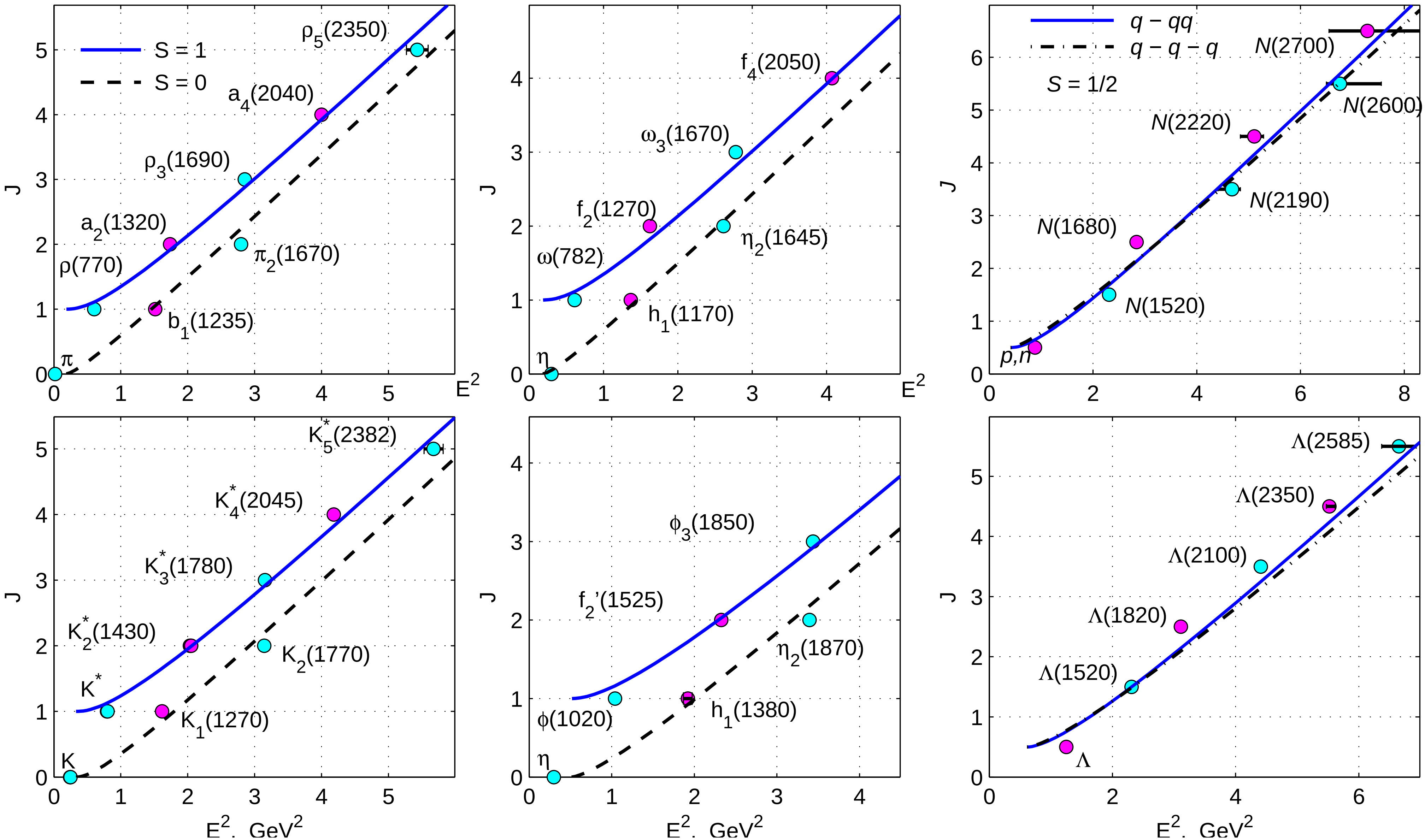
Energy and angular momentum of states (6) :

$$E = \frac{\pi\gamma\theta}{\Omega} + \sum_{j=1}^2 \frac{m_j}{\sqrt{1-v_j^2}} + \Delta E_{SL}, \qquad \Delta E_{SL} = \sum_{j=1}^2 \big[1-(1-v_j^2)^{1/2}\big](\Omega\cdot s_j),$$

$$J = \frac{\pi\gamma\theta}{2\Omega^2} + \frac{1}{2\Omega}\sum_{j=1}^2 \frac{m_j v_j^2}{\sqrt{1-v_j^2}} + S.$$

Rotational states of all string models (here q-$\bar{q}$ and q-q-q) generate Regge trajectories

for mesons and baryons



Stability problem for the model q-q-q

We substitute the general solution of string oscillatory equation (3)

$$X^\mu(\tau,\sigma)=\frac{1}{2}[\Psi^\mu_{j+}(\tau+\sigma)+\Psi^\mu_{j-}(\tau-\sigma)],\quad \sigma\in[\sigma_j,\sigma_{j+1}],\qquad \dot\Psi^2_{j\pm}(\tau)=0 \qquad (7)$$

into boundary conditions (4), (5). We obtain the system of ordinary (nonlinear) differential equations with deviating arguments :

$$\frac{d}{d\tau}\frac{\dot\Psi^\mu_{1+}(\tau)+\dot\Psi^\mu_{1-}(\tau)}{\big(2\dot\Psi_{1+}(\tau),\dot\Psi_{1-}(\tau)\big)^{1/2}}=\frac{\gamma}{2m_1}\Big[\dot\Psi^\mu_{1+}(\tau)-\dot\Psi^\mu_{1-}(\tau)\Big],$$

$$\frac{d}{d\tau}\frac{(1+\dot\sigma_2)\dot\Psi^\mu_{1+}(+_2)+(1+\dot\sigma_2)\dot\Psi^\mu_{1-}(-_2)}{\big[2(1-\dot\sigma_2^2)\big(\dot\Psi_{1+}(+_2),\dot\Psi_{1-}(-_2)\big)\big]^{1/2}}=\frac{\gamma}{m_2}\frac{d}{d\tau}\Big[\Psi^\mu_{2+}(+_2)-\dot\Psi^\mu_{1+}(+_2)\Big],$$

$$\frac{d}{d\tau}\frac{\dot\Psi^\mu_{2+}(+)+\dot\Psi^\mu_{2-}(-)}{\big(2\dot\Psi_{2+}(+),\dot\Psi_{2-}(-)\big)^{1/2}}=\frac{\gamma}{2m_3}\Big[\dot\Psi^\mu_{2-}(-)-\dot\Psi^\mu_{2+}(+)\Big],$$

$$\Psi^\mu_{1+}(+_2)+\Psi^\mu_{1-}(-_2)=\Psi^\mu_{2+}(+_2)+\Psi^\mu_{2-}(-_2).$$

Here $\quad(\pm_2)\equiv\big(\tau\pm\sigma_2(\tau)\big),\quad(\pm)\equiv(\tau\pm\pi).$

For the rotational state (6) with $\Psi^\mu_{1\pm}(\tau) = \underline{\Psi}^\mu_{2\pm}(\tau) = \Omega^{-1}\big[e^\mu_0\theta\tau + e^\mu(\tau \pm \phi_1/\theta)\big]$ we substitute small disturbances

$$\Psi^\mu_{j\pm}(\tau) = \underline{\Psi}^\mu_{j\pm}(\tau) + \psi^\mu_{j\pm}(\tau), \qquad \sigma_2(\tau) = \underline{\sigma}_2 + \delta_2(\tau), \qquad \underline{\sigma}_2 = \frac{\pi - 2\phi_1}{2\theta}$$

into equations of motion, omitting squares $\psi_{j\pm}$ and δ_2.

The linearized system for small disturbances $\psi_{j\pm}$ and δ_2 is

$$\psi^\mu_{1+}(+_2) + \psi^\mu_{1-}(-_2) = \psi^\mu_{2+}(+_2) + \psi^\mu_{2-}(-_2), \qquad \big(\dot{\underline{\Psi}}_{j\pm}, \dot\psi_{j\pm}\big) = 0,$$

$$\dot\psi^\mu_{1+}(\tau) + \dot\psi^\mu_{1-}(\tau) - \underline{U}^\mu_1\big(\underline{U}_1, \dot\psi_{1+}(\tau) + \dot\psi_{1-}(\tau)\big) = Q_1\big[\psi^\mu_{1+}(\tau) - \psi^\mu_{1-}(\tau)\big],$$

$$\dot\psi^\mu_{2+}(+) + \dot\psi^\mu_{2-}(-) - \underline{U}^\mu_3\big(\underline{U}_3, \dot\psi_{2+}(+) + \dot\psi_{2-}(-)\big) = Q_3\big[\psi^\mu_{2-}(-) - \psi^\mu_{2+}(+)\big],$$

$$\dot\psi^\mu_{1+}(+_2) + \dot\psi^\mu_{1-}(-_2) - 2a_0\big[\dot\delta_2 e^\mu(\tau) + \theta\delta_2 \acute{e}^\mu(\tau)\big] =$$

$$= \frac{e^\mu_0}{2a_0}\Big[\big(\dot{\underline{\Psi}}_{1+}(+_2), \dot\psi_{1-}(-_2)\big) + \big(\dot{\underline{\Psi}}_{1-}(-_2), \dot\psi_{1+}(+_2)\big)\Big] + 2Q_2\big[\psi^\mu_{2+}(+_2) - \psi^\mu_{1+}(+_2)\big]$$

Here $a_0 = \theta/\Omega$,

$$Q_j = \frac{\gamma}{m_j}\sqrt{\underline{\dot x}^2_j(\tau)} = \frac{\gamma a_0}{m_j}\sqrt{1 - v_j^2}, \quad \underline{U}^\mu_j(\tau) = \frac{e^\mu_0 - \epsilon_j v_j \acute{e}^\mu(\tau)}{\sqrt{1 - v_j^2}}, \quad \epsilon_1 = -1, \quad \epsilon_3 = 1,$$

Subsystem of 4 equations with $\psi^3_{j\pm}$ has solutions $\psi^3_{j\pm} = B^3_{j\pm} e^{-i\xi\tau}$ if and only if

$$\begin{vmatrix} -i\xi - Q_1 & -i\xi + Q_1 & 0 & 0 \\ 0 & 0 & (-i\xi + Q_3)\, e^{-i\pi\xi} & (-i\xi - Q_3)\, e^{i\pi\xi} \\ (-i\xi + 2Q_2)\, e^{-i\underline{\sigma}_2\xi} & -i\xi e^{i\underline{\sigma}_2\xi} & 2Q_2 e^{-i\underline{\sigma}_2\xi} & 0 \\ e^{-i\underline{\sigma}_2\xi} & e^{i\underline{\sigma}_2\xi} & -e^{-i\underline{\sigma}_2\xi} & -e^{i\underline{\sigma}_2\xi} \end{vmatrix} = 0 \quad \Longleftrightarrow$$

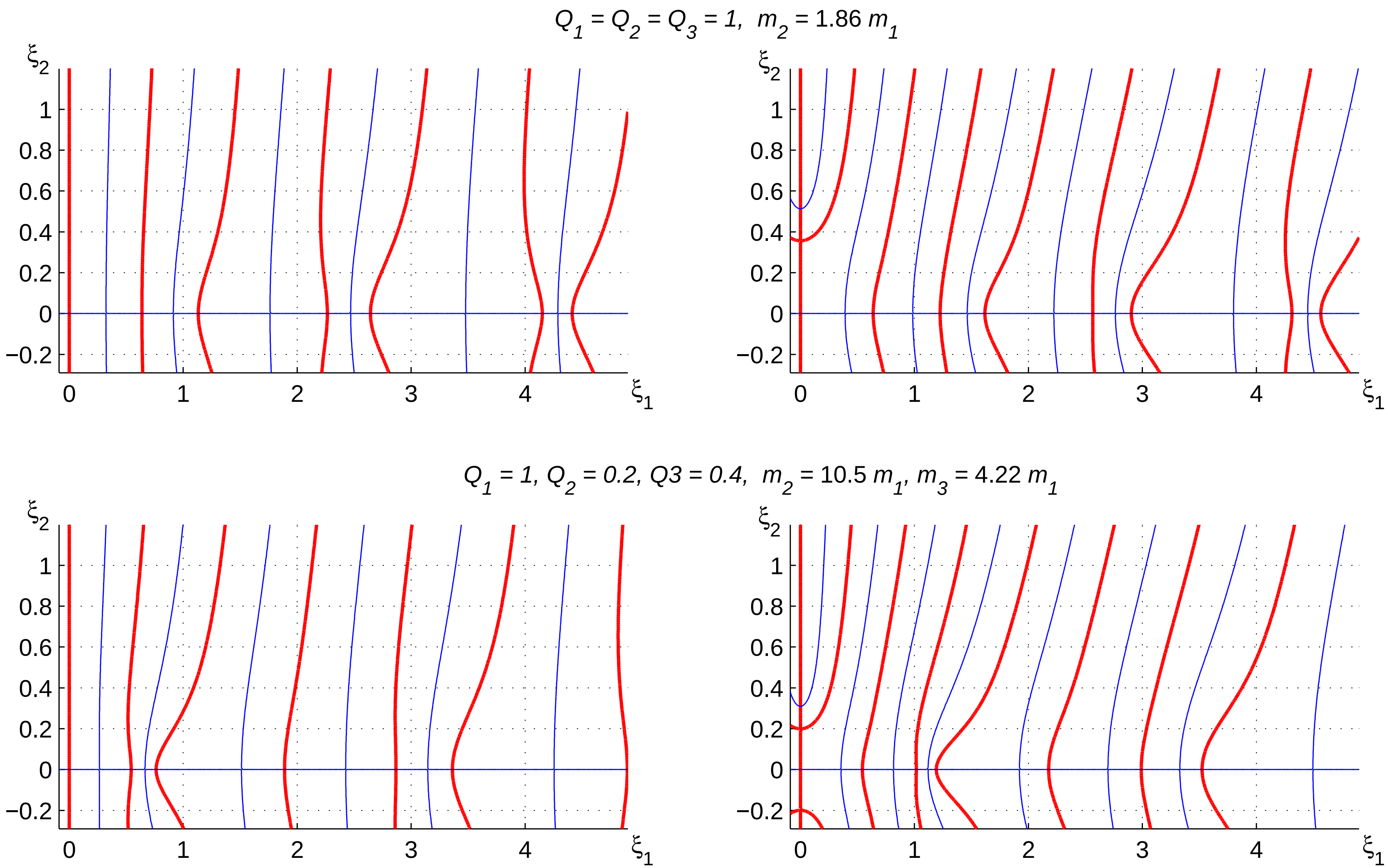
$$Q_2\big[(Q_1Q_3 - \xi^2)\sin\pi\xi + (Q_1 + Q_3)\,\xi\cos\pi\xi\big] + \xi(Q_1\tilde{c}_2 - \xi\tilde{s}_2)(Q_3\tilde{c}_3 - \xi\tilde{s}_3) = 0. \quad (8)$$

The similar condition for small oscillations in the rotational plane (for projections $\psi_{j\pm}$ onto basic vectors e_0, e, $\acute{e}$, for example, $\big(\psi_{j\pm}, e\big) = B_{j\pm} e^{-i\xi\tau}$) is the following spectral equation:

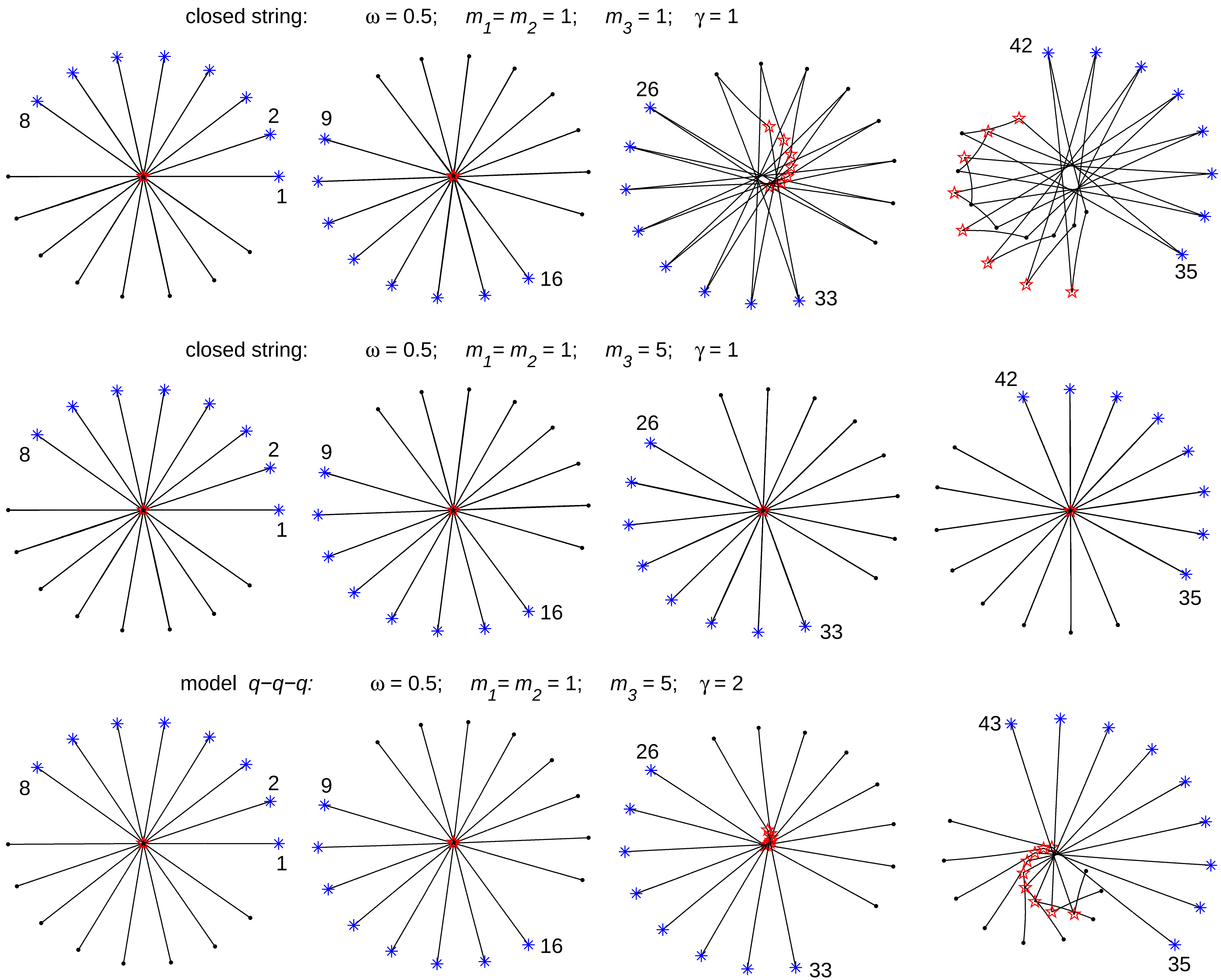
$$\frac{\xi}{Q_2} \cdot \frac{\xi^2 - \theta^2}{\xi^2 - \theta^2} = \frac{(Q_1^2\varkappa_1\tilde{c}_2 - \xi^2) - 2Q_1\xi\tilde{s}_2}{(Q_1^2\varkappa_1\tilde{s}_2 - \xi^2) + 2Q_1\xi\tilde{c}_2} + \frac{(Q_3^2\varkappa_3\tilde{c}_3 - \xi^2) - 2Q_3\xi\tilde{s}_3}{(Q_3^2\varkappa_3\tilde{s}_3 - \xi^2) + 2Q_3\xi\tilde{c}_3}. \quad (9)$$

$$\varkappa_j = 1 + v_j^{-2}, \quad \tilde{c}_2 = \cos\underline{\sigma}_2\xi, \quad \tilde{s}_2 = \sin\underline{\sigma}_2\xi, \quad \tilde{c}_3 = \cos(\pi - \underline{\sigma}_2)\xi, \quad \tilde{s}_3 = \sin(\pi - \underline{\sigma}_2)\xi.$$

Roots of Eqs. (8) and (9) $\big[$in the form $F(\xi) = 0\big]$ on the complex plane of $\xi = \xi_1 + i\xi_2$ are cross points of zero level lines for $\operatorname{Re} F(\xi)$ and $\operatorname{Im} F(\xi)$.



Zero level lines for real part (blue) and imaginary part (red) for Eq. (8) (left) and (9) (right) for $\xi = \xi_1 + i\xi_2$. Exponential growth $\psi \sim Be^{-i\xi_1\tau}e^{\xi_2\tau}$.



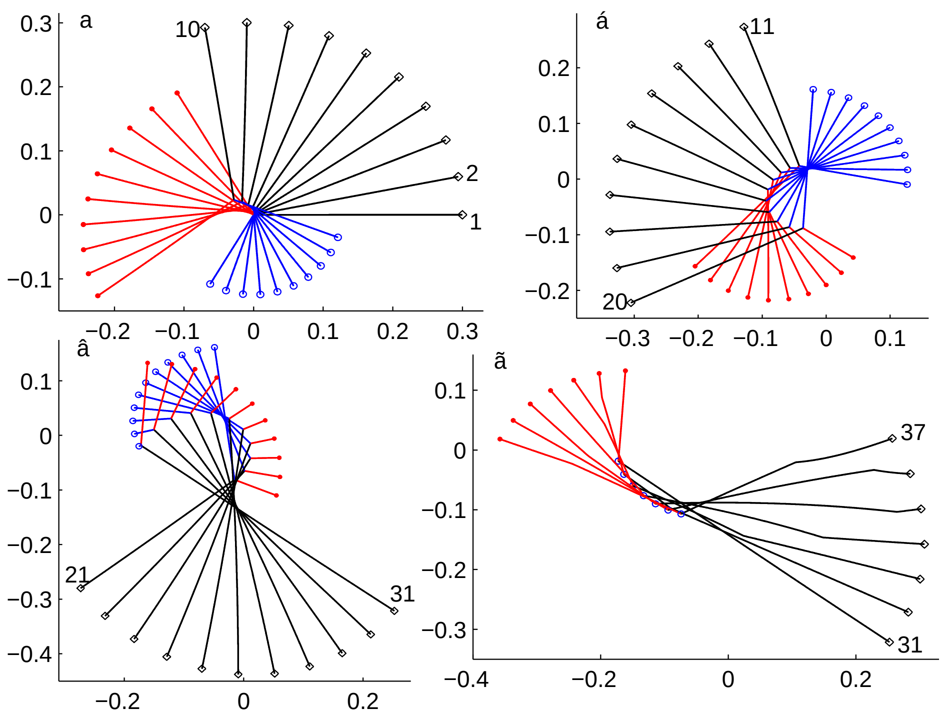
closed string: ω = 0.5; m1= m2 = 1; m3 = 1; γ = 1
8
2
1
9
16
26
33
42
35
closed string: ω = 0.5; m1= m2 = 1; m3 = 5; γ = 1
8
2
1
9
16
26
33
42
35
model q−q−q: ω = 0.5; m1= m2 = 1; m3 = 5; γ = 2
8
2
1
9
16
26
33
43
35

String model Y

Dynamical equations (for 3 world sheets $X_j^\mu(\tau_j,\sigma)$, $j=1,\,2,\,3$, $\sigma\in[0,\pi]$):

$$\frac{\partial^2 X_j^\mu}{\partial\tau_j^2}-\frac{\partial^2 X_j^\mu}{\partial\sigma^2}=0,\qquad m_j\frac{dU_j^\mu(\tau_j)}{d\tau_j}+\gamma X_j^{\prime\mu}(\tau_j,\pi)=0,\qquad U_j^\mu(\tau_j)=\frac{\dot x_j^\mu(\tau_j)}{\sqrt{\dot x_j^2(\tau_j)}},$$

$$X_1^\mu\big(\tau,0\big)=X_2^\mu\big(\tau_2(\tau),0\big)=X_3^\mu\big(\tau_3(\tau),0\big),\qquad \sum_{j=1}^{3}X_j^{\prime\mu}\big(\tau_j(\tau),0\big)\,\dot\tau_j(\tau)=0.$$



Rotational states of the Y configuration

$$\underline{X}_j^\mu(\tau_j,\sigma)=\Omega^{-1}\big[\theta\tau_j e_0^\mu+\sin(\theta\sigma)\cdot e^\mu(\tau_j+\Delta_j)\big],$$

$$\Delta_j=2\pi(j-1)/(3\theta).$$

G. S. Sharov, Phys. Rev. D **62**, 094015 (2000); arXiv: hep-ph/0004003.

G. 't Hooft, Minimal strings for baryons, arXiv: hep-th/0408148.

Analysis of small disturbances for the model Y

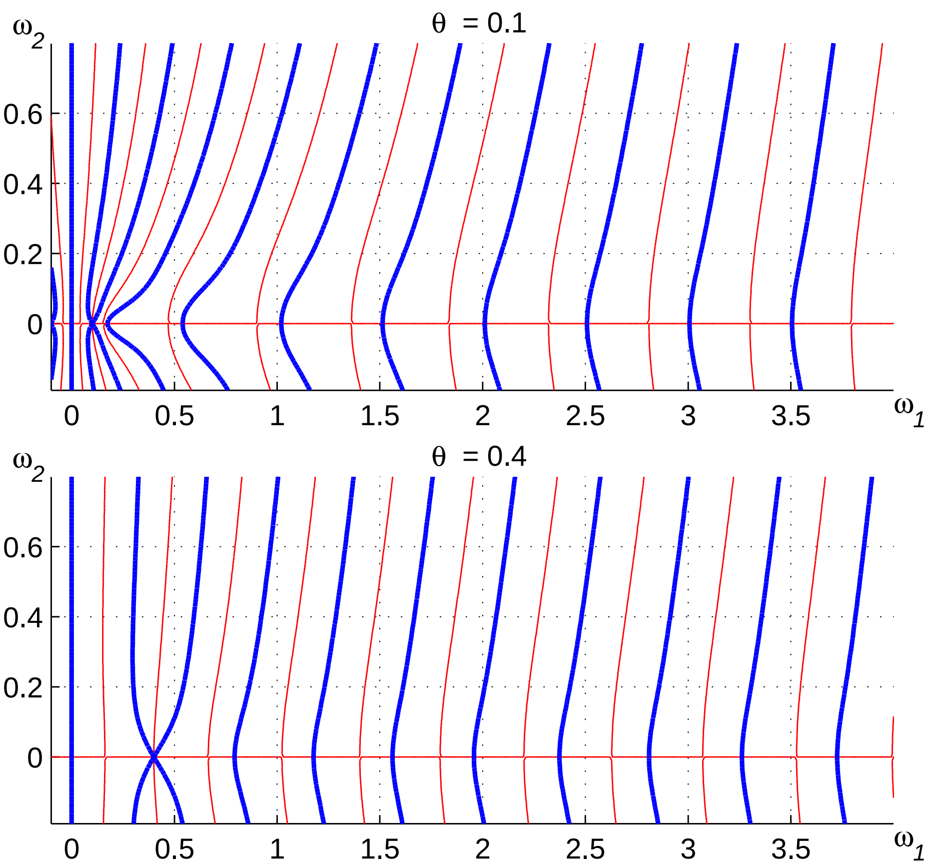
$$U_j^\mu(\tau_j) = \underline{U}_j^\mu(\tau_j) + u_j^\mu(\tau_j), \qquad \dot\Psi^\mu_{j\pm}(\tau_j \pm \pi) = \frac{m_j}{\gamma}\Big[\sqrt{-\dot U_j^2(\tau_j)}\, U_j^\mu(\tau_j) \mp \dot U_j^\mu(\tau_j)\Big]$$

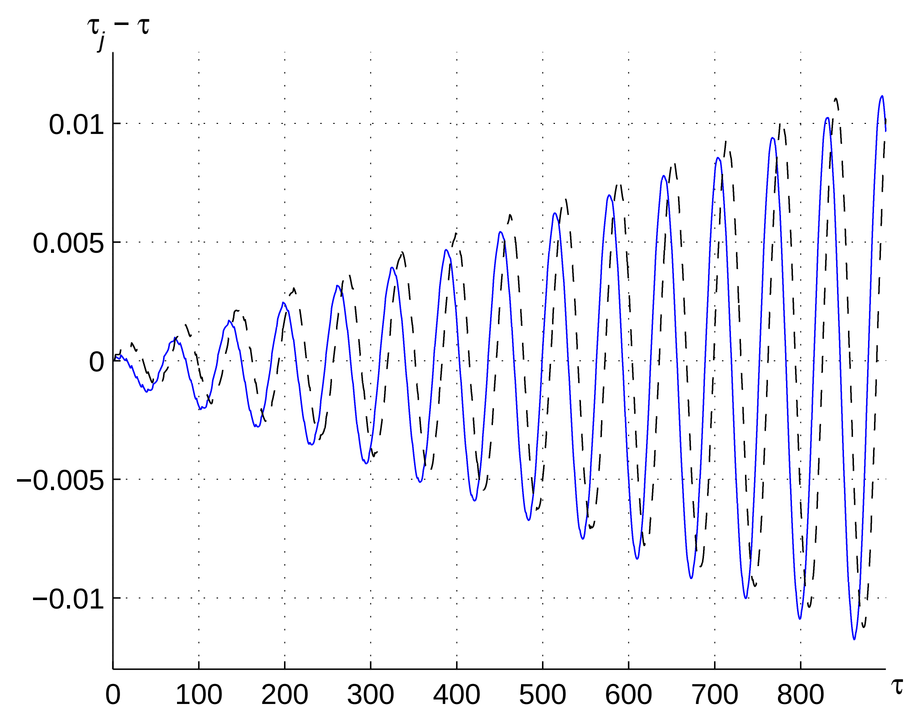
and $\tau_j(\tau) = \tau + \delta_j(\tau)$ $(j = 2, 3)$ substitute into dynamical equations.

Spectral equation of small oscillations in the rotational plane for $m_1 = m_2 = m_3$ is

$$(\xi^2 - \theta^2)\left(\frac{\xi^2 - q}{2Q\xi} + \tan \pi\xi\right)\left(\frac{\xi^2 - q}{2Q\xi} - \cot \pi\xi\right) = 0.$$

Here $Q = \dfrac{\theta v_1}{\sqrt{1 - v_1^2}}$, $q = \theta^2 \dfrac{1 + v_1^2}{1 - v_1^2}$.

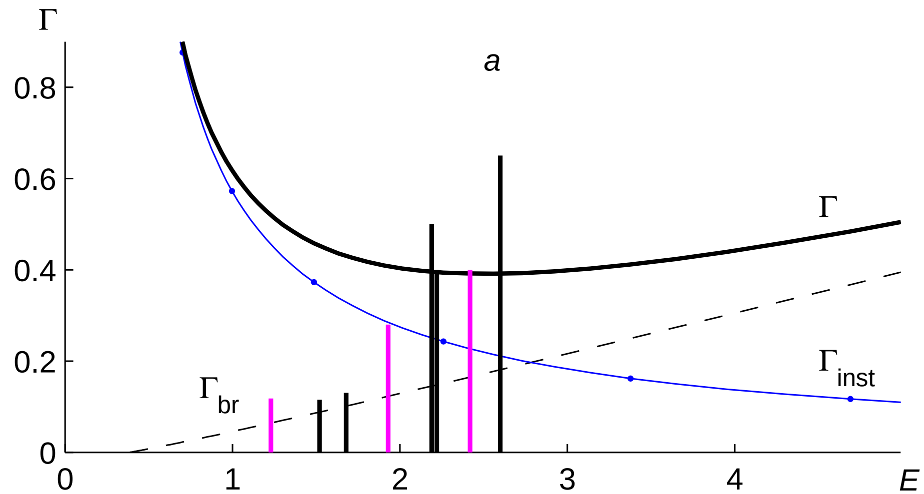




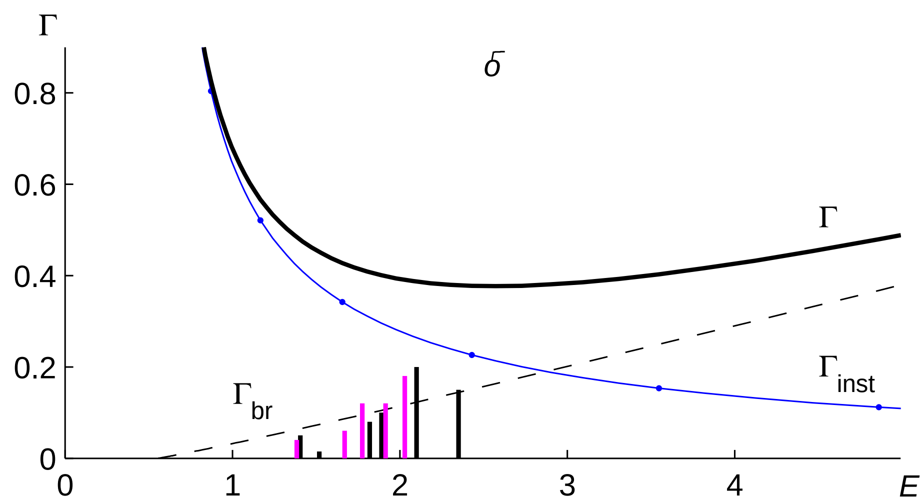
Rotational instability and hadron's width

For the linear model q-q-q width of a hadron state

$$\Gamma = \Gamma_{br} + \Gamma_{inst}, \qquad \Gamma_{br} \simeq 0.1 \cdot E_{str} = 0.1\frac{\pi\gamma\theta}{\Omega}, \qquad \Gamma_{inst} \simeq \frac{\xi_2^*}{a_0} = \frac{\xi_2^*\Omega}{\theta}.$$



for N and Δ baryons,



for Λ and Σ baryons

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