Dephasing of Cooper pairs and subgap electron transport in superconducting hybrids

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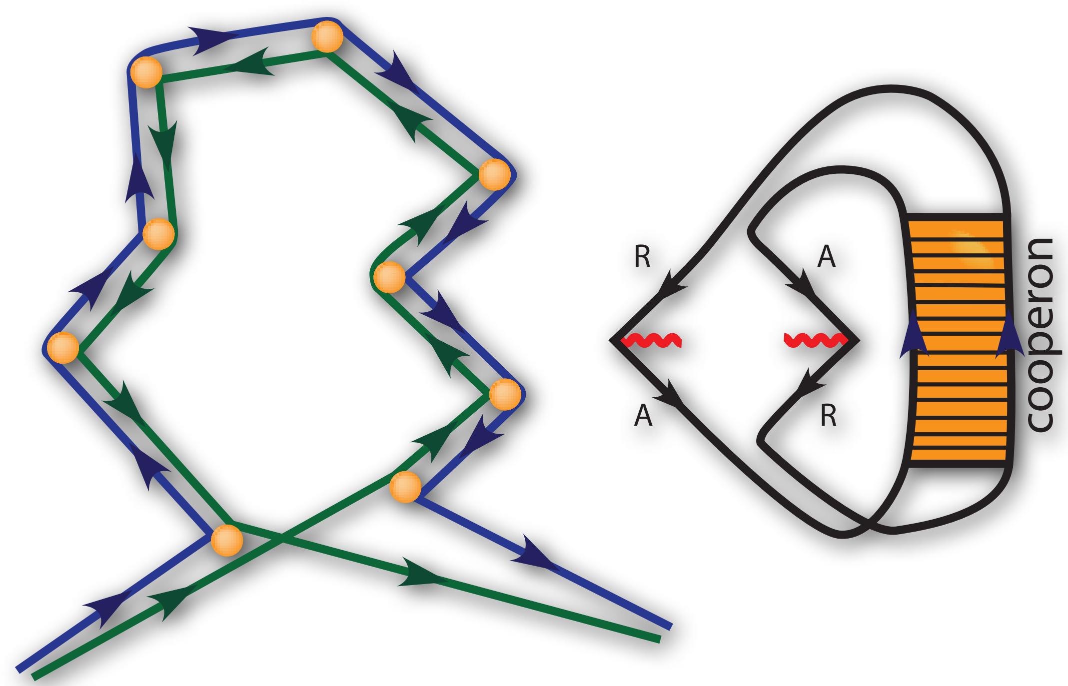
Outline

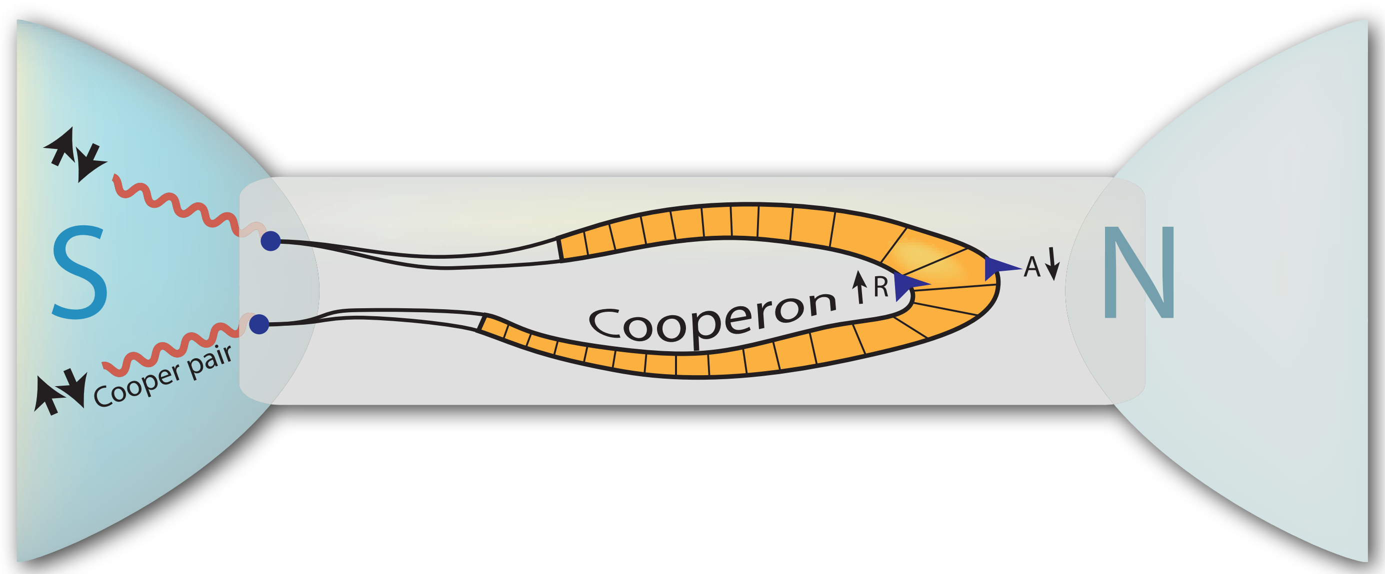
- Introduction
- Weak localization
- Superconducting hybrids
- Cooperon dephasing and spin structure
- Quasi-1d SN hybrid
- Conclusions

Weak localization correction



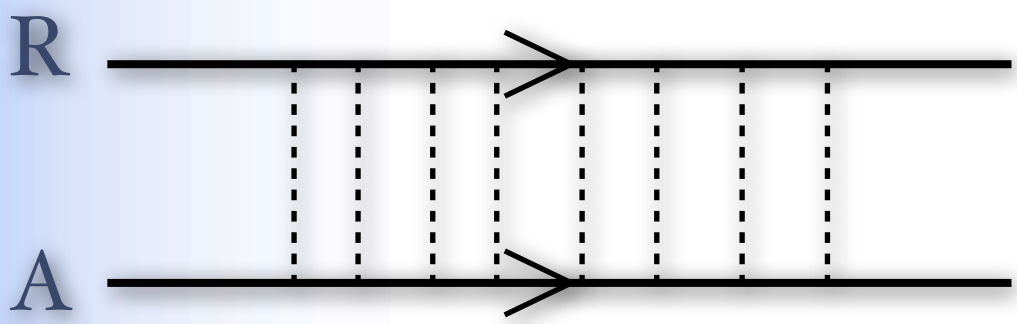
$$\frac{\delta\sigma(\omega)}{\sigma_0} = -\frac{1}{\pi\nu}\Re\int\frac{d^d\mathbf{p}}{(2\pi)^d}\frac{1}{D\mathbf{p}^2 - i\omega}$$

Andreev current in SN structure



$$I = \frac{\pi T}{2\nu e^3 (R_I \Gamma)^2} \int_\Gamma d^2\mathbf{r} d^2\mathbf{r}' \int d\tau \mathrm{Im} \frac{\mathcal{C}(\mathbf{r},\mathbf{r}';\tau) e^{ieV\tau}}{\sinh(\pi T \tau)},$$

Cooperon



$$\sim \frac{1}{D\mathbf{p}^2 - i\omega + 1/\tau_\phi}$$

Dephasing time

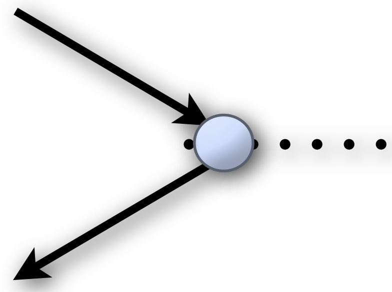
$$L_\phi = \sqrt{D\tau_\phi}$$

Only trajectories with length smaller than this scale contribute to the Cooperon.

Cooperon and interactions

Keldysh diagram technique

$$\hat{G}(\mathbf{r},\mathbf{r}';\varepsilon) = \begin{pmatrix} G^R(\mathbf{r},\mathbf{r}';\varepsilon) & G^K(\mathbf{r},\mathbf{r}';\varepsilon) \\ 0 & G^A(\mathbf{r},\mathbf{r}';\varepsilon) \end{pmatrix}$$

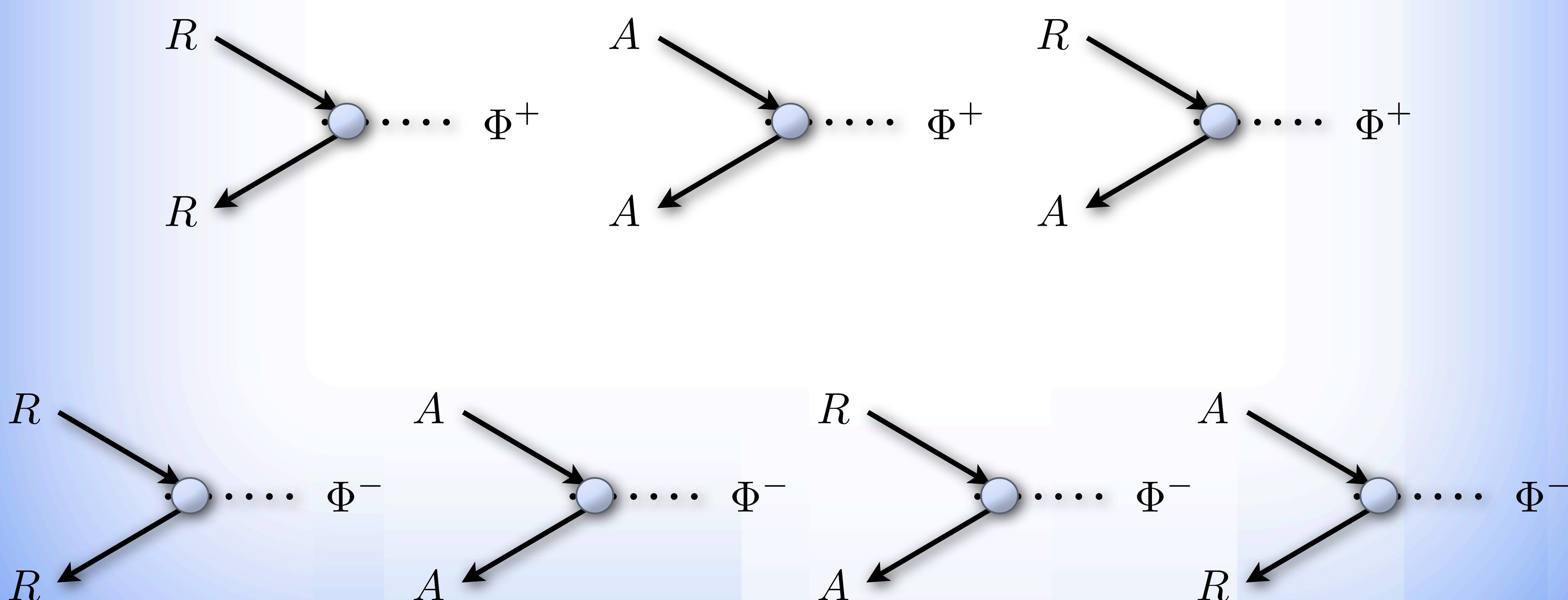


$$\Phi^{\pm} = \frac{\Phi^F \pm \Phi^B}{\sqrt{2}}$$

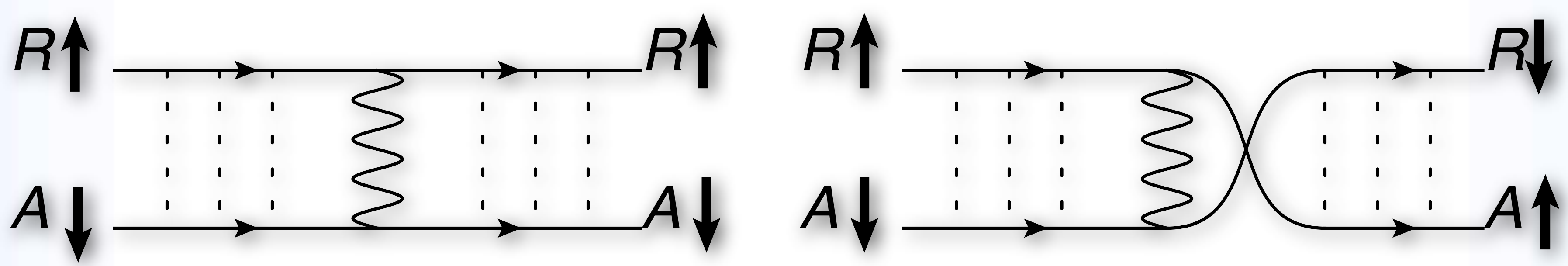
$$\hat{\gamma}^{+} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \hat{\gamma}^{-} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{G}(\mathbf{r},\mathbf{r}';\varepsilon) = \begin{pmatrix} 1 & F_\varepsilon \\ 0 & -1 \end{pmatrix} \begin{pmatrix} G^R(\mathbf{r},\mathbf{r}';\varepsilon) & 0 \\ 0 & G^A(\mathbf{r},\mathbf{r}';\varepsilon) \end{pmatrix} \begin{pmatrix} 1 & F_\varepsilon \\ 0 & -1 \end{pmatrix}$$

Cooperon and interactions



Cooperon and interactions



$$\mathcal{C}_s \sim |\uparrow\uparrow\rangle,\ |\downarrow\downarrow\rangle, |\uparrow\uparrow\rangle,\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \quad \Rightarrow \quad \tau_\phi^s$$

$$\mathcal{C}_{as} \sim \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \quad \Rightarrow \quad \tau_\phi^{as}$$

τ_ϕ^{as} finite at zero temperature even in first order of perturbation theory

From the practical point of view it is sufficient to keep only Φ^+ field in estimation of τ_ϕ^{as}

Non-linear sigma-model

$$S_w[\check{Q}, \mathbf{A}, \Phi] = \frac{i\pi\nu}{4}\,\mathrm{Tr}[D(\check{\partial}\check{Q})^2 - 4\check{\Xi}\partial_t\check{Q} + 4i\check{\Phi}\check{Q}] - \frac{i\pi}{4e^2 R_I \Gamma}\,\mathrm{Tr}_\Gamma[\check{Q}_{\mathrm{sc}}, \check{Q}]$$

$$\check{\partial}\check{Q} = \partial_{\mathbf{r}}\check{Q} - i[\check{\Xi}\check{\mathbf{A}}, \check{Q}], \quad \check{\Xi} = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix}$$

$$\check{Q}^2 = \check{1}\delta(t - t')$$

$$\check{\Phi} = \begin{pmatrix} \Phi^+\hat{1} & \Phi^-\hat{1} \\ \Phi^-\hat{1} & \Phi^+\hat{1} \end{pmatrix}, \quad \check{\mathbf{A}} = \begin{pmatrix} \mathbf{A}^+\hat{1} & \mathbf{A}^-\hat{1} \\ \mathbf{A}^-\hat{1} & \mathbf{A}^+\hat{1} \end{pmatrix}$$

$$\check{Q}_{\mathrm{sc}}(t, t') = \begin{pmatrix} \hat{\sigma}_y & 0 \\ 0 & \hat{\sigma}_y \end{pmatrix} \delta(t - t')$$

Non-linear sigma-model: K-gauge

$$\check{Q}(\mathbf{r},t,t') \to e^{i\check{\Xi}\check{\mathcal{K}}(\mathbf{r},t)}\check{Q}(\mathbf{r},t,t')e^{-i\check{\Xi}\check{\mathcal{K}}(\mathbf{r},t')}$$

$$\check{Q}_N = \check{\mathcal{U}} \circ \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & -\hat{\sigma}_z \end{pmatrix} \check{\mathcal{U}}, \qquad \check{\mathcal{U}}(t-t') = \begin{pmatrix} \delta(t-t'-0)\hat{1} & -\frac{iT}{\sinh(\pi T(t-t'))}\hat{1} \\ 0 & -\delta(t-t'+0)\hat{1} \end{pmatrix}$$

$$\Phi(\mathbf{r},t) \to \Phi_{\mathcal{K}}(\mathbf{r},t) = \Phi(\mathbf{r},t) - \partial_t \mathcal{K}(\mathbf{r},t) \qquad \mathbf{A}(\mathbf{r},t) \to \mathbf{A}_{\mathcal{K}}(\mathbf{r},t) = \mathbf{A}(\mathbf{r},t) - \partial_{\mathbf{r}} \mathcal{K}(\mathbf{r},t)$$

In order to eliminate linear terms in both electromagnetic potentials and deviations from the N-metal saddle point the transformed electromagnetic potentials should satisfy

$$\Phi^+_{\mathcal{K}}(\mathbf{r},t) = D\partial_{\mathbf{r}}\mathbf{A}^+_{\mathcal{K}}(\mathbf{r},t) - 2iDT\int dt' \coth(\pi T(t-t'))\partial_{\mathbf{r}}\mathbf{A}^-_{\mathcal{K}}(\mathbf{r},t'),$$

$$\Phi^-_{\mathcal{K}}(\mathbf{r},t) = -D\partial_{\mathbf{r}}\mathbf{A}^-_{\mathcal{K}}(\mathbf{r},t)$$

Non-linear sigma-model

$$S_A = -\frac{i}{32}\left(\frac{\pi}{e^2 R_I \Gamma}\right)^2 \langle \mathrm{Tr}_\Gamma[\check{Q}_{\mathrm{sc}}, \check{Q}]\, \mathrm{Tr}_\Gamma[\check{Q}_{\mathrm{sc}}, \check{Q}]\rangle_Q$$

$$\check{Q} \approx \check{Q}_0 + i\check{Q}_0 \circ \mathcal{U} \circ \check{W} \circ \mathcal{U} - \frac{1}{2}\check{Q}_0 \circ \check{\mathcal{U}} \circ \check{W} \circ \check{W} \circ \check{\mathcal{U}}$$

$$\check{W} = \begin{pmatrix} 0 & c_1(\mathbf{r},t,t') & d_1(\mathbf{r},t,t') & 0 \\ \bar{c}_1(\mathbf{r},t',t) & 0 & 0 & d_2(\mathbf{r},t,t') \\ \bar{d}_1(\mathbf{r},t',t) & 0 & 0 & c_2(\mathbf{r},t,t') \\ 0 & \bar{d}_2(\mathbf{r},t',t) & \bar{c}_2(\mathbf{r},t',t) & 0 \end{pmatrix}$$

$$c_s(\mathbf{r},t,t') = \frac{c_1(\mathbf{r},t,t') + c_2(\mathbf{r},t',t)}{\sqrt{2}} \qquad c_{as}(\mathbf{r},t,t') = \frac{c_1(\mathbf{r},t,t') - c_2(\mathbf{r},t',t)}{\sqrt{2}}$$

Andreev subgap current

$$I = \frac{\pi T}{2\nu e^3 (R_I \Gamma)^2} \int_\Gamma d^2\mathbf{r} d^2\mathbf{r}' \int d\tau \,\mathrm{Im} \frac{\langle \mathcal{P}(\mathbf{r},\mathbf{r}',\tau;t) e^{ieV\tau}\rangle_\Phi}{\sinh(\pi T \tau)}$$

$$\mathcal{P}(\mathbf{r},\mathbf{r}',\tau;t) = \frac{\theta(\tau) e^{i\mathcal{K}^+(\mathbf{r},t-\tau) - i\mathcal{K}^+(\mathbf{r},t)}}{2}$$

$$\times \int_{\mathbf{x}(0)=\mathbf{r}'}^{\mathbf{x}(\tau)=\mathbf{r}} \mathcal{D}\mathbf{x} e^{-\int_0^\tau dt' \left(\frac{(\dot{\mathbf{x}}(t'))^2}{2D} - \frac{i}{2}(\Phi^+(\mathbf{x}(t'),t-(t'+\tau)/2) - \Phi^+(\mathbf{x}(t'),t+(t'-\tau)/2)) \right)}$$

$$\langle \mathcal{P}(\mathbf{r},\mathbf{r}',\tau;t)\rangle_\Phi = \mathcal{D}(\mathbf{r},\mathbf{r}';\tau) e^{-f(\mathbf{r},\mathbf{r}',\tau)}$$

Quasi-1d structure

$$G = \frac{\pi T}{4\nu e^2 R_I^2} \int_0^\infty d\tau^2 \frac{\mathcal{D}(0,0;\tau)\cos(eV\tau)}{\sinh(\pi T\tau)} e^{-f(0,0,\tau)}$$

$$\mathcal{D}(0,0;\tau) = \frac{1}{2La^2}\vartheta_2(0, e^{-\tau/\tau_D})$$

$$f(0,0,\tau) \simeq \frac{8}{g}\ln\left(\frac{\tau}{\tau_{RC}}\right) + \frac{\tau}{\tau_\phi^{as}} + \sqrt{\frac{\pi\tau\tau_c}{4\tau_\varphi^2}}\ln\left(\frac{\tau_c}{\tau}\right)$$

$g = 4\pi\nu Da^2/L \gg 1$ $\quad$ $\tau_{RC} = RC$ $\quad$ $\tau_D = 2L^2/(\pi^2 D)$ $\quad$ $\tau_\phi^{as} = 2\pi\nu a^2\sqrt{2D\tau_c}$

Quasi-1d structure

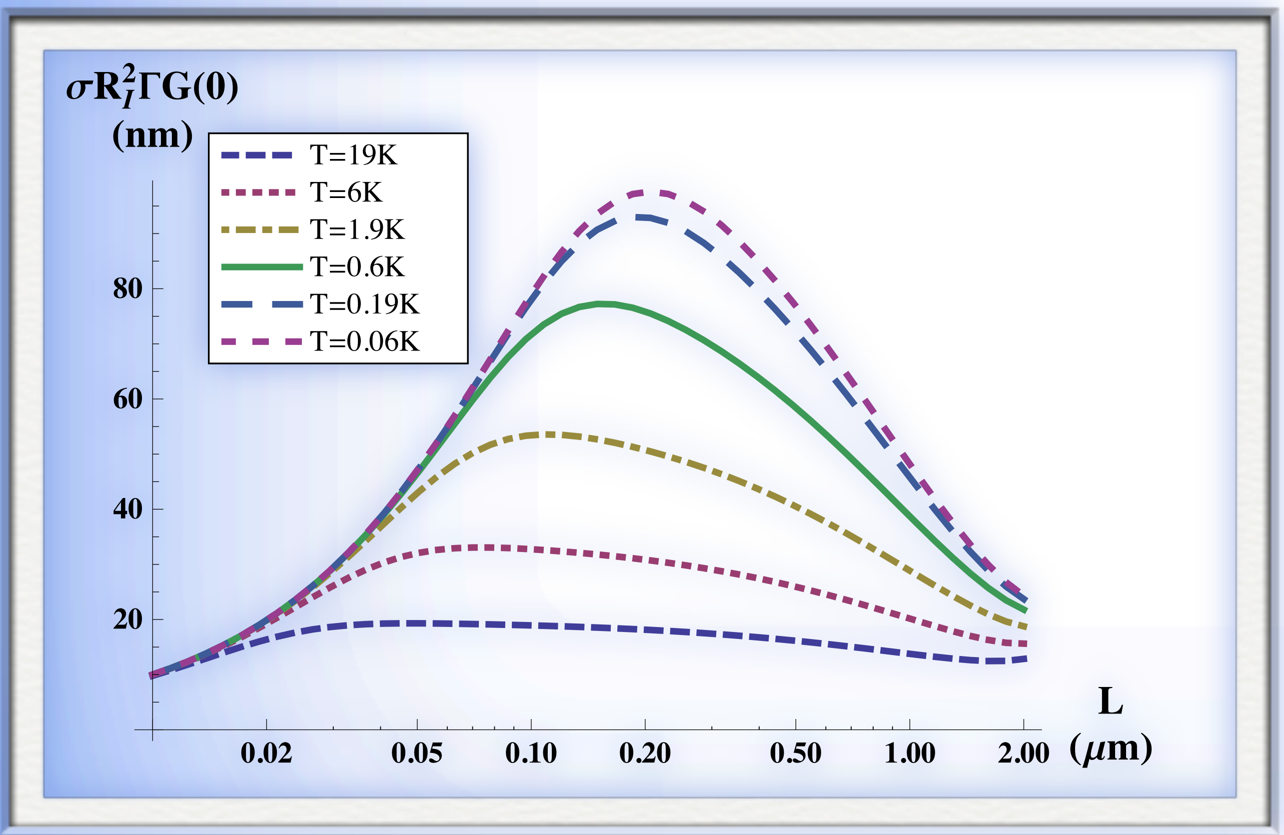
For the case of small voltages and $T \ll 1/\tau_\phi^{as}$ we obtained

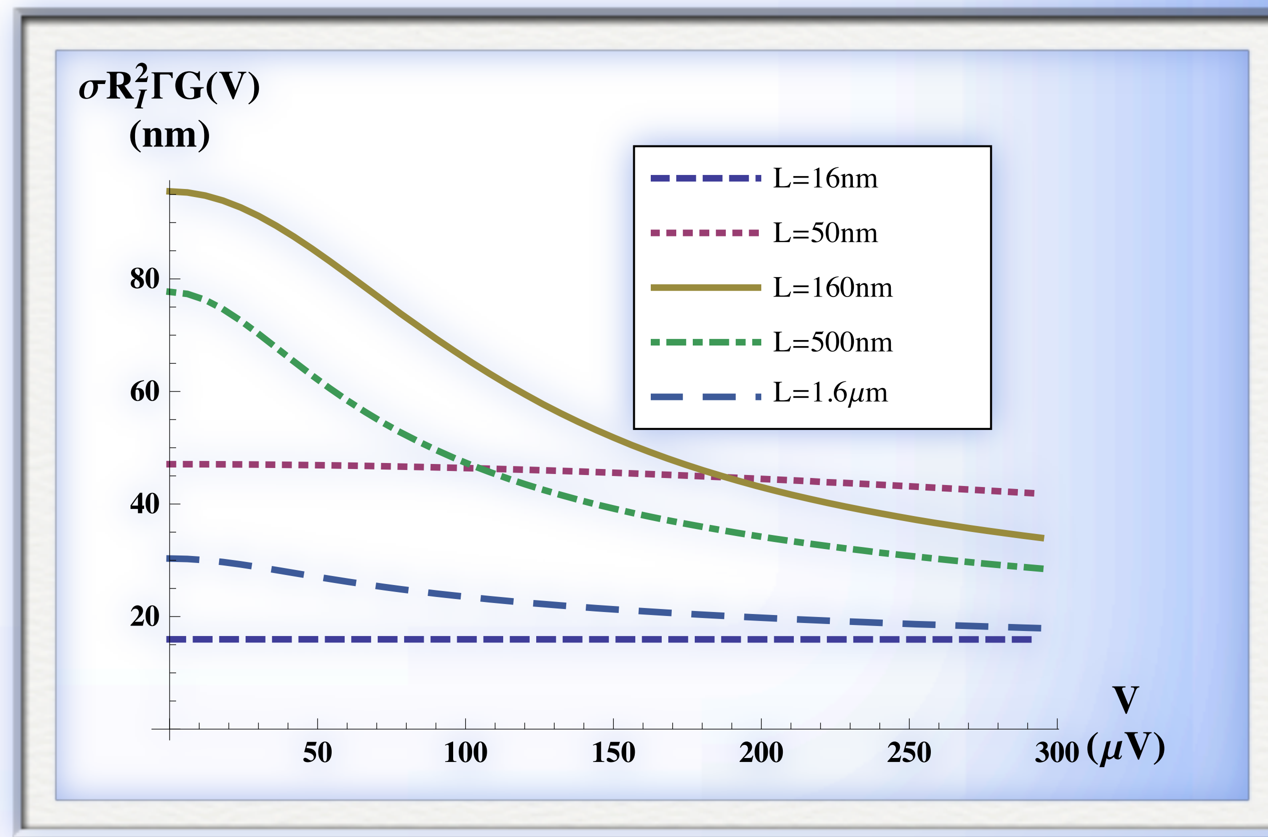
$$G(0) \simeq \begin{cases} \frac{1}{\sigma R_I^2 a^2} \left(\frac{4\tau_{RC}}{\tau_D}\right)^{8/g} \frac{2L\zeta\left(2-\frac{16}{g}\right)\left(2^{2-16/g}-1\right)}{\pi^2}, & L \ll L_\varphi, \\ \frac{1}{\sigma R_I^2 a^2} \frac{L_\varphi}{\sqrt{2\pi}} \left(\frac{4\tau_{RC}}{\tau_\phi^{as}}\right)^{8/g} \Gamma\left(\frac{1}{2}-\frac{8}{g}\right), & L \gg L_\varphi, \end{cases}$$

For the case of zero temperature and $L \gg L_\varphi = \sqrt{D\tau_\phi^{as}}$ the result is

$$G(V) \simeq \frac{1}{\sigma R_I^2 a^2} \frac{L_\varphi}{\sqrt{2\pi}} \left(\frac{4\tau_{RC}}{\tau_\varphi}\right)^{8/g} \mathrm{Re} \frac{\Gamma\left(\frac{1}{2}-\frac{8}{g}\right)}{(1+ieV\tau_\varphi)^{1/2-8/g}}$$

Quasi-1d structure





Conclusions

In conclusion, we have demonstrated that electron-electron interactions yield dephasing of Cooper pairs penetrating from a superconductor into a diffusive normal metal. At low T this phenomenon imposes fundamental limitations on the proximity effect in NS hybrids restricting the penetration length of superconducting correlations into the N-metal to a temperature independent value - dephasing length.This new length scale can be probed by measuring the subgap conductance in NS systems.

Thank you for your attention ☺