Dark Energy from Instantons

Leonid Marochnik

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Cosmological Constant 1

- The constraint on the equation-of-state parameter of dark energy

$$w = p_{de} / \varepsilon_{de} \qquad w = -1.08 \pm 0.1$$

Riess et al. (2011)

•Although lambda-term is consistent with the observational value of $w \approx -1$ there are well-known problems with that.

Cosmological Constant 2

- “Old cosmological constant problem”: Why is the lambda-term value measured from observations of the order 10^{-122} of vacuum energy density?

- “Coincidence problem”: Why is the acceleration happening during the contemporary epoch of matter domination?

- Apparently, there is also a discrepancy between redshifts of transition points that come from observational data and from lambda-term theory

Dark Energy of Instanton Origin 1

- Observational constraint on the equation-of-state parameter of dark energy tells us only that the equation of state of dark energy is close to

$$p = -\varepsilon$$

This does not mean that it is necessarily due to

$$\Lambda \neq 0$$

Such an equation of state can exist due to other reasons (a well-known example is a scalar field).

- **Assumption that the dark energy is of instanton origin seems to be able to resolve all 3 issues**

Dark Energy of Instanton Origin 2

- Instantons are Euclidean solutions that mediate tunneling between two vacua

- The idea of the present work to employ gravitational instantons of some type to tunnel “something” to the Lorentzian space of real time is not new

- In application to quantum cosmology, the basic idea was to explain the origin of inflationary Universe of Lorentzian signature by tunneling from a Riemannian space of Euclidean signature or from “nothing”.

Dark Energy of Instanton Origin 3

- Tryon (1973): Our Universe could be a vacuum fluctuation, that “appear from nowhere” and “…such an event need not have violated any of the conventional laws of physics”;
- Zeldovich (1981a): Quantum creation of the Universe
- Atkatz & Pagel (1982) “…the Universe arises as a result of quantum-mechanical barrier penetration”
- “No boundary proposal” by Hartle &Hawking (1983)
- Birth of inflationary Universe by tunneling from “nothing” by Vilenkin (1982, 1983) and Grishchuk & Zeldovich (1982).

Dark Energy of Instanton Origin 4

This old idea can get a “second wind” due to a mechanism of tunneling based on quantum metric fluctuations, which is able to address both the birth of dark energy and inflationary Universe

Dark Energy of Instanton Origin 5

- The mechanism proposed is quite universal in the sense that it does not depend on the model of the scalar field but is based on natural quantum fluctuations of the space metric.

- It is able to give birth to both a flat inflationary Universe **to start its cosmological evolution** and to dark energy in the contemporary epoch of aging emptying Universe **to finish its cosmological evolution.**

Dark Energy of Instanton Origin 6

- Self-consistent solutions to the equations of quantum gravity in the one-loop approximation in imaginary time
- Analytically continuation these to the Lorentzian space of real time.
- Assumption that these solutions do exist in the space of real time

Dark Energy of Instanton Origin 7

•In a sense, this is one of possible concretizations of Hawking's idea about the reality of imaginary time

•We can only partially overcome the present lack of a consistent quantum theory of gravity by using the one-loop approximation in which it is finite and mathematically consistent (Marochnik, Usikov & Vereshkov, Found.Phys., 38, 546, 2008a- MUV 2008a).

Self-Consistent Equations of One-Loop Quantum 1 Gravity

Faddeev – Popov gauged path integral

Factorization of classical and quantum variables

(allowing the existence of a self–consistent system of equations for gravitons, ghosts and macroscopic geometry)

Exponential parameterization

Transition to the one–loop approximation

(taking into account the fact that contributions of ghost fields to observables cannot be eliminated in any way)

Choice of ghost sector, satisfying the condition of one–loop finiteness of the theory off the mass shell.

Self-Consistent Equations of One-Loop Quantum Gravity 2

- Path integral (Faddeev-Popov), exponential parameterization, synchronous gauge (automatically provides finiteness)

$$R_i^k - \frac{1}{2}\delta_i^k R = \kappa < \Psi \mid \hat{T}^k_{i(grav)} + \hat{T}^k_{i(ghost)} \mid \Psi >$$

Quantum generalization of Lifshits (1946) equation for classical gravitational waves

$$\ddot{\psi}_{\mathbf{k}\sigma} + 3H\dot{\psi}_{\mathbf{k}\sigma} + \frac{k^2}{a^2}\psi_{\mathbf{k}\sigma} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathit{gravitons}$$

$$\ddot{\theta}_{\mathbf{k}} + 3H\dot{\theta}_{\mathbf{k}} + \frac{k^2}{a^2}\theta_{\mathbf{k}} = 0 \ldots\ldots\ldots\ldots\ldots\ldots \mathit{ghosts}$$

Canonical commutation relations for gravitons and anticommutation relations for ghosts

$$\frac{a^3}{4\kappa}\left[\dot{\psi}^{+}_{\mathbf{k}\sigma}, \psi_{\mathbf{k}'\sigma'}\right]_- = -i\hbar\delta_{\mathbf{k}bfk'}\delta_{\sigma\sigma'},$$

(see MUV, 2008a for details)

$$\frac{a^3}{8\kappa}\left[\dot{\bar{\theta}}_{\mathbf{k}}, \theta_{\mathbf{k}'}\right]_+ = -\frac{a^3}{8\kappa}\left[\dot{\theta}_{\mathbf{k}}, \bar{\theta}_{\mathbf{k}'}\right]_+ = -i\hbar\delta_{\mathbf{k}\mathbf{k}'}.$$

Stress tensor of graviton and ghosts should be obtained by solving operator equations of motion and averaging over a quantum ensemble $\mid \Psi >$

$$3\frac{a'^2}{a^4}=\kappa\varepsilon_g=\frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk\Big(\sum_\sigma <\Psi_g\,|\,\hat{\psi}'^{+}_{\mathbf{k}\sigma}\hat{\psi}'_{\mathbf{k}\sigma}+k^2\hat{\psi}^{+}_{\mathbf{k}\sigma}\hat{\psi}_{\mathbf{k}\sigma}\,|\,\Psi_g>$$

$$-2<\Psi_{gh}\,|\,\bar{\theta}'_{\mathbf{k}}\theta'_{\mathbf{k}}+k^2\bar{\theta}_{\mathbf{k}}\theta_{\mathbf{k}}\,|\,\Psi_{gh}>\Big)$$

$$2\frac{a''}{a^3}-\frac{a'^2}{a^4}=-\kappa p_g=-\frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk\Big(\sum_\sigma <\Psi_g\,|\,\hat{\psi}'^{+}_{\mathbf{k}\sigma}\hat{\psi}'_{\mathbf{k}\sigma}-\frac{k^2}{3}\hat{\psi}^{+}_{\mathbf{k}\sigma}\hat{\psi}_{\mathbf{k}\sigma}\,|\,\Psi_g>$$

$$-2<\Psi_{gh}\,|\,\bar{\theta}'_{\mathbf{k}}\theta'_{\mathbf{k}}-\frac{k^2}{3}\bar{\theta}_{\mathbf{k}}\theta_{\mathbf{k}}\,|\,\Psi_{gh}>\Big)$$

$$\hat{\phi}''_{\vec{k},\sigma}+\Big(k^2-\frac{a''}{a}\Big)\hat{\phi}_{\vec{k},\sigma}=0 \qquad \hat{\psi}_{\mathbf{k}\sigma}=\frac{1}{a}\hat{\phi}_{\mathbf{k}\sigma}$$

$$\hat{\vartheta}''_{\vec{k}}+\Big(k^2-\frac{a''}{a}\Big)\hat{\vartheta}_{\vec{k}}=0 \qquad \hat{\theta}_{\mathbf{k}}=\frac{1}{a}\hat{\vartheta}_{\mathbf{k}}$$

Self-Consistent De Sitter Gravitational Instanton 1

- **In real time, this system of equations is unable to form a self-consistent De Sitter solution**

- **In imaginary time, it forms a self-consistent De Sitter gravitational instanton**

Self-Consistent De Sitter Instanton

Self-consistent solution
in imaginary time $t = -i\tau$ $\eta = -i\upsilon$

$$a_S = a_0 \exp(H_\tau \tau) = -(H_\tau \upsilon)^{-1}$$

$$\hat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a}\sqrt{\frac{2\kappa\hbar}{k}}\left(\hat{Q}_{\mathbf{k}\sigma} g_k\right) \qquad \hat{\vartheta}_{\mathbf{k}} = \frac{1}{a}\sqrt{\frac{2\kappa\hbar}{k}}\left(\hat{q}_{\mathbf{k}} g_k\right)$$

$$\xi = k\upsilon \qquad g(\xi) = \left(1 + \frac{1}{\xi}\right) e^{-\xi}$$

Self-Consistent De Sitter Gravitational Instanton 3

This gravitational instanton does exist if Hubble function satisfies condition

$$H_{\tau}{}^{2} = \frac{8\pi^{2}}{\kappa\hbar(< n_{gh} > - < n_{g} >)} \qquad H_{\tau}{}^{2} \neq 0$$

$$H_{\tau}{}^{2} = 0 \qquad H_{\tau}{}^{2} \neq \frac{8\pi^{2}}{\kappa\hbar(< n_{gh} > - < n_{g} >)}$$

Ghosts

- In real time, ghosts are fictitious particles, which appear to compensate for the spurious effect of vacuum polarization of fictitious fields of inertia.

- The gravitational effect of gravitons is proportional to a difference $(< n_g > - < n_{gh} >) \geq 0$

- Figuratively speaking this means those ghosts cannot be “materialized” in real time.

Ghosts 2

- In imaginary time, the self-consistent De Sitter solution is proportional to $(<n_{gh}>-<n_g>)$

- Figuratively speaking, this means those ghosts are “materialized” to form the De Sitter instanton

- **The remarkable fact is that the passage back to a space of real time “de-materializes” ghosts**

- **In the Lorentzian space of real time, the solution is again proportional to** $(<n_g>-<n_{gh}>)\geq 0$ **and ghosts again are fictitious particles as they must be**

Analytical continuation to the Lorentzian space of real time 1

$$a(t) = a_0 \exp(Ht) = -(H\eta)^{-1}$$

$$H = \pm\left(\frac{8\pi^2}{\kappa\hbar < n_g > - < n_{gh} >}\right)^{1/2} = \pm i H_\tau$$

$$H = \pm\left(\frac{8\pi^2}{\kappa\hbar N}\right)^{1/2} \qquad N \approx < n_g > - < n_{gh} >$$

$$H_\tau{}^2 \tau^2 \equiv H^2 t^2$$

Analytical continuation to the Lorentzian space of real time 3

- For Poisson distributions

$$H^2 = \frac{8\pi^2}{\kappa\hbar N}$$

$$N^{-1} \sim < (\Delta N / N)^2 >$$

$$H^2 \sim < (\Delta N / N)^2 >$$

“Old cosmological constant problem”

- Hubble H is defined by the number of gravitons

N that were tunneled into the De Sitter space by instantons carrying out the tunneling.

- In the case of the birth of dark energy, for the observational value of the Hubble constant (Riess et al. 2011), one gets

$$N \approx 10^{122}$$

which has nothing to do with vacuum energy

Tunneling 1

- Using a mathematical analogy between graviton-ghost equations and stationary Schrödinger equation, solutions to these can be thought of in terms of quantum tunneling

$$\hat{\phi}''_{\vec{k},\sigma} + (k^2 - \frac{a''}{a})\hat{\phi}_{\vec{k},\sigma} = 0 \qquad \hat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a}\hat{\phi}_{\mathbf{k}\sigma}$$

$$\hat{\vartheta}''_{\vec{k}} + (k^2 - \frac{a''}{a})\hat{\vartheta}_{\vec{k}} = 0 \qquad \hat{\theta}_{\mathbf{k}} = \frac{1}{a}\hat{\vartheta}_{\mathbf{k}}$$

Tunneling 2

- In these equations,

$$x = k\eta$$

plays the role of the spatial coordinate of Schrödinger equation

- The role of "one-dimensional potential" plays

$$a'' / a$$

Tunneling 3

- Whether gravitons belongs to the Lorentzian or to the Euclidean space is governed by the sign of k^2 ($k^2 > 0$ is for real and $k^2 < 0$ is for imaginary time).
- Super-horizon gravitons and ghosts (x<<1) do not “feel” the difference between Lorentzian and Euclidean signatures and can belong to each of these.
- The boundary x=0 plays the role of a classically impenetrable barrier dividing these topologically non–equivalent spaces

Analytical continuation to the Lorentzian space of real time 2

$$\hat{\phi}_{\mathbf{k}\sigma} = \frac{1}{a_S}\sqrt{\frac{2\kappa\hbar}{k}}\left[\hat{c}_{\mathbf{k}\sigma} f(x)\right], \qquad \eta = \int dt / a$$

$$\hat{\vartheta}_{\mathbf{k}} = \frac{1}{a_S}\sqrt{\frac{2\kappa\hbar}{k}}\left[\hat{\alpha}_{\mathbf{k}} f(x)\right],$$

$$f(x) = (1 - \frac{i}{x})e^{-ix} \qquad x = k\eta$$

Birth of Inflationary Universe

Tunneling of gravitons from Euclidean space of imaginary time (from "nothing") creates the De Sitter empty flat Universe in real time

Equation of State 1

$$\varepsilon_g = -p_g = \frac{3\hbar N}{8\pi^2} H^4$$

This equation of state is superficially similar to that which comes from quantum conformal anomalies. As was shown by Starobinsky (1980), quantum corrections to the Einstein equations due to zero oscillations can provide a self-consistent De Sitter solution in the vicinity of Planck's value of curvature. In such a case, the equation of state is $\varepsilon \sim nhH^4$

(Zeldovich, 1981b) where the number of types of elementary particles n is of the order of ~100

Equation of State 2

- Conformal anomalies that arise due to regularization and renormalization procedures do not apply to this work, which deals with the equations of quantum gravity that are finite in the one-loop approximation.
- In the finite one-loop quantum gravity, the effect of conformal anomalies is exactly zero, and the De Sitter solution can be formed only by graviton-ghost instantons (MUV 2008b, section XII). In contrast to the conformal anomaly parameter n, the parameter N is arbitrary and can be a huge number.

Birth of Dark Energy 1

The Universe consists of approximately 75% of dark energy and 25% of dark matter plus ordinary matter in the contemporary epoch

Birth of Dark Energy 2

- In the presence of matter

$$3\frac{{a'_\eta}^2}{a^4} = \kappa(\varepsilon_g + \varepsilon_m) \qquad \kappa\varepsilon_m = \frac{3C_m}{a^3}$$

$$3C_m = \kappa\varepsilon_m a^3 = \kappa\varepsilon_{0m} {a_0}^3 = const$$

Birth of Dark Energy 3

- In imaginary time $$3\frac{a'^2}{a^4} = -\kappa(\varepsilon_{inst} + \varepsilon_m)$$

- **Solutions to this equation can exist only if**

$$\varepsilon_{inst} < 0$$

i.e. **again under the condition that ghosts are "materialized" in the Euclidean space**.

(this condition is necessary but not sufficient)

Birth of Dark Energy 4

In the presence of matter, solutions to (4.2) can exist only after the scale factor exceeds the threshold

$$a \geq a_{threshold}$$

which is defined by the condition

$$\varepsilon_m = -\varepsilon_{inst} = \varepsilon_{de}$$

Birth of Dark Energy 5

- Observational data are consistent with the De Sitter expansion law so that $\varepsilon_{de} \approx const$

- From this fact and (4.1) it follows

$$1 + z_{threshold} = (\frac{\varepsilon_{de}}{\varepsilon_m})^{1/3} = (\frac{\Omega_{de}}{\Omega_m})^{1/3}$$

Birth of Dark Energy 6

- Assuming for the sake of definiteness that

$$\Omega_{de} / \Omega_m \approx 1/3$$

(which is in the range of WMAP data of Spergel et al., 2007), one gets

$$1 + z_{threshold} \approx 1.44$$

A good agreement with the observational value

$$1 + z_{transition} \approx 1.43 \pm 0.07$$

(Riess et al., 2007; see also Shapiro and Turner, 2006, for more discussion).

Birth of Dark Energy 7

- In case of lambda-term and the same assumption

$$\Omega_{de} / \Omega_m \approx 1/3$$

- Transition point redshift is

$$1 + z_{transition}{}^{(\Lambda)} = \left(\frac{2\varepsilon_\Lambda}{\varepsilon_m}\right)^{1/3} = \left(\frac{2\Omega_\Lambda}{\Omega_m}\right)^{1/3} \approx 1.82$$

Coincidence of “one-dimentional potentials” 1

- To analytically continue the self-consistent solution to real time, one has to do that again for graviton and ghost mode functions across the barrier x=0 in the presence of matter.
- The remarkable fact is that “one-dimensional potentials” are the same for both matter dominated background and De Sitter background

Coincidence of “one-dimentional potentials” 2

- Grishchuk (1975)

$$a = const \cdot \eta^{-\beta} \qquad a''/a = \beta(\beta+1)/\eta^2$$

$$a''/a = 2/\eta^2 \qquad \beta = -2 \qquad \beta = 1$$

- Due to the identity of equations for matter-dominated and De Sitter backgrounds, the boundary conditions for tunneling are naturally satisfied at the barrier

Coincidence” problem

- The combination of the two facts, the **existence of a threshold and coincidence of “one-dimensional potentials”** distinguishes the matter-dominated epoch from others

Tunneling back to “nothing”

- With a decrease in the contribution of matter, the Universe is increasingly emptied, and conditions for the tunneling are again approaching that for an empty space.
- Because of the identity $H_{\tau}{}^{2}\tau^{2} \equiv H^{2}t^{2}$ nothing prevents an empty Universe from tunneling back to “nothing” in the end of its cosmological evolution.

Cosmological Scenario 1

- A flat inflationary Universe could have been formed by tunneling from “nothing”. After that it should evolve in accordance to inflation scenarios that are beyond the scope of this paper. Then the standard Big Bang cosmology starts and lasts as long as the Universe begins to become empty.

Cosmological Scenario 2

- As the Universe ages and is emptied, the same mechanism of tunneling that gave rise to the empty Universe at the beginning, gives now birth to dark energy. This mechanism is switched on after the energy density of matter has dropped below a critical level.

Cosmological Scenario 3

- After that, to the extent that the space to empty, expansion takes place faster and faster, and gradually it becomes again exponentially fast (de Sitter). continues The identity (3.16) provides a possibility for the empty Universe (that has completed its cosmological evolution) to be able to tunnel back to “nothing”.
- After that, the entire scenario can be repeated indefinitely.

References

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