$\mathcal{N}=4$ SYM low-energy effective action in $\mathcal{N}=3$ and $\mathcal{N}=4$ harmonic superspaces

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References

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Low-energy effective action

$$\Gamma = \Gamma[\Phi]$$

$$\mathcal{N} = 4 \text{ SYM multiplet } \Phi = \begin{cases} X^I, \ I = 1, \ldots, 6 \\ F_{mn} = \partial_m A_n - \partial_n A_m \\ \psi^A_\alpha, \ A = 1, 2, 3, 4 \end{cases}$$

- In Coulomb branch, Φ is massless and Abelian; massive multiplets are integrated out
- Leading terms in the derivative expansion

$$\Gamma = \Gamma_0 + \Gamma_2 + \Gamma_4 + \Gamma_6 + \ldots$$

$\Gamma_0 \equiv 0$ (effective scalar potential vanishes)
$\Gamma_2 = S_2$ – free classical action
Γ_4 – four-derivative terms, leading
Γ_6 – six-derivative terms, next-to-leading

Some bosonic terms in Γ_4

- F^4/X^4 term

$$\int d^4x \frac{F^{\alpha\beta}F_{\alpha\beta}\bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}}}{(X^I X^I)^2}$$

- Wess-Zumino term

$$\varepsilon_{IJKLMN}\int \frac{1}{|X|^6} X^I dX^J \wedge dX^K \wedge dX^L \wedge dX^M \wedge dX^N$$

Some bosonic terms in Γ_6

- F^6/X^8 term

$$\int d^4x \frac{F^2\bar{F}^2(F^2+\bar{F}^2)}{(X^I X^I)^4}$$

Our goal:

To find manifestly $\mathcal{N}=4$ (or $\mathcal{N}=3$) supersymmetric description of Γ_4 which takes into account all terms of the four-derivative order.

We present solutions for Γ_4 in $\mathcal{N}=4$ and $\mathcal{N}=3$ harmonic superspaces

Γ_4 in $\mathcal{N}=2$ harmonic superspace [Buchbinder, Ivanov 2002]

$$\mathcal{N}=4: \quad \begin{cases} W & - \ \mathcal{N}=2 \text{ gauge superfield strength} \\ q_a^{\pm} & - \ \text{hypermultiplet, } a=1,2 \end{cases}$$

"Hidden" $\mathcal{N}=2$ SUSY

$$\delta W = \frac{1}{2}\bar\epsilon^{\dot\alpha a}\bar D^-_{\dot\alpha} q^+_a\,, \quad \delta q^+_a = \frac{1}{4}(\epsilon^\beta_a D^+_\beta W + \bar\epsilon^{\dot\alpha}_a \bar D^+_{\dot\alpha}\bar W)$$

- Non-holomorphic potential [Dine, Seiberg 1997]

$$\int d^4x d^8\theta \, \ln W \ln \bar W$$

- Buchbinder-Ivanov action

$$\Gamma_4 \sim \int d^4x d^8\theta \left[\ln W \ln \bar W + (X-1)\frac{\ln(1-X)}{X} + \mathrm{Li}_2(X) - 1\right]$$

$$X = -2\frac{q^{+a}q^-_a}{\bar W W}$$

$\mathcal{N}=4$ Harmonic Superspace

$$(x^m, \theta_{i\alpha}, \bar\theta^i_{\dot\alpha}, u)$$

$$u \in USp(4)\ :\quad uu^\dagger = 1\,,\quad u\Omega u^{\rm T} = \Omega\,,\quad (\Omega^{\rm T} = -\Omega)$$

$$u = (u_i^{(+,0)}, u_i^{(-,0)}, u_i^{(0,+)}, u_i^{(0,-)}) \qquad i = 1,2,3,4$$

Covariant harmonic derivatives form usp(4) algebra

$$D^{(\pm\pm,0)}, D^{(0,\pm\pm)}, D^{(\pm,\pm)}, D^{(\pm,\mp)}, S_1, S_2$$

Grassmann variables

$$\theta^{(\pm,0)}_\alpha = u^{(\pm,0)i}\theta_{i\alpha},\quad \theta^{(0,\pm)}_\alpha = u^{(0,\pm)i}\theta_{i\alpha},\quad \bar\theta^{(\pm,0)}_{\dot\alpha} = u_i^{(\pm,0)}\bar\theta^i_{\dot\alpha},\quad \bar\theta^{(0,\pm)}_{\dot\alpha} = u_i^{(0,\pm)}\bar\theta^i_{\dot\alpha}$$

Analytic subspace (contains 8 Grassmann variables)

$$(\zeta, u) = (x^m_A, \theta^{(\pm,0)}_\alpha, \bar\theta^{(0,\pm)}_{\dot\alpha}, u)$$

$$x^m_A = x^m - i\theta^{(0,-)}\sigma^m\bar\theta^{(0,+)} + i\theta^{(0,+)}\sigma^m\bar\theta^{(0,-)} - i\theta^{(+,0)}\sigma^m\bar\theta^{(-,0)} + i\theta^{(-,0)}\sigma^m\bar\theta^{(+,0)}$$

Short covariant spinor derivatives

$$D^{(0,\pm)}_\alpha = \pm\frac{\partial}{\partial\theta^{(0,\mp)\alpha}}\,,\quad \bar D^{(\pm,0)}_{\dot\alpha} = \pm\frac{\partial}{\partial\bar\theta^{(\mp,0)\dot\alpha}}$$

$\mathcal{N}=4$ superfield strengths

$$\bar{W}_{ij} = \frac{1}{2}\varepsilon_{ijkl}W^{kl}$$

$$D^i_\alpha W^{kj} + D^j_\alpha W^{ik} = 0, \quad \bar{D}_{i\dot\alpha}W^{jk} = \frac{1}{3}(\delta^j_i \bar{D}_{l\dot\alpha}W^{lk} - \delta^k_l \bar{D}_{l\dot\alpha}W^{lj})$$

Harmonic projections

$$W^{ij} \to W^{(+,+)}, W^{(-,-)}, W^{(+,-)}, W^{(-,+)}, W_1, W_2$$

e.g.

$$W_1 = u_i^{(0,+)}u_j^{(0,-)}W^{ij}, \qquad W_2 = u_i^{(+,0)}u_j^{(-,0)}W^{ij}$$

Constraints for W_1:

$$\widetilde{W}_1 = W_1$$

$$D^{(0,\pm)}_\alpha W_1 = \bar{D}^{(\pm,0)}_{\dot\alpha} W_1 = 0$$

$$D^{(\pm\pm,0)}W_1 = D^{(0,\pm\pm)}W_1 = 0$$

$$D^{(+,+)}D^{(+,+)}W_1 = 0$$

Low-energy effective action

$$\Gamma_4 = \int d^4x d^8\theta \, \mathcal{H}(W/\Lambda), \quad [\Lambda] = 1$$

Scale invariance $\Lambda \frac{d}{d\Lambda}\Gamma_4 = 0 \Rightarrow$

$$\boxed{\Gamma_4 \propto \int d^4x d^8\theta \, \ln(W/\Lambda)}$$

- Contains F^4/X^4 term in components
- Contains Wess-Zumino term
- Contains also all other bosonic and fermionic terms of the four-derivative order

$\mathcal{N}=3$ harmonic superspace

$$(x^m, \theta_{i\alpha}, \bar\theta^i_{\dot\alpha}, u) \qquad i=1,2,3$$

SU(3) harmonics

$$u = (u_i^1, u_i^2, u_i^3) \in SU(3): \quad u^\dagger u = 1, \quad \det u = 1$$

Harmonic derivatives $(D^1_2,\ D^2_3,\ D^1_3,\ D^2_1,\ D^3_2,\ D^3_1,\ S_1,\ S_2)$

$$D^I_J = u^I \frac{\partial}{\partial u^J} - \bar u_J \frac{\partial}{\partial \bar u_I}$$

Grassmann variables

$$\theta_{I\alpha} = \bar u^i_I \theta_{i\alpha}\,, \qquad \bar\theta^I_{\dot\alpha} = u^I_i \bar\theta^i_{\dot\alpha}\,, \quad I=1,2,3$$

Analytic subspace

$$(\zeta, u) = (x^m_A, \theta_{2\alpha}, \theta_{3\alpha}, \bar\theta^1_{\dot\alpha}, \bar\theta^2_{\dot\alpha}, u)$$

$$x^m_A = x^m - i\theta_1\sigma^m\bar\theta^1 + i\theta_3\sigma^m\bar\theta^3$$

Short Grassmann derivatives:

$$D^1_\alpha = \frac{\partial}{\partial\theta^\alpha_1}\,, \qquad \bar D_{3\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{3\dot\alpha}}$$

$\mathcal{N} = 3$ superfield strength

Superfield constraints:

$$D^i_\alpha W_{jl} = \frac{1}{2}(\delta^i_j D^k_\alpha W_{kl} - \delta^i_l D^k_\alpha W_{kj})$$

$$\bar{D}_{i\dot{\alpha}} W_{jk} + \bar{D}_{j\dot{\alpha}} W_{ik} = 0$$

Harmonic projections

$$W_{ij} \to (W_{12}, W_{23}, W_{13}), \quad \bar{W}^{ij} \to (\bar{W}^{12}, \bar{W}^{23}, \bar{W}^{13})$$

e.g.

$$W_{23} = \bar{u}^i_2 \bar{u}^j_3 W_{ij}\,, \qquad \bar{W}^{12} = u^1_i u^2_j \bar{W}^{ij}$$

Analyticity properties

$$D^1_\alpha W_{23} = \bar{D}_{3\dot{\alpha}} W_{23} = 0$$

$$D^1_\alpha \bar{W}^{12} = \bar{D}_{3\dot{\alpha}} \bar{W}^{12} = 0$$

Conjugation

$$\widetilde{W_{23}} = \bar{W}^{12}$$

Classical action

$\mathcal{N}=3$ gauge fields

$$D^I_J \to \nabla^I_J = D^I_J + V^I_J$$

V^1_2, V^2_3 and V^1_3 are analytic

$$D^1_\alpha(V^1_2, V^2_3, V^1_3) = 0 \qquad \bar{D}_{3\dot\alpha}(V^1_2, V^2_3, V^1_3) = 0$$

Classical (abelian) $\mathcal{N}=3$ SYM action

$$S_2 = \int d\zeta({}^{33}_{11})[V^1_2(D^2_3 - D^1_3 V^2_3) + V^2_3(D^1_2 V^1_3 - D^1_3 V^1_2) + V^1_3(D^1_2 V^2_3 - D^2_3 V^1_2 - V^1_3)]$$

Expressing superfield strengths in terms of gauge potentials

$$\bar{W}^{12} = -\frac{1}{4} D^{1\alpha} D^1_\alpha V^2_1 \,, \quad W_{23} = \frac{1}{4} \bar{D}_{3\dot\alpha} \bar{D}^{\dot\alpha}_3 V^3_2$$

Effective action in $\mathcal{N}=3$ superspace

$$\Gamma_4 = \int d\zeta({}^{33}_{11}) du \, \mathcal{H}^{11}_{33}(\bar{W}^{12}, W_{23})$$

- Analytic measure $d\zeta({}^{33}_{11})du$ is charged $\Rightarrow \mathcal{H}^{11}_{33}$ is also charged
- Superfield strengths $\bar{W}^{12}$ and W_{23} are also charged

Sample action [Ivanov, Zupnik 2001]

$$\int d\zeta({}^{33}_{11}) du \bar{W}^{12}\bar{W}^{12}W_{23}W_{23} \sim \int d^4x \, F^{\alpha\beta}F_{\alpha\beta}\bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}} + \dots$$

- Γ_4 should be superconformal $\Rightarrow$ need dimensionful and charged objects to construct the effective Lagrangian

Scalar fields φ^i, $\bar{\varphi}_i$

$$\bar{W}^{12} = \bar{u}^i_3 \bar{\varphi}_i + \ldots\,, \qquad W_{23} = u^1_i \varphi^i + \ldots$$

In the Coulomb branch the scalar fields have vevs,

$$c^i = \langle \varphi^i \rangle\,, \quad \bar{c}_i = \langle \bar{\varphi}_i \rangle$$

Introduce the objects

$$c^I = u^I_i c^i\,, \qquad \bar{c}_I = \bar{u}^i_I \bar{c}_i$$

and

$$\bar{\omega}^{12} = \bar{W}^{12} - \bar{c}_3\,, \qquad \omega_{23} = W_{23} - c^1$$

so that

$$\langle \bar{\omega}^{12} \rangle = \langle \omega_{23} \rangle = 0$$

Look for the effective action in the form

$$\Gamma_4 = \int d\zeta({}^{33}_{11}) \left(\frac{\bar{\omega}^{12}\omega_{23}}{c^i \bar{c}_i} \right)^2 H\left(\frac{\bar{\omega}^{12} c^3}{c^i \bar{c}_i}, \frac{\omega_{23}\bar{c}_1}{c^i \bar{c}_i} \right)$$

Superconformal invariance gives unique solution:

$$\Gamma_4 = \int d\zeta\binom{33}{11}\left(\frac{\bar{\omega}^{12}\omega_{23}}{c^i\bar{c}_i}\right)^2 H\left(\frac{\bar{\omega}^{12}c^3}{c^i\bar{c}_i}, \frac{\omega_{23}\bar{c}_1}{c^i\bar{c}_i}\right)$$

$$\boxed{H(x,y) = \frac{\ln(1+x+y)}{x^2y^2} + \frac{1}{xy(1+x+y)} - \frac{\ln(1+x)}{x^2y^2} - \frac{\ln(1+y)}{x^2y^2}}$$

$$x = \frac{\bar{\omega}^{12}c^3}{c^i\bar{c}_i}\,, \qquad y = \frac{\omega_{23}\bar{c}_1}{c^i\bar{c}_i}$$

Features

- Effective Lagrangian depends on vevs c^i and $\bar{c}_i$. However, we proved that Γ_4 is independent of c^i and $\bar{c}_i$,

$$\Gamma_4[\bar{W}^{12}, W_{23}; c'^i, \bar{c}'_i] = \Gamma_4[\bar{W}^{12}, W_{23}; c^i, \bar{c}_i]$$

- In components Γ^4 contains both F^4/X^4 and Wess-Zumino terms, as well as all terms which are necessary by supersymmetry

Bonus: $\mathcal{N}=3$ supersymmetry is off-shell because the superfield strengths are expressed in terms of gauge superfields

$$\bar{W}^{12} = -\frac{1}{4}(D^1)^2 V_1^2\,, \qquad W_{23} = \frac{1}{4}(\bar{D}_3)^2 V_2^3$$

- Consider the effective action

$$\Gamma = S_2 + \Gamma_4 + \ldots$$

- After eliminating auxiliary fields, uncover the F^6/X^8 term,

$$\int d^4x \frac{F^2 \bar{F}^2 (F^2 + \bar{F}^2)}{(\varphi^i \bar{\varphi}_i)^4}$$

(analogous to [Ivanov, Zupnik 2001] in the non-superconformal case)

Open problems

- Develop background field method in $\mathcal{N}=3$ harmonic superspace and derive Γ_4 perturbatively
- Apply $\mathcal{N}=3$ harmonic superspace for studying Wilson loops and amplitudes in the $\mathcal{N}=4$ SYM theory