CMB Imprints of a Pre-Inflationary Climbing Phase

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❖ E. Dudas, N. Kitazawa, AS, PLB 694 (2010) 80 [arXiv:1009.0874 [hep-th]].

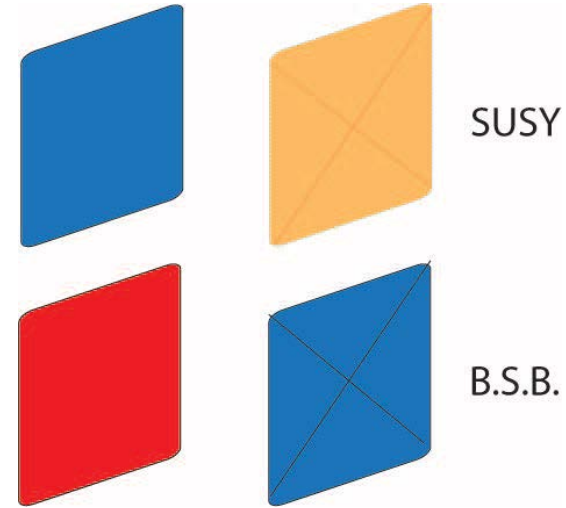❖ E. Dudas, N. Kitazawa, S. Patil, AS, arXiv:1202.6630 [hep-th], to appear in JCAP

Ginzburg Conference, Moscow, May 2012

String Theory & SUSY Breaking

Brane SUSY Breaking :
 (string-scale)

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Aldazabal, Uranga, 1999)
(Angelantonj, 1999)

- SUSY : D9(T>0,Q>0) + O9_ (T<0,Q<0) → SO(32)

- BSB : anti-D9(T>0,Q<0) + O9_+ (T>0,Q>0) → USp(32)

SUSY

B.S.B.

Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ \left[R - \frac{1}{2}(\partial\phi)^2 - 2\alpha\, e^{\frac{3\phi}{2}} \right] - \frac{1}{12}\, e^{\phi} H^2 \right\}$$

A climbing scalar in D dim's

- $\gamma < 1$?   Both signs of speed should be allowed

a. "Climbing" solution (φ climbs, descends → lim. τ-speed):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) + \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$t = 0.001$

b. "Descending" solution (φ descends → lim. τ- speed):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) + \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$t = 0.001$

Limiting τ- speed (LM attractor): $v_l = -\dfrac{\gamma}{\sqrt{1-\gamma^2}}$

(Lucchin, Matarrese, 1985)

$\gamma \to 1$: v_l diverges & LM attractor and descending solution disappear

- Beyond $\gamma = 1$?   only climbing

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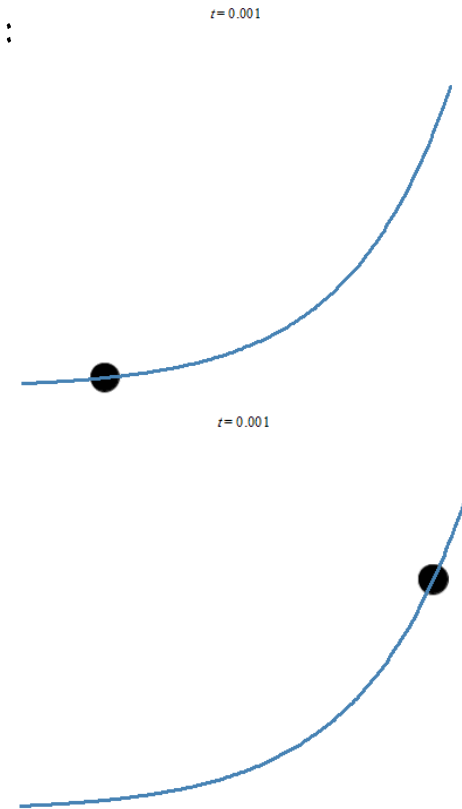
A climbing scalar in D dim's

- Consider the action for gravity and a scalar ϕ, in Einstein frame:

$$S \; = \; \frac{1}{2\kappa^2} \int d^D x \, \sqrt{-g} \, \left[R \, - \, \frac{1}{2} \, (\partial \phi)^2 \, - \, V(\phi) \, + \, \dots \right] ,$$

- Look for **cosmological solutions** of the type

$$ds^2 \; = \; - \, e^{2B(t)} \, dt^2 \, + \, e^{2A(t)} \, d\mathbf{x} \cdot d\mathbf{x}$$

(Halliwell, 1987)
....
(Dudas,,Mourad, 2000)
(Russo, 2004)

- Make the convenient **gauge choice** $\quad V(\phi) \, e^{2B} = M^2$

- Let: $\quad \beta = \sqrt{\dfrac{D-1}{D-2}} \, , \qquad \tau \, = \, M \, \beta \, t \, , \qquad \varphi = \dfrac{\beta \, \phi}{\sqrt{2}} \, , \qquad a \, = \, (D-1) \, A$

- In expanding phase : $\quad \ddot{\varphi} \, + \, \dot{\varphi} \, \sqrt{1 + \dot{\varphi}^2} \, + \, \left(1 + \dot{\varphi}^2 \right) \, \dfrac{1}{2V} \, \dfrac{\partial V}{\partial \varphi} \; = \; 0$

damping $\qquad$ potential

A climbing scalar in D dim's

- With exponential potentials
$$V(\varphi) = M^2 \, e^{2\gamma\varphi}$$

the equation for φ becomes
$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \gamma\left(1 + \dot{\varphi}^2\right) = 0$$

- γ=1:
$$\dot{\varphi} = \frac{1}{2(\tau - \tau_0)} - \frac{1}{2}(\tau - \tau_0) \quad , \quad \dot{a} = \frac{1}{2(\tau - \tau_0)} + \frac{1}{2}(\tau - \tau_0)$$

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau - \tau_0| - \frac{1}{2}(\tau - \tau_0)^2\right]$$

$$a(\tau) = a_0 + \frac{1}{2}\left[\log|\tau - \tau_0| + \frac{1}{2}(\tau - \tau_0)^2\right]$$

Integration constants:
- "Big Bang time"
- φ(τ) and a(τ) at some (later) reference time.

φ SPEED close to Big Bang FIXED : CLIMBING SCALAR!

Newtonian Analogy

Particle subject to damping and constant force :

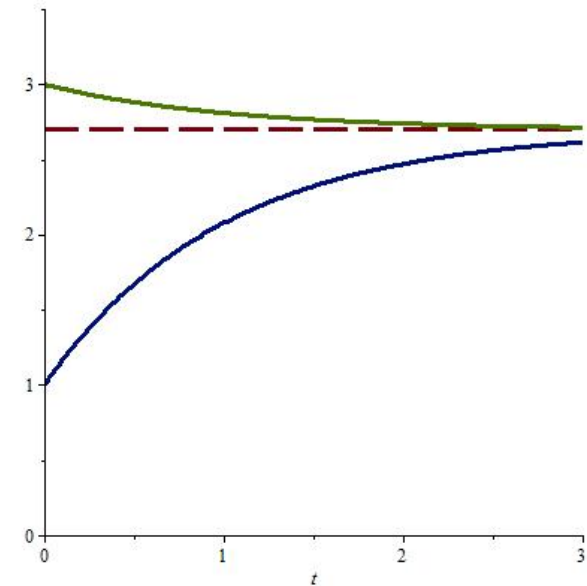$$m \, \frac{d^2Y}{dt^2} + \beta \, \frac{dY}{dt} = f$$

- Limiting speed:

$$v_{lim} = \frac{f}{\beta}$$

$$\frac{dY}{dt} = v_{lim} + (v_0 - v_{lim}) \, e^{-\frac{\beta t}{m}}$$

Two classes of solutions:

- approach v_{lim} from **below** or from **above**

As β is reduced, v_{lim} increases **without limit**. The "Upper branch" is eventually **lost altogether**, leaving way to a single "undamped" solution.

String Theory Realizations

- **Other dimensions D :**   (γ = 1 for **ALL** D>10, up to volume stabilization)
- **Non-Critical Strings :**   (from D>10 (super)critical + small exponent)

- **10D tadpole and KKLT :**   *(Kachru, Kallosh, Linde, Trivedi, 2003)*

  a)   No-scale reduction: *(Cremmer, Ferrara, Kounnas, Nanopoulos, 1983)*
  (Witten, 1985)

$$g_{IJ}^{(10)} = e^{\sigma}\delta_{IJ} \; , \; g_{\mu\nu}^{(10)} = e^{-3\sigma} g_{\mu\nu}^{(4)} \; , \; C_{IJ} = \epsilon_{IJ} \; a_2 \qquad s = e^{3\sigma} e^{\frac{\phi}{2}} \; , \; t = e^{\sigma} e^{-\frac{\phi}{2}}$$

  b)   Two-form duality and "tadpole uplift": $\quad s = e^{\Phi_s} \; , \; t = e^{\frac{1}{\sqrt{3}} \, \Phi_t}$

$$S_4 \; = \; \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi_s)^2 - \frac{1}{2}(\partial\Phi_t)^2 - 2\,\alpha\,e^{-\sqrt{3}\Phi_t} + \cdots \right]$$

String Theory Realizations

Assume : s stabilized by fluxes

$$T = e^{\frac{\Phi_t}{\sqrt{3}}} + i\,\frac{\theta}{\sqrt{3}}$$

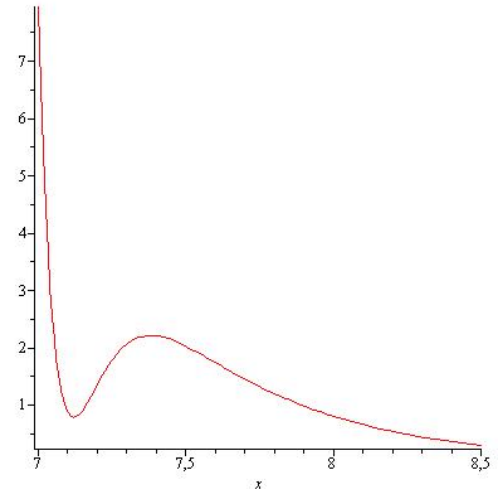$$W = W_0 + a\,e^{-bT}\,, \qquad K = -3\,\ln(T + \bar{T})$$

$$V_F = e^K\,\left[|D_T W|^2 - 3|W|^2\right] = \qquad \textit{(Cremmer, Ferrara, Girardello, vanProeyen, 1983)}$$

$$\frac{b}{(T+\bar{T})^2}\left\{a\,\bar{W}_0 e^{-bT} + \bar{a}\,W_0 e^{-b\bar{T}} + \frac{|a|^2}{3}\left[6 + b(T+\bar{T})\right]e^{-b(T+\bar{T})}\right\}$$

Uplift : $$\boxed{V = V_F + \frac{c}{(T+\bar{T})^3}}$$

$$\boxed{S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\,(\partial\Phi_t)^2 - \frac{1}{2}\,e^{-\frac{2}{\sqrt{3}}\Phi_t}\,(\partial\theta)^2 - V(\Phi_T,\theta)\right]}$$

Climbing in the KKLT System

Performing the redefinitions $\quad \Phi_t = \dfrac{2}{\sqrt{3}}\, x\, , \;\; \theta = \dfrac{2}{\sqrt{3}}\, y\, , \;\; \tau = M\, \sqrt{\dfrac{3}{2}}\, t$
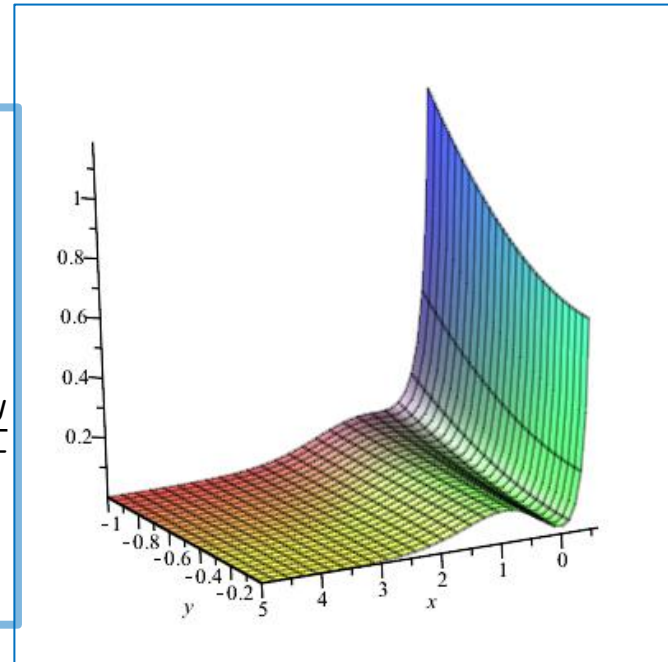
Working in the metric

$$ds^2 = -e^{2B(t)}\, dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x}$$

Choosing the gauge

$$V(\Phi_t, \theta)\, e^{2B} = M^2$$

$$\frac{d^2x}{d\tau^2} + \frac{dx}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \frac{1}{2V}\frac{\partial V}{\partial x}\left[1+\left(\frac{dx}{d\tau}\right)^2\right]$$

$$+ \; \frac{1}{2V}\frac{\partial V}{\partial y}\frac{dx}{d\tau}\frac{dy}{d\tau} + \frac{2}{3}\, e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2 = 0\, ,$$

$$\frac{d^2y}{d\tau^2} + \frac{dy}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \left(\frac{1}{2V}\frac{\partial V}{\partial x} - \frac{4}{3}\right)\frac{dx}{d\tau}\frac{dy}{d\tau}$$

$$+ \; \frac{1}{2V}\frac{\partial V}{\partial y}\left[e^{\frac{4x}{3}}+\left(\frac{dy}{d\tau}\right)^2\right] = 0$$

$$V(x,y) = \frac{c}{8}\, e^{-2x} - \frac{b}{2}\, e^{-\frac{4x}{3} - b\, e^{\frac{2x}{3}}}\left[(Re\; a\overline{W_0})\cos\frac{2by}{3} + (Im\; a\overline{W_0})\sin\frac{2by}{3} + \frac{|a|^2}{3}\left(3 + b\, e^{\frac{2x}{3}}\right)\, e^{-b\, e^{\frac{2x}{3}}}\right]$$

Climbing in the KKLT System

Leave aside momentarily the potential well → only uplift (γ =1 in KKLT)

- "Polar" variables :

$$\frac{dx}{d\tau} = r\,w\;,\qquad e^{-\frac{2x}{3}}\,\frac{dy}{d\tau} = r\,\sqrt{1-w^2}$$

- Dynamical γ_{eff} (= γw) :

$$\frac{dr}{d\tau} + r\,\sqrt{1+r^2} - \gamma\,w\left(1+r^2\right) = 0$$

$$\frac{dw}{d\tau} + (1-w^2)\left(\frac{2}{3}\,r - \frac{\gamma}{r}\right) = 0$$

1) **Late-time attractors** ("TOO FAST" FOR INFLATION)

2) **Right after Big Bang :**

$$\frac{dr}{d\tau} + (\epsilon - w)\,r^2 \approx 0\;,\qquad \frac{dw}{d\tau} + \frac{2}{3}\,r(1-w^2) \approx 0$$

- **Eqs. combine to:**

$$\dot{r} \approx -2\,\epsilon\,r^2$$

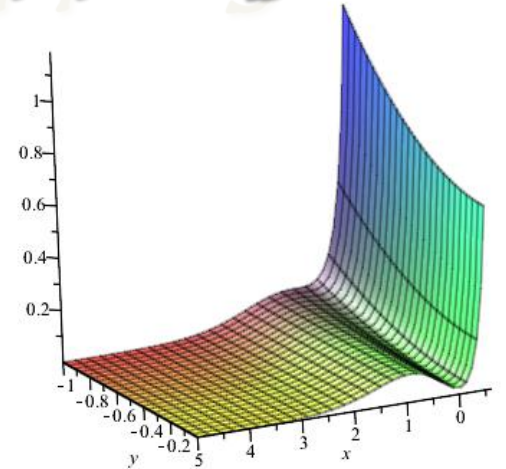$$r \approx \frac{1}{2\,\epsilon\,\tau}\;,\; w \approx -\epsilon$$

$$r\,w \approx -\frac{1}{2\,\tau}$$ Climbing scalar !

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KKLT Moduli Trapping

Climbing can yield **moduli trapping**

For a **range of parameters** the cosmological evolution can **trap** a climbing scalar inside the tiny KKLT potential well
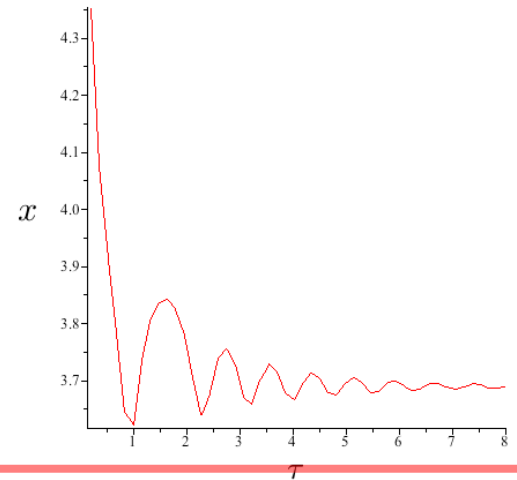
Analytic approach: approximate wells by piece-wise exponentials
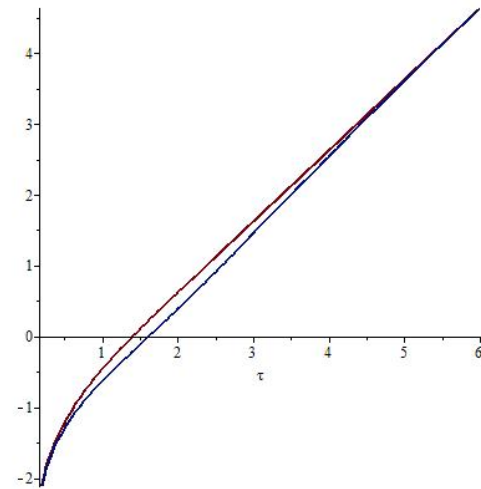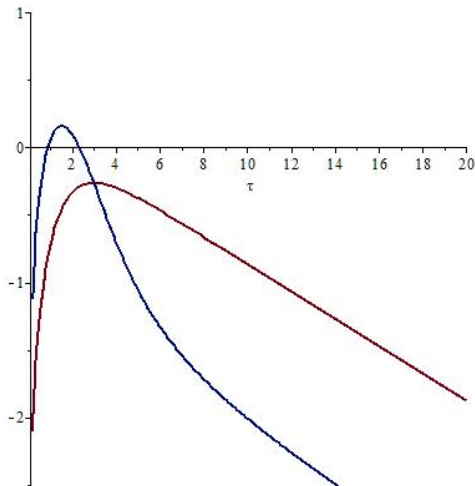
Climbing + Inflation

Combine two exponentials :

    a.   "Hard" exponential of BSB
    b.   "Soft" exponential (e.g. from non-BPS branes)

$$V(\phi) = \frac{M^2}{2\kappa^2}\left(e^{2\,\varphi} + e^{2\,\gamma\,\varphi}\right)$$

(Sen , 1998)
(Dudas, Mourad, AS 2001)

- ▪ "Hard" exponential: makes initial climbing phase inevitable
- ▪ "Soft" exponential: drives inflation during subsequent descent

The Mukhanov-Sasaki Equation

Schroedinger-like equation for **scalar (or tensor)** fluctuations :

$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\mathbf{x}\cdot d\mathbf{x}\right)$$

$$Scalar : z(\eta) = a^2(\eta)\,\frac{\phi_0'(\eta)}{a'(\eta)}$$

$$Tensor : z(\eta) = a$$

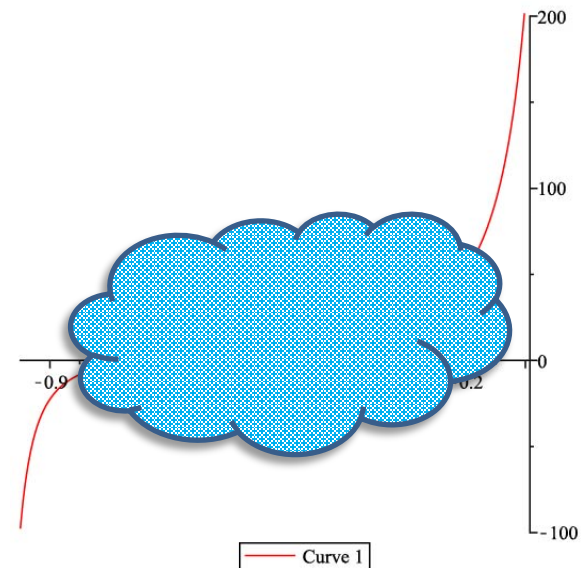$$W_s = \frac{1}{z}\,\frac{d^2 z}{d\eta^2}$$

"MS Potential" : determined by the background

$$\textbf{I}\text{nitial Singularity}: W_s \underset{\eta \to -\eta_0}{\widetilde{}} -\frac{1}{4}\frac{1}{(\eta + \eta_0)^2}$$

$$\textbf{LM Inflation}: W_s \underset{\eta \to 0}{\widetilde{}} -\frac{\nu^2 - \frac{1}{4}}{\eta^2}$$

$$\left[\nu = \frac{3}{2}\frac{1 - \gamma^2}{1 - 3\gamma^2}\right]$$

$$P(k) \sim k^3 \left|\frac{v(-\epsilon)}{z(-\epsilon)}\right|^2$$

Curve 1

Numerical Power Spectra
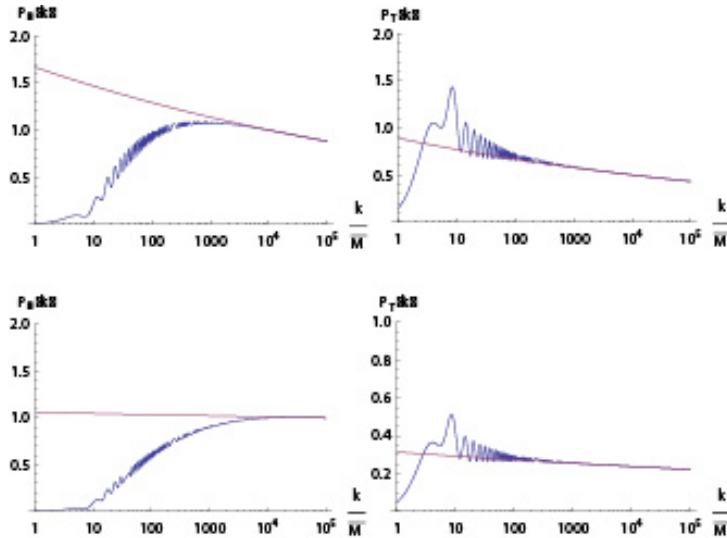
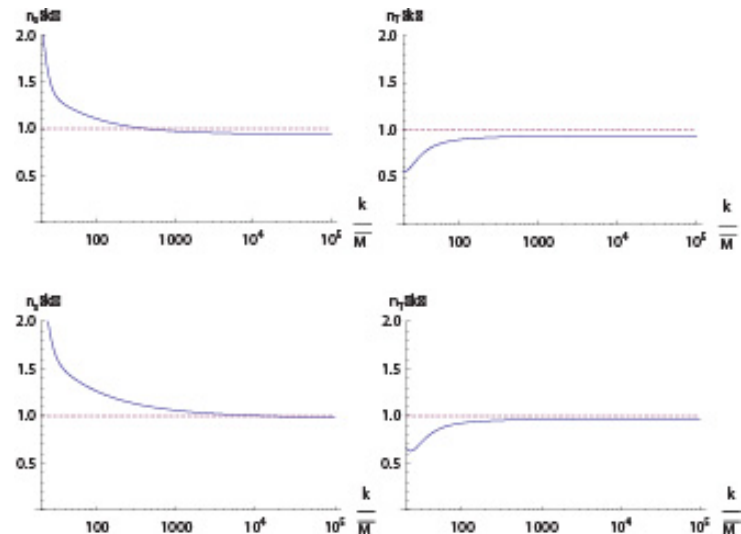$$\epsilon_\phi(k) \equiv -\frac{\dot{H}}{H^2}\,, \qquad \eta_\phi(k) \equiv \frac{V_{\phi\phi}}{V}$$

$$n_S(k) \;-\; 1 \;=\; 2\big[\eta_\phi(k) \;-\; 3\,\epsilon_\phi(k)\big]$$
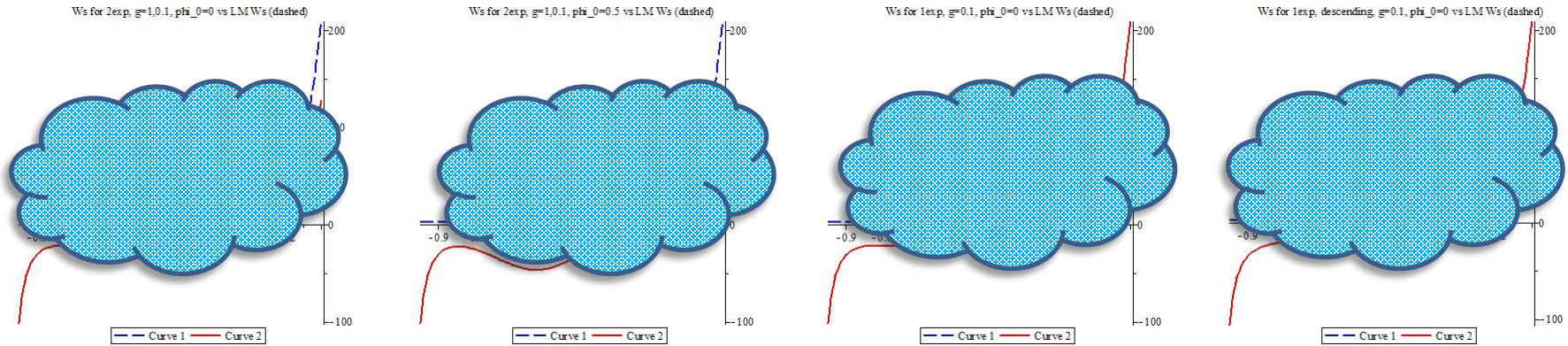$$n_T(k) \;-\; 1 \;=\; -\,2\,\epsilon_\phi(k)$$

$$P_{S,T} \;\sim\; \int \frac{dk}{k}\; k^{n_{S,T}-1}$$

QUALITATIVE ANALYSIS

- Dynamics yields $\epsilon_\phi(k)$, $\eta_\phi(k)$

- Slow-roll relations estimate $n_{S,T}(k)$

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Analytic Power Spectra

Ws for 2exp, g=1,0.1, phi_0=0 vs LM Ws (dashed)

Ws for 2exp, g=1,0.1, phi_0=0.5 vs LM Ws (dashed)

Ws for 1exp, g=0.1, phi_0=0 vs LM Ws (dashed)

Ws for 1exp, descending, g=0.1, phi_0=0 vs LM Ws (dashed)

WKB:
$$v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}} \exp\left(\int_{-\eta^\star}^{-\epsilon} \sqrt{|W_s(y) - k^2|}\, dy\right)$$
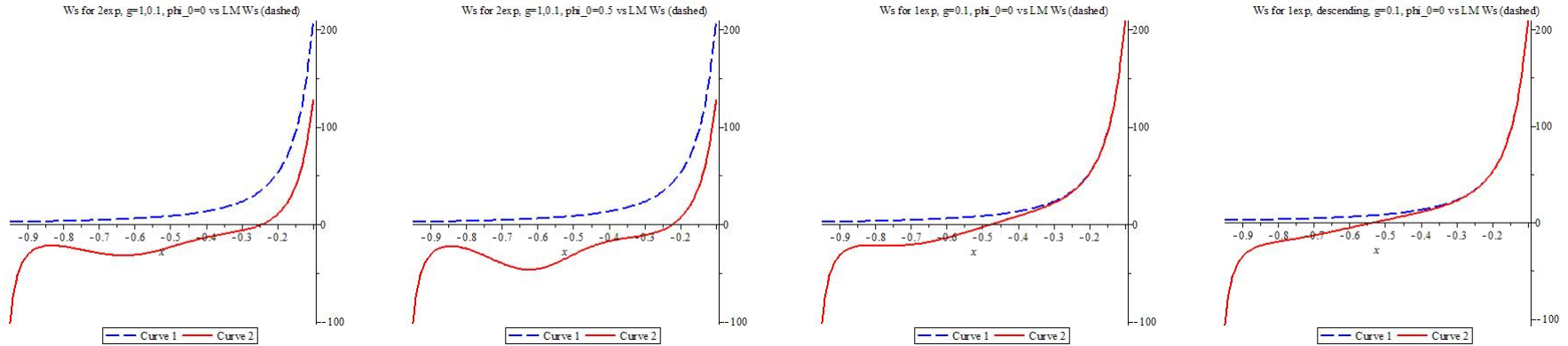
Analytically:
- **Bessel** functions (LM attractor)
- **Coulomb** (or Whittaker) functions
- [**Heun** functions]

$$P(k) \sim k^{3-2\nu}$$

$$\frac{d^2 v_k}{d\eta^2} + \left[k^2 - \frac{(1-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0^2} - \frac{(2-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0\,\eta} - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] v_k = 0$$

Analytic Power Spectra

Ws for 2exp, g=1,0.1, phi_0=0 vs LM Ws (dashed)

Ws for 2exp, g=1,0.1, phi_0=0.5 vs LM Ws (dashed)

Ws for 1exp, g=0.1, phi_0=0 vs LM Ws (dashed)

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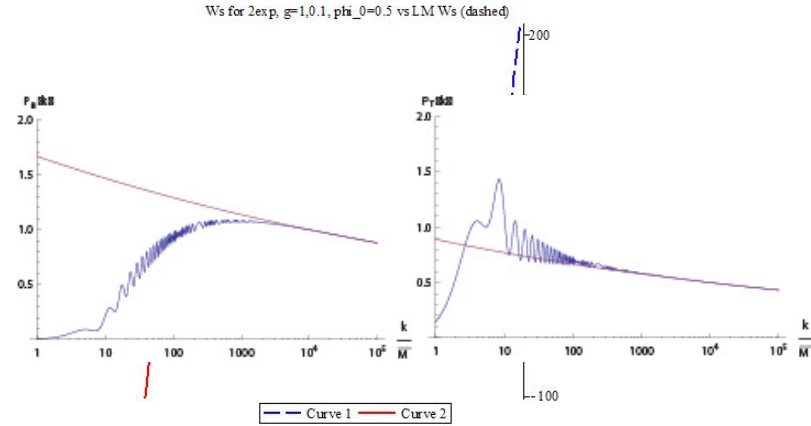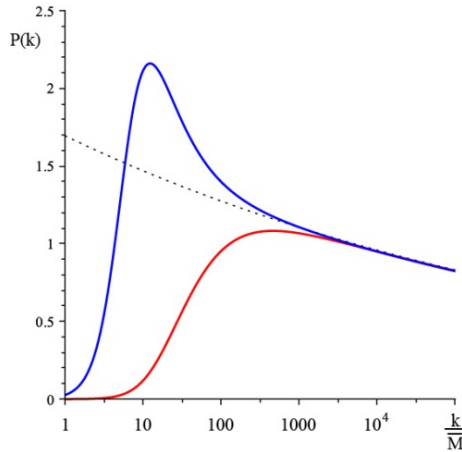
Analytically:
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- [**Heun** functions]

$$P(k) \sim k^{3-2\nu}$$

$$\frac{d^2 v_k}{d\eta^2} + \left[k^2 - \frac{(1-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0^2} - \frac{(2-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0\,\eta} - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] v_k = 0$$

Analytic Power Spectra

Letting :

$$\Delta = \sqrt{(k\eta_0)^2 - (1-a)\left(\nu^2 - \frac{1}{4}\right)}, \quad \rho = -\frac{\eta\Delta}{\eta_0}$$

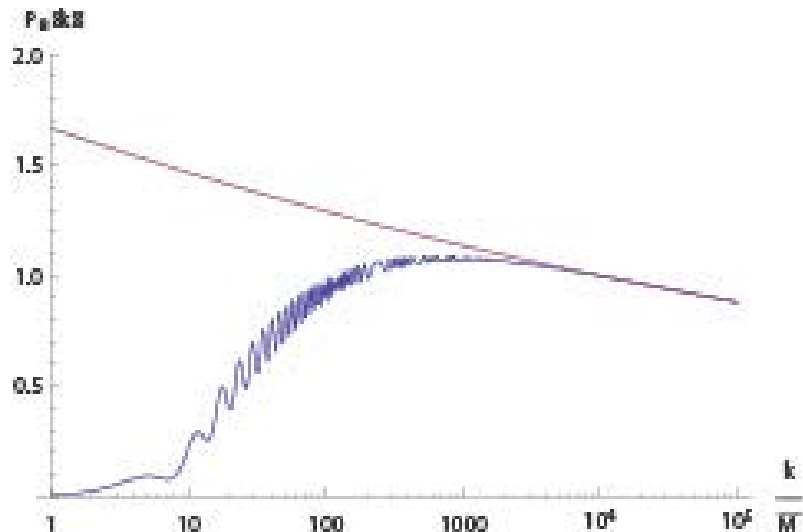$$L = \nu - \frac{1}{2}, \quad \alpha = -\frac{(2-a)\left(\nu^2 - \frac{1}{4}\right)}{2\Delta}$$

MS → Coulomb :

$$\frac{d^2 v_k}{d\rho^2} + \left[1 - \frac{2\alpha}{\rho} - \frac{L(L+1)}{\rho^2}\right] v_k = 0$$

$$P(k) \sim \frac{k^3 \exp\left(-\frac{(2-a)\left(\frac{2-3\gamma^2}{(1-3\gamma^2)^2}\right)}{2\sqrt{(k\eta_0)^2 + (a-1)\left(\frac{2-3\gamma^2}{(1-3\gamma^2)^2}\right)}}\right)}{\left|\Gamma\left(L+1+\frac{i(2-a)\left(\frac{2-3\gamma^2}{(1-3\gamma^2)^2}\right)}{2\sqrt{(k\eta_0)^2 + (a-1)\left(\frac{2-3\gamma^2}{(1-3\gamma^2)^2}\right)}}\right)\right|^2 \left[(k\eta_0)^2 + (a-1)\left(\frac{2-3\gamma^2}{(1-3\gamma^2)^2}\right)\right]^\nu}$$
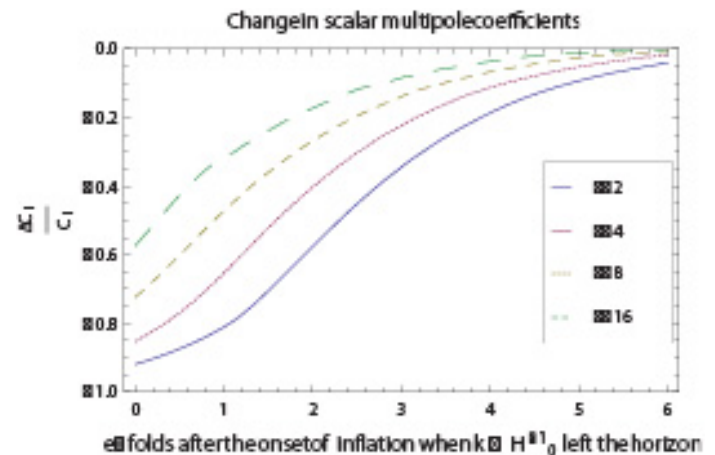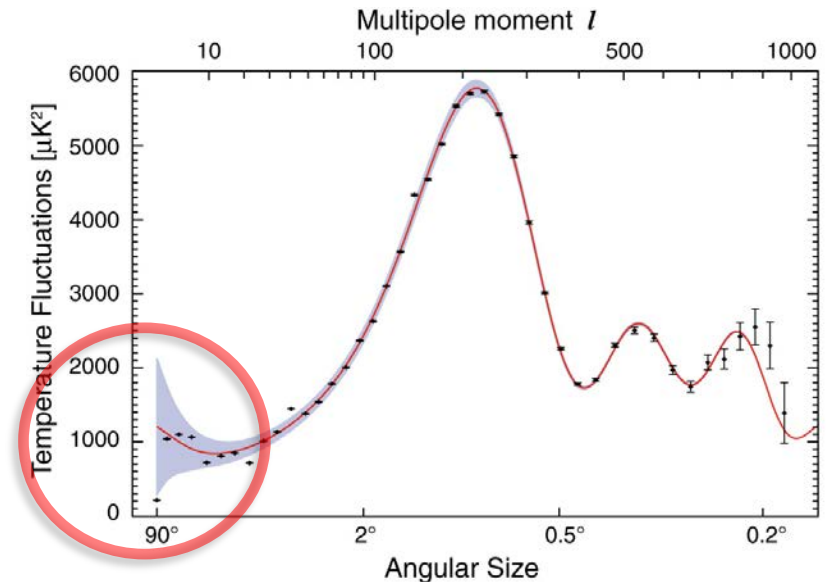
An Observable Window ?

- WMAP7 power spectrum :

- Depression of scalar multipoles :

$$vs \quad \left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell + 1}}$$

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Summary & Outlook

- String Theory ➔ **logarithmic slope** of certain potentials that accompany the

  (string-scale) Brane SUSY Breaking mechanism

- Naturally weak coupling : **climbing** can induce **moduli trapping**
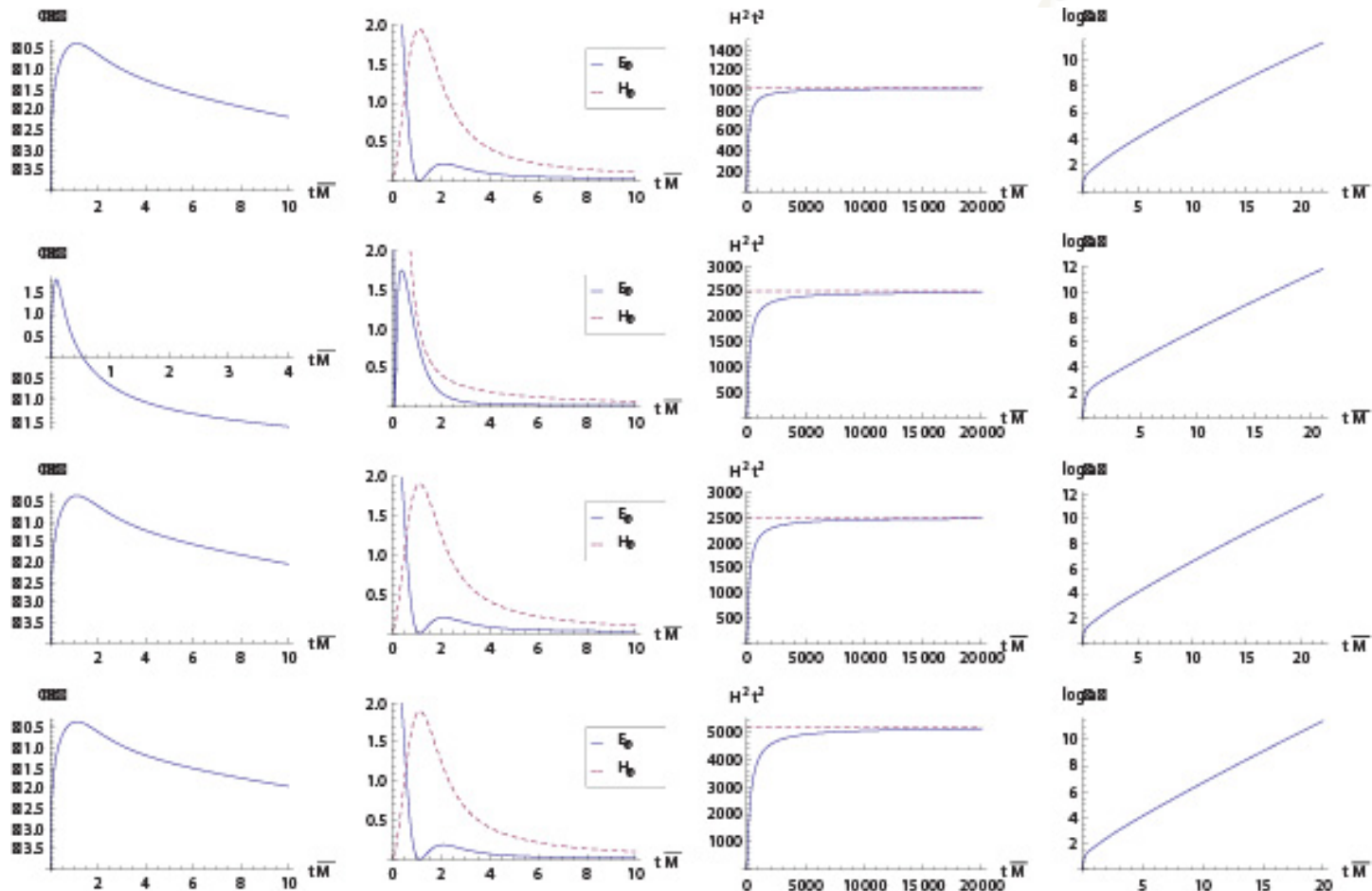
  &/or **inject inflation**

COMBINING "mild exponential" for **inflation** and "hard exponential" of **BSB** :

- STRONG IR depression of **scalar** power spectrum (over ~ 6 e-folds)
- [MILDER IR enhancement of **tensor** spectrum]
- Models where inflation begins **well off** the attractor (new CMB codes?)
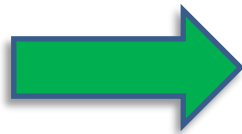
Signs in the sky of pre-inflationary climbing ?

IF inflationary epoch was "short enough"

Numerical Power Spectra

$$\epsilon_\phi \equiv -\frac{\dot{H}}{H^2}, \quad \eta_\phi \equiv \frac{V_{\phi\phi}}{V}$$

$$n_S - 1 = 2(\eta_\phi - 3\epsilon_\phi),$$
$$n_T - 1 = -2\epsilon_\phi$$

$$P_{S,T} \sim \int \frac{dk}{k} k^{n_{S,T}-1}$$

A non-BPS D3 brane : Inflation

The Sugimoto model has a **stable** non-BPS D3 brane

(Sen , 1998)
(Dudas, Mourad, AS 2001)

$$V \sim e^{-\sqrt{3}\Phi_t} + e^{-\frac{\sqrt{3}\Phi_t}{2}}$$

IF Φ_s is somehow stabilized (as assumed in the KKLT setting) → for Φ_t :

- "critical" exponential (**climbing**);
- "subcritical" exponential (eventual **inflation**)

- **INFLATION**:
$$\mathcal{I} = \frac{d^2 A}{dt^2} + \frac{dA}{dt}\left(\frac{dA}{dt} - \frac{dB}{dt}\right) > 0$$

- **One-field** :
$$\dot{\varphi}^2 < \frac{1}{D-2} \to \gamma < \frac{1}{\sqrt{D-1}}$$

- **KKLT**: **NO** inflation by itself