Braided Hopf Algebras as the Framework for Logarithmic Conformal Field Theories

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LCFT are used in describing:

- Two-dimensional percolation and self-avoiding walks (central charge $c = 0$). Disordered critical points.

  J. Cardy, Logarithmic correlations in quenched random magnets and polymers, cond-mat/9911024.

  V. Gurarie and A.W.W. Ludwig, Conformal Field Theory at central charge $c = 0$ and two-dimensional critical systems with quenched disorder, hep-th/0409105.

- Transitions between plateaux in the integer quantum Hall effect or spin quantum Hall effect.

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- AdS/CFT correspondence (massless limit of nonlinear sigma models with non-compact target spaces).

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Let $L_0 \sim z\frac{\partial}{\partial z}$ act nondiagonally:

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Solution:

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Typical feature of LCFTs: extended chiral algebras

- Gaberdiel–Kausch model and its $(1,p)$ generalizations.
  $p = 3$:

$$W^- = e^{-\sqrt{6}\varphi},$$

$$W^0 = \frac{1}{2}\,\partial^3\varphi\,\partial^2\varphi + \frac{1}{4}\,\partial^4\varphi\,\partial\varphi + \frac{3}{2}\sqrt{\frac{3}{2}}\,\partial^2\varphi\,\partial^2\varphi\,\partial\varphi + \sqrt{\frac{3}{2}}\,\partial^3\varphi\,\partial\varphi\,\partial\varphi \\ + 3\,\partial^2\varphi\,\partial\varphi\,\partial\varphi\,\partial\varphi + \frac{3}{5}\sqrt{\frac{3}{2}}\,\partial\varphi\,\partial\varphi\,\partial\varphi\,\partial\varphi\,\partial\varphi + \frac{1}{20\sqrt{6}}\,\partial^5\varphi,$$

and

$$W^+ = \Big(-\sqrt{\frac{3}{2}}\,\partial^4\varphi - 39\,\partial^2\varphi\,\partial^2\varphi + 18\,\partial^3\varphi\,\partial\varphi \\ + 12\sqrt{6}\,\partial^2\varphi\,\partial\varphi\,\partial\varphi - 18\,\partial\varphi\,\partial\varphi\,\partial\varphi\,\partial\varphi\Big)e^{\sqrt{6}\varphi}$$

Typical feature of LCFTs: extended chiral algebras

- The (p', p) series of models (FGST).

The simplest case $(2, 3)$:

Recent progress in understanding the (2,3) model:

Gaberdiel–Runkel–Wood

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$$\begin{aligned}
W^{+} = \Big(& \tfrac{35}{27}\left(\partial^4\varphi\right)^2 + \tfrac{56}{27}\,\partial^5\varphi\,\partial^3\varphi + \tfrac{28}{27}\,\partial^6\varphi\,\partial^2\varphi + \tfrac{8}{27}\,\partial^7\varphi\,\partial\varphi - \tfrac{280}{9\sqrt{3}}\left(\partial^3\varphi\right)^2\partial^2\varphi \\
& - \tfrac{70}{3\sqrt{3}}\,\partial^4\varphi\left(\partial^2\varphi\right)^2 - \tfrac{280}{9\sqrt{3}}\,\partial^4\varphi\,\partial^3\varphi\,\partial\varphi - \tfrac{56}{3\sqrt{3}}\,\partial^5\varphi\,\partial^2\varphi\,\partial\varphi - \tfrac{28}{9\sqrt{3}}\,\partial^6\varphi\left(\partial\varphi\right)^2 \\
& + \tfrac{35}{3}\left(\partial^2\varphi\right)^4 + \tfrac{280}{3}\,\partial^3\varphi\left(\partial^2\varphi\right)^2\partial\varphi + \tfrac{280}{9}\left(\partial^3\varphi\right)^2\left(\partial\varphi\right)^2 + \tfrac{140}{3}\,\partial^4\varphi\,\partial^2\varphi\left(\partial\varphi\right)^2 \\
& + \tfrac{56}{9}\,\partial^5\varphi\left(\partial\varphi\right)^3 - \tfrac{140}{\sqrt{3}}\left(\partial^2\varphi\right)^3\left(\partial\varphi\right)^2 - \tfrac{560}{3\sqrt{3}}\,\partial^3\varphi\,\partial^2\varphi\left(\partial\varphi\right)^2 - \tfrac{70}{3\sqrt{3}}\,\partial^4\varphi\left(\partial\varphi\right)^4 \\
& + 70\left(\partial^2\varphi\right)^2\left(\partial\varphi\right)^4 + \tfrac{56}{3}\,\partial^3\varphi\left(\partial\varphi\right)^5 - \tfrac{28}{\sqrt{3}}\,\partial^2\varphi\left(\partial\varphi\right)^6 + \left(\partial\varphi\right)^8 - \tfrac{1}{27\sqrt{3}}\,\partial^8\varphi\Big)e^{2\sqrt{3}\varphi}
\end{aligned}$$

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\begin{aligned}
W^{-} = \Big(&\tfrac{217}{192}\left(\partial^5\varphi\right)^2 - \tfrac{2653}{3456}\,\partial^6\varphi\,\partial^4\varphi - \tfrac{23}{384}\,\partial^7\varphi\,\partial^3\varphi - \tfrac{11}{1152}\,\partial^8\varphi\,\partial^2\varphi - \tfrac{1}{768}\,\partial^9\varphi\,\partial\varphi - \tfrac{1225}{64\sqrt{3}}\,\partial^4\varphi\left(\partial^3\varphi\right)^2 \\
&- \tfrac{13475}{576\sqrt{3}}\left(\partial^4\varphi\right)^2\partial^2\varphi + \tfrac{2695}{64\sqrt{3}}\,\partial^5\varphi\,\partial^3\varphi\,\partial^2\varphi + \tfrac{2555}{192\sqrt{3}}\,\partial^5\varphi\,\partial^4\varphi\,\partial\varphi - \tfrac{2891}{576\sqrt{3}}\,\partial^6\varphi\left(\partial^2\varphi\right)^2 - \tfrac{1351}{192\sqrt{3}}\,\partial^6\varphi\,\partial^3\varphi\,\partial\varphi \\
&- \tfrac{103}{192\sqrt{3}}\,\partial^7\varphi\,\partial^2\varphi\,\partial\varphi - \tfrac{13}{384\sqrt{3}}\,\partial^8\varphi\left(\partial\varphi\right)^2 + \tfrac{3535}{32}\left(\partial^3\varphi\right)^2\left(\partial^2\varphi\right)^2 - \tfrac{735}{16}\left(\partial^3\varphi\right)^3\partial\varphi - \tfrac{3395}{54}\,\partial^4\varphi\left(\partial^2\varphi\right)^3 \\
&+ \tfrac{245}{24}\,\partial^4\varphi\,\partial^3\varphi\,\partial^2\varphi\,\partial\varphi + \tfrac{12635}{576}\left(\partial^4\varphi\right)^2\left(\partial\varphi\right)^2 + \tfrac{245}{12}\,\partial^5\varphi\left(\partial^2\varphi\right)^2\partial\varphi + \tfrac{105}{32}\,\partial^5\varphi\,\partial^3\varphi\left(\partial\varphi\right)^2 \\
&- \tfrac{2443}{288}\,\partial^6\varphi\,\partial^2\varphi\left(\partial\varphi\right)^2 - \tfrac{19}{96}\,\partial^7\varphi\left(\partial\varphi\right)^3 - \tfrac{13405}{144\sqrt{3}}\left(\partial^2\varphi\right)^5 + \tfrac{8225}{24\sqrt{3}}\,\partial^3\varphi\left(\partial^2\varphi\right)^3\partial\varphi - \tfrac{105\sqrt{3}}{4}\left(\partial^3\varphi\right)^2\partial^2\varphi\left(\partial\varphi\right)^2 \\
&+ \tfrac{665}{24\sqrt{3}}\,\partial^4\varphi\left(\partial^2\varphi\right)^2\left(\partial\varphi\right)^2 + \tfrac{245}{2\sqrt{3}}\,\partial^4\varphi\,\partial^3\varphi\left(\partial\varphi\right)^3 - \tfrac{245}{8\sqrt{3}}\,\partial^5\varphi\,\partial^2\varphi\left(\partial\varphi\right)^3 - \tfrac{91}{24\sqrt{3}}\,\partial^6\varphi\left(\partial\varphi\right)^4 \\
&+ \tfrac{16205}{144}\left(\partial^2\varphi\right)^4\left(\partial\varphi\right)^2 + \tfrac{385}{4}\,\partial^3\varphi\left(\partial^2\varphi\right)^2\left(\partial\varphi\right)^3 + \tfrac{525}{8}\left(\partial^3\varphi\right)^2\left(\partial\varphi\right)^4 + \tfrac{35}{3}\,\partial^4\varphi\,\partial^2\varphi\left(\partial\varphi\right)^4 - 7\,\partial^5\varphi\left(\partial\varphi\right)^5 \\
&+ \tfrac{665}{3\sqrt{3}}\left(\partial^2\varphi\right)^3\left(\partial\varphi\right)^4 + \tfrac{105\sqrt{3}}{2}\,\partial^3\varphi\,\partial^2\varphi\left(\partial\varphi\right)^5 - \tfrac{35}{3\sqrt{3}}\,\partial^4\varphi\left(\partial\varphi\right)^6 + \tfrac{455}{6}\left(\partial^2\varphi\right)^2\left(\partial\varphi\right)^6 + 5\,\partial^3\varphi\left(\partial\varphi\right)^7 \\
&+ \tfrac{25}{\sqrt{3}}\,\partial^2\varphi\left(\partial\varphi\right)^8 + \left(\partial\varphi\right)^{10} - \tfrac{1}{13824\sqrt{3}}\,\partial^{10}\varphi\Big)e^{-2\sqrt{3}\varphi}
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Long-lasting desire:

Find a dual, algebraic description:

identfy algebraic objects that capture essential pieces of LCFT models.

Feigin, Gainutdinov, AS, Tipunin,
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More recent:

AS, Tipunin 1101.5810; AS 1109.1767, 1109.5919.

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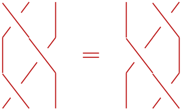

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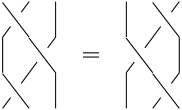

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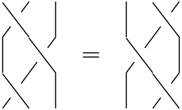

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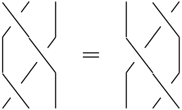

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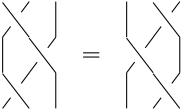

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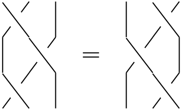

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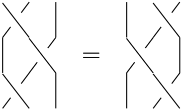

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Nichols algebras

Examples:

- q-deformed root systems at roots of unity (Lusztig's book).

- And many more.

Alternative description (Woronowicz):

$$\mathfrak{B}(X) = \bigoplus_{n\geqslant 0} X^{\otimes n}/\ker(\mathfrak{S}_n),$$

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Particular cases:

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- $\Psi : X \otimes X \to X \otimes X$ such that

$$x_i \otimes x_j \mapsto q_{ij}\, x_j \otimes x_i.$$

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Bold conjecture in extreme form:

- Every finite-dimensional Nichols algebra with diagonal braiding corresponds to a Logarithmic CFT.
- The representation category of the extended symmetry algebra reralized in a LogCFT model is equivalent to the category of Yetter–Drinfeld $\mathfrak{B}(X)$-modules.

Plan of the talk:

1. From LogCFT to Nichols algebras
2. and back.

Conclusions:

- Each LogCFT is naturally mapped into a Nichols algebra.
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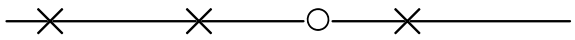
From LogCFT to Nichols algebras

- dressed/screened vertex operators



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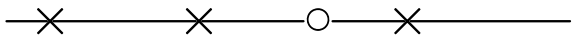

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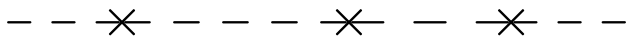

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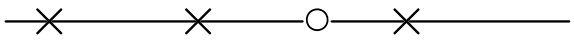

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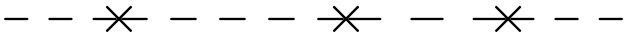

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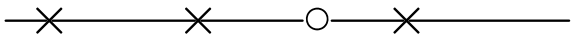 $= X \otimes X \otimes Y \otimes X$

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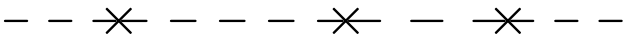 $= X \otimes X \otimes X,$

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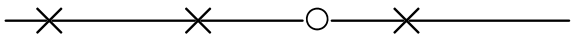 $= X \otimes X \otimes Y \otimes X$

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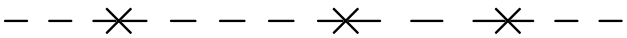 $= X \otimes X \otimes X,$

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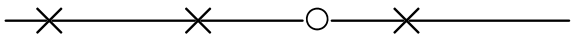

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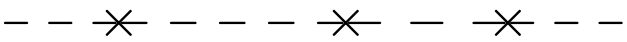

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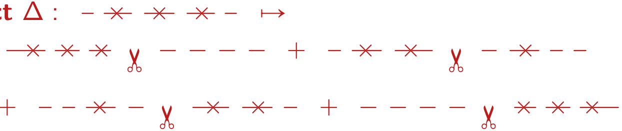

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The Nichols algebra $\mathfrak{B}(X)$ is whatever is generated by single crosses.

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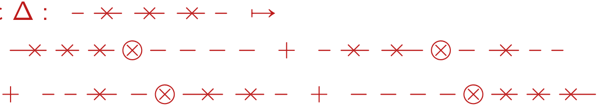

meaning $X^{\otimes n} \to \bigoplus_{i=0}^{n} X^{\otimes i} \otimes X^{\otimes (n-i)}$

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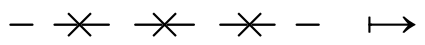
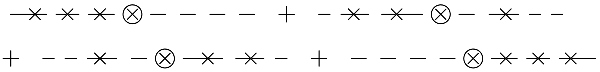
- **Coproduct** Δ : 



meaning $X^{\otimes n} \to \bigoplus_{i=0}^{n} X^{\otimes i} \otimes X^{\otimes(n-i)}$

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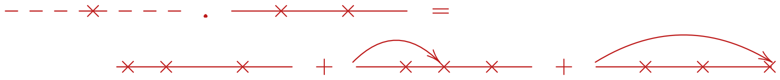

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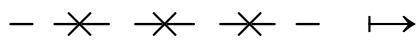
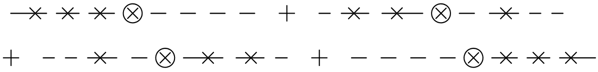
- **Coproduct** Δ : 



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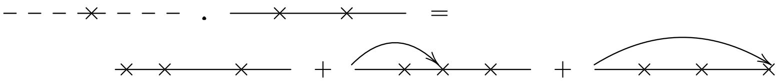
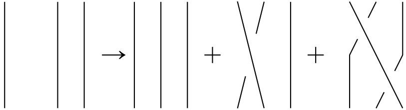


or in alternative notation

 $= (\mathrm{id} + \Psi_1 + \Psi_2\Psi_1)(X \otimes X \otimes X)$,

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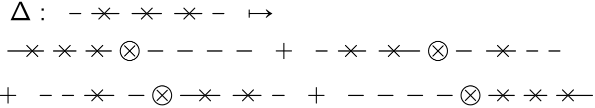

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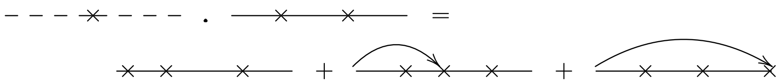

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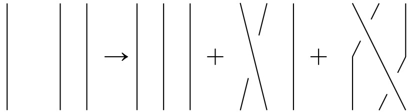

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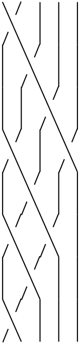
Antipode $\mathcal{S} : \mathfrak{B}(X) \to \mathfrak{B}(X)$

The antipode acts by half-twist,

e.g., $\mathcal{S}_5 : X^{\otimes 5} \to X^{\otimes 5}$ is

$\mathcal{S}_5 = -$ 

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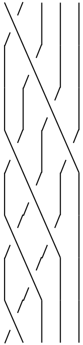
$\mathcal{S}_5 = -$ 

All braided Hopf algebra axioms are satisfied.

The Nichols algebra of screenings: (co)modules

Hopf bimodules of $\mathfrak{B}(X)$ are spanned by 

$\mathfrak{B}(X)$ action and coaction:

- left action is

$= (\mathrm{id} + \Psi_1 + \Psi_2\Psi_1)(X \otimes Y \otimes X),$

- right action similarly
- left action: again by deconcatenation

$\delta_L :$

- right coaction: similarly.

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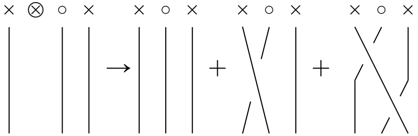

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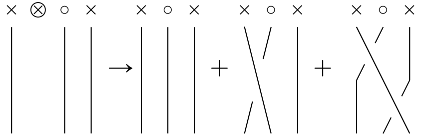

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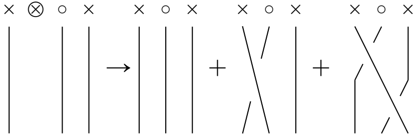

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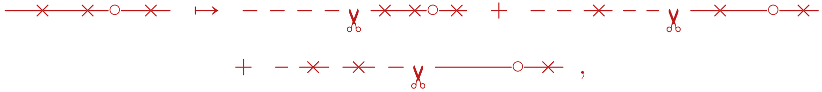

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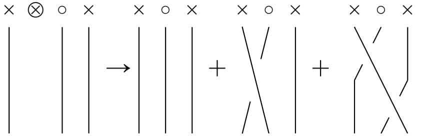

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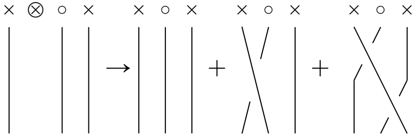

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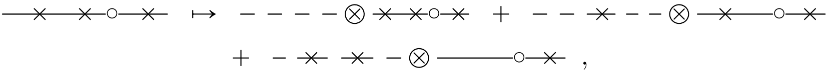
δ_{L} :  ,

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Left–left Yetter–Drinfeld modules are right coinvariants in Hopf bimodules:

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They carry the left adjoint action and satisfy the axiom

Graphic notation:

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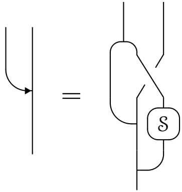

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product coproduct left action left coaction right action right coaction

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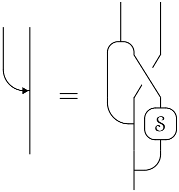

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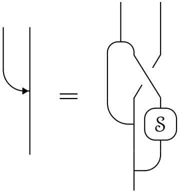

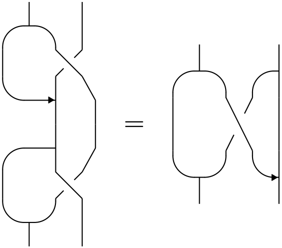

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product | coproduct | left action | left coaction | right action | right coaction

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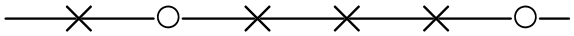
More general Yetter–Drinfeld $\mathfrak{B}(X)$-modules:

—multivertex modules, e.g.,



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Summary:

Given a braided vector space X ("screenings"), we define

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Given a braided vector space X ("screenings") and another braided vector space Y ("vertex operators"), we define

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Rank-1 Nichols algebra $\mathfrak{B}_p$

Primitive root of unity $\mathfrak{q} = e^{\frac{i\pi}{p}}, \quad p \geqslant 2.$

$\mathfrak{B}_p$ is linearly spanned by

$$F(r) = \;-\!\times\!-\!\times\;\times\!- \qquad (r \text{ crosses}), \quad 0 \leqslant r \leqslant p-1,$$

with braiding $\quad \Psi(F(r) \otimes F(s)) = \mathfrak{q}^{2rs} F(s) \otimes F(r).$

Product:

$$F(r)\,F(s) = \left\langle {r+s \atop r} \right\rangle F(r+s),$$

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Coproduct:

by deconcatenation.

Vertices:

$$V^a \text{ with braiding} \quad \Psi(V^a \otimes V^b) = \mathfrak{q}^{\frac{ab}{2}} V^b \otimes V^a.$$

Then category equivalence follows, 1109.5919

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From Nichols algebras to LogCFTs

Diagonal braiding, $x_i \otimes x_j \mapsto q_{ij}\, x_j \otimes x_i$, $1 \leqslant i,j \leqslant \theta$.

1. Construct screenings in θ-boson representation:

$$F_j \equiv F_{\alpha_j} = \oint e^{\alpha_j.\varphi} \text{ with } \quad \begin{aligned} e^{i\pi\alpha_j.\alpha_j} &= q_{j,j}, \\ e^{2i\pi\alpha_k.\alpha_j} &= q_{k,j}q_{j,k}, \quad k \neq j, \end{aligned}$$

and find the Virasoro algebra $\quad T_\xi(z) = \frac{1}{2}\partial\varphi(z).\partial\varphi(z) + \xi.\partial^2\varphi(z)$
such that the F_j indeed have dimension 1.

2. Recall the generalized Cartan matrix $(a_{i,j})_{1\leqslant i,j\leqslant\theta}$,

with $a_{i,i} = 2$ and

$$q_{i,i}^{a_{i,j}} = q_{i,j}q_{j,i} \text{ or } q_{i,i}^{1-a_{i,j}} = 1 \text{ for each pair } i \neq j,$$

3. Impose conditions on scalar products:

$$a_{i,j}\alpha_i.\alpha_i = 2\alpha_i.\alpha_j \quad \text{or} \quad (1-a_{i,j})\alpha_i.\alpha_i = 2$$

$\Rightarrow$ Virasoro central charge is invariant under Weyl groupoid action

$$\mathfrak{R}^{(k)}(q_{i,j}) = q_{i,j}q_{i,k}^{-a_{k,j}}q_{k,j}^{-a_{k,i}}q_{k,k}^{a_{k,i}a_{k,j}}.$$

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From Nichols algebras to LogCFTs

Diagonal braiding, $x_i \otimes x_j \mapsto q_{ij}\, x_j \otimes x_i$, $1 \leqslant i, j \leqslant \theta$.

1. Construct screenings in θ-boson representation:

$$F_j \equiv F_{\alpha_j} = \oint e^{\alpha_j.\varphi} \text{ with } \quad \begin{aligned} e^{i\pi\alpha_j.\alpha_j} &= q_{j,j}, \\ e^{2i\pi\alpha_k.\alpha_j} &= q_{k,j}q_{j,k}, \quad k \neq j, \end{aligned}$$

and find the Virasoro algebra $\quad T_\xi(z) = \frac{1}{2}\partial\varphi(z).\partial\varphi(z) + \xi.\partial^2\varphi(z)$
such that the F_j indeed have dimension 1.

2. Recall the generalized Cartan matrix $(a_{i,j})_{1\leqslant i,j\leqslant\theta}$,

with $a_{i,i} = 2$ and

$$q_{i,i}^{a_{i,j}} = q_{i,j}q_{j,i} \text{ or } q_{i,i}^{1-a_{i,j}} = 1 \text{ for each pair } i \neq j,$$

3. Impose conditions on scalar products:

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Nichols algebras → LogCFT beyond Virasoro. Rank 2

- A list of 20+ entries (Heckenberger)
- The braiding matrix:
  $$(q_{i,j}) = \begin{pmatrix} e^{\frac{2i\pi}{p}} & (-1)^j e^{-\frac{i\pi}{p}} \\ (-1)^j e^{-\frac{i\pi}{p}} & e^{\frac{2i\pi}{p}} \end{pmatrix}.$$
- Let $|p| \geqslant 3 \iff$ none of the screenings is fermionic.
  Then necessarily $m = n = 0$ and the Virasoro central charge is
  $c = 50 - \dfrac{24}{k+3} - 24(k+3)$, where $k + 3 = \frac{1}{p} - j$.
- Presentation of the Nichols algebra:
  $$\mathfrak{B}(X) = T(X)/\big([F_1, F_1, F_2],\ [F_1, F_2, F_2],\ F_1^p,\ [F_1, F_2]^p,\ F_2^p\big)$$
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  $$q_{12}q_{21}q_{22} = 1, \quad q_{11}q_{12}q_{21} = 1, \quad q_{12}q_{21} \text{ is a } p\text{th root of unity.}$$

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$$W_3 = \partial\varphi_\alpha\partial\varphi_\alpha\partial\varphi_\alpha + \frac{3}{2}\partial\varphi_\alpha\partial\varphi_\alpha\partial\varphi_\beta - \frac{3}{2}\partial\varphi_\alpha\partial\varphi_\beta\partial\varphi_\beta - \partial\varphi_\beta\partial\varphi_\beta\partial\varphi_\beta$$

$$-\frac{9(p-1)}{2p}\partial^2\varphi_\alpha\partial\varphi_\alpha - \frac{9(p-1)}{4p}\partial^2\varphi_\alpha\partial\varphi_\beta + \frac{9(p-1)}{4p}\partial^2\varphi_\beta\partial\varphi_\alpha + \frac{9(p-1)}{2p}\partial^2\varphi_\beta\partial\varphi_\beta$$

$$+ \frac{9(p-1)^2}{4p^2}\partial^3\varphi_\alpha - \frac{9(p-1)^2}{4p^2}\partial^3\varphi_\beta.$$

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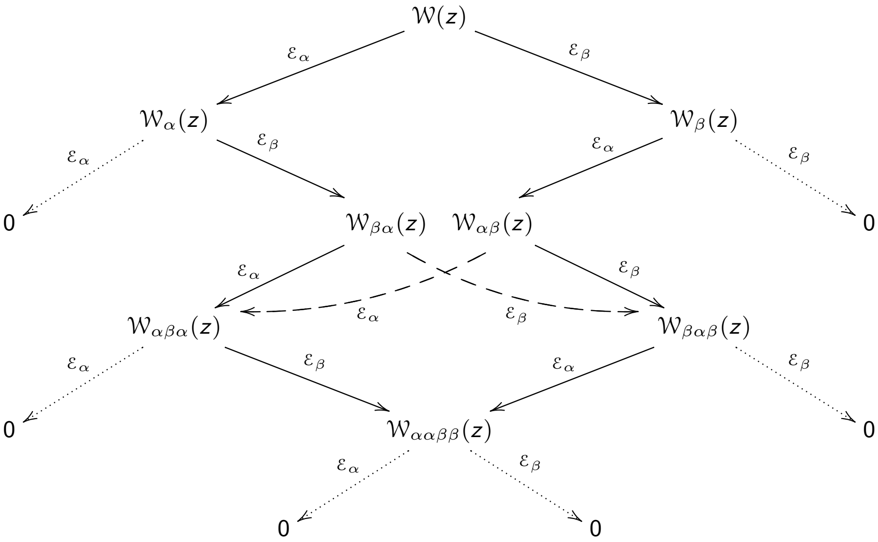
  They produce an *octuplet* structure similar to Gaberdiel–Kausch's *triplet* structure.

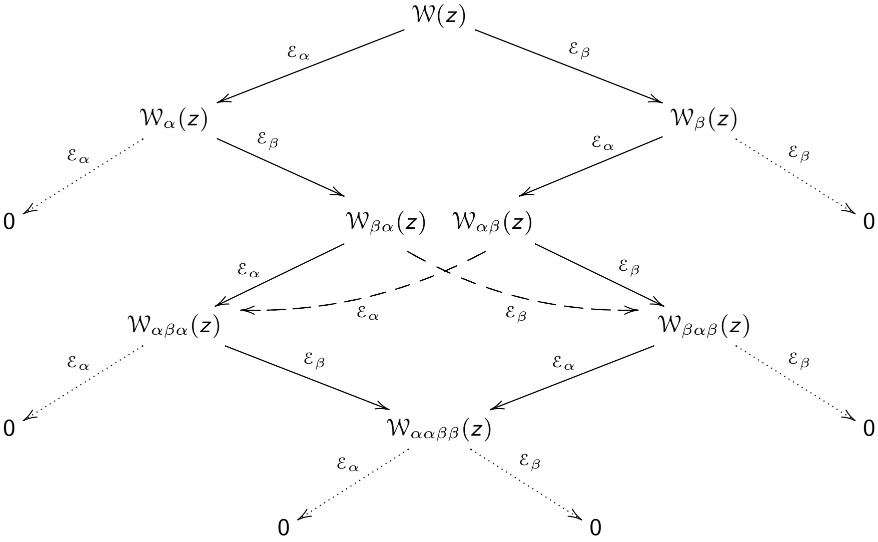
The octuplet algebra

$\mathcal{W}(z) = e^{(p\alpha+p\beta).\varphi(z)}$ is a W_3-primary with conformal dimension $3p-2$.



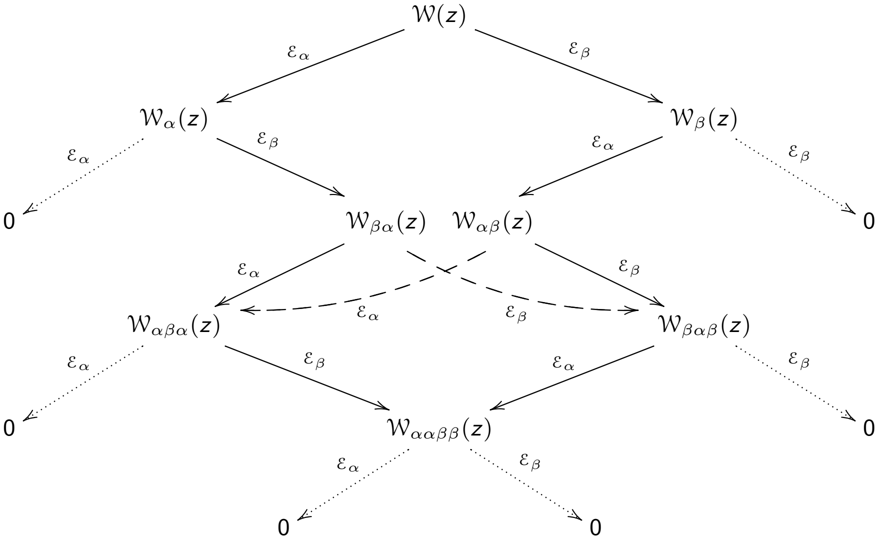
The octuplet algebra

Other elements of the ideal are also hidden here.



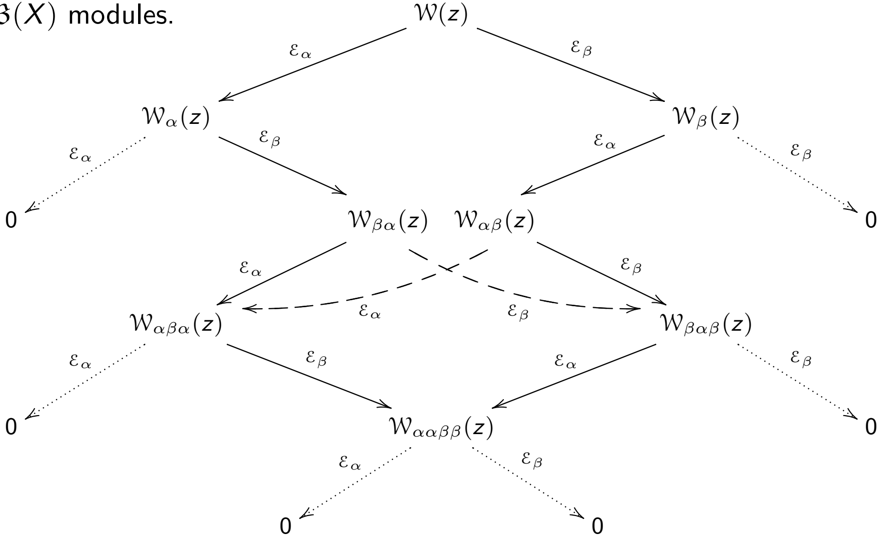
The octuplet algebra

A “logarithmic” extension of the W_3 algebra



The octuplet algebra

A "logarithmic" extension of the W_3 algebra, whose representation category is conjecturally equivalent to the category of Yetter–Drinfeld $\mathfrak{B}(X)$ modules.



Conclusions:

- **Each LogCFT is naturally mapped into a Nichols algebra.**
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How much of the LogCFT content can be extracted from Nichols algebras:

- The spectrum of primary fields
- The space of torus amplitudes
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2. The space of torus amplitudes (center of ${}^{\mathfrak{B}(X)}_{\mathfrak{B}(X)}\mathcal{YD}$).
3. Projective module structure (projectives in ${}^{\mathfrak{B}(X)}_{\mathfrak{B}(X)}\mathcal{YD}$).
4. Modular transformations of characters and pseudocharacters (Lyubashenko's mapping class group action).
5. Fusion (monoidal structure of ${}^{\mathfrak{B}(X)}_{\mathfrak{B}(X)}\mathcal{YD}$).

Conclusions:

- **Each LogCFT is naturally mapped into a Nichols algebra.**
- **In the simplest cases studied, the representation categories are equivalent.**
- **First steps of the reconstruction Nichols → LogCFT are quite encouraging.**

Realistic (or semi-realistic) prospects.

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Thank you.