On Radial Reduction and Cubic Interaction for Higher Spins in (A)dS space

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We present a new modified version of the radial reduction formalism to obtain a cubic interaction of higher spin gauge fields in d+1 dimensional (A)dS space from the corresponding cubic interaction in a flat d+2 dimensional background.

Plan

- T. Biswas and W. Siegel RR and HS gauge invariance : contradiction
- New modified RR
- Interaction and modified RR
- Operator algebra and way to construct interacting massless higher spins in a direct AdS invariant way with AdS covariant derivatives.

**Ginzburg Conference
Moscow 31-May-12**

Some References

- T. Biswas and W. Siegel, ``Radial dimensional reduction: Anti-de Sitter theories from flat,'' JHEP **0207** (2002) 005 [hep-th/0203115].

- K. Hallowell and A. Waldron, ``Constant curvature algebras and higher spin action generating functions,'' Nucl. Phys. B **724** (2005) 453 [hep-th/0505255].

- E. Joung and M. Taronna, ``Cubic interactions of massless higher spins in (A)dS: metric-like approach,'' Nucl. Phys. B 861 (2012) 145 [arXiv:1110.5918 [hep-th]].

- E. Joung, L. Lopez and M. Taronna, ``On the cubic interactions of massive and partially-massless higher spins in (A)dS,'' arXiv:1203.6578 [hep-th].

Notation

We utilize instead of symmetric tensors such as $h^{(s)}_{\mu_1\mu_2\ldots\mu_s}(z)$ polynomials homogeneous in the vector a^{μ_i} of degree s at the base point "z"

$$h^{(s)}(z;a) = \sum_{\mu_i} (\prod_{i=1}^{s} a^{\mu_i}) h^{(s)}_{\mu_1\mu_2\ldots\mu_s}(z).$$

$$Tr: h^{(s)}(z;a) \Rightarrow Trh^{(s-2)}(z;a) = \frac{1}{s(s-1)}\Box_a h^{(s)}(z;a),$$

$$Grad: h^{(s)}(z;a) \Rightarrow Gradh^{(s+1)}(z;a) = (a\nabla)h^{(s)}(z;a),$$

$$*_a = \frac{1}{(s!)^2}\prod_{i=1}^{s} \overleftarrow{\partial}_a^{\mu_i} \overrightarrow{\partial}_{\mu_i}^a.$$

$$Div: h^{(s)}(z;a) \Rightarrow Divh^{(s-1)}(z;a) = \frac{1}{s}(\nabla\partial_a)h^{(s)}(z;a).$$

$$(a\nabla)f^{(m-1)}(z;a) *_a g^{(m)}(z;a) = -f^{(m-1)}(z;a) *_a \frac{1}{m}(\nabla\partial_a)g^{(m,n)}(z;a)$$

Duality relations

$$a^2 f^{(m-2)}(a) *_a g^{(m)}(a^m) = f^{(m-2)}(a,b) *_a \frac{1}{m(m-1)}\Box_a g^{(m)}(a).$$

Radial Reduction (RR)
Idea

**Gauge invariant
Interacting HS theory
in d+2 dimensional flat
space**

RR

→
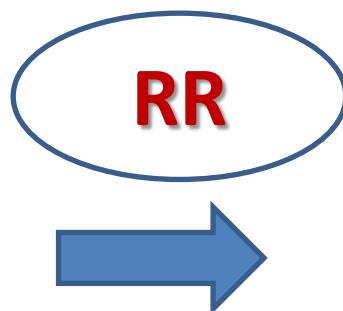
**Gauge invariant
Interacting HS theory
in d+1 dimensional AdS space**

(RR)Curvilinear Coordinates

**Curvilinear coordinates
in d+2 dimensional flat space**

$$X^{d+2} = \frac{1}{2}e^u[r + \frac{1}{r}(\pm L^2 + x^i x^j \eta_{ij})]$$

$$X^{d+1} = \frac{1}{2}e^u[r - \frac{1}{r}(\pm L^2 - x^i x^j \eta_{ij})]$$

$$X^i = e^u L \frac{x^i}{r}, \quad i = 1, 2, \ldots d$$

Corresponding flat metric

$$ds^2 = L^2 e^{2u}[\mp du^2 + \frac{1}{r^2}(\pm dr^2 + dx^i dx^j \eta_{ij})]$$

$$ds^2 = e^{2u}[-du^2 + g_{\mu\nu}(x)dx^\mu dx^\nu] = G_{uu}(u)du^2 + G_{\mu\nu}(u,x)dx^\mu dx^\nu$$

$$x^\mu = (r, x^i) \qquad\qquad g_{\mu\nu}(x) = \frac{1}{r^2}\delta_{\mu\nu}$$

(RR)Covariant Derivatives

Coordinates $X^A = (u, x^\mu)$ where $x^\mu = (r, x^i)$ AdS_{d+1} part

Metric
$$ds^2 = e^{2u}[-du^2 + g_{\mu\nu}(x)dx^\mu dx^\nu] = G_{uu}(u)du^2 + G_{\mu\nu}(u,x)dx^\mu dx^\nu$$

Christoffel symbols
$$\Gamma^u_{uu} = 1, \quad \Gamma^u_{\mu\nu} = g_{\mu\nu}, \quad \Gamma^\mu_{uv} = \delta^\mu_v, \quad \Gamma^u_{u\mu} = \Gamma^\mu_{uu} = 0,$$
$$\Gamma^\mu_{\nu\lambda} = \Gamma^{\mu(AdS)}_{\nu\lambda} = \frac{1}{2}g^{\mu\rho}(\partial_\nu g_{\lambda\rho} + \partial_\lambda g_{\nu\rho} - \partial_\rho g_{\nu\lambda}),$$

Covariant derivatives

$$\tilde{\nabla}_A h^{(s)}(X;a) = [\tilde{\nabla}_A h_{A_1 A_2 \dots A_s}(X)]a^{A_1} a^{A_2} \dots a^{A_s}$$

Operators in auxiliary a space.

$$\tilde{\nabla}_A = (\nabla_u, D_\mu)$$
$$\nabla_u = \partial_u - a^u \partial_{a^u} - a^\mu \partial_{a^\mu}$$
$$D_\mu = \nabla^{AdS}_\mu - a^u \partial_{a^\mu} - a_\mu \partial_{a^u}$$
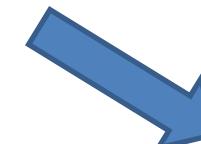
$$\partial_u h^{(s)}(X;a) = (\partial_u h_{A_1 A_2 \dots A_s})a^{A_1} a^{A_2} \dots a^{A_s}$$
$$\nabla^{AdS}_\mu h^{(s)}(X;a) = (\nabla^{AdS}_\mu h_{A_1 A_2 \dots A_s})a^{A_1} a^{A_2} \dots a^{A_s}$$

This covariant representation for derivatives gives us a simple tool for radial reduction of tensors and derivatives and extraction of corrections in an elementary algebraic way.

In Fronsdal's formulation the initial field
in d+2 dimensional flat space is double traceless

$$\Box^2_{a^A} h^{(s)}(X^A; a^A) = 0$$

Rewriting this condition in the curvilinear coordinates

$$e^{-4u}[-\partial^2_{a^u} + g^{\mu\nu}(x)\partial_{a^\mu}\partial_{a^\nu}]^2 h^{(s)}(u, x^\mu; a^u, a^\mu) = 0$$

- They solved this constraint as a set of four unconstrained
d+1 dimensional tensor fields with spins s, s-1,s-2,s-3 .
- Then they perform a **Weyl rescaling** of fields

$$h^{(s)}(u, x^\mu; a^\mu) => e^{(s-d/2)u} h^{(s)}(u, x^\mu; a^\mu)$$

to cancel   u-dependence in the measure of free action which means effectively :

$$ds^2 : e^{2u}[-du^2 + g_{\mu\nu}(x)dx^\mu dx^\nu] \Rightarrow -du^2 + g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$\int d^{d+2}X = \int du\, e^{(d+2)u} \sqrt{g}$$

• After that the authors make a Wick rotation of the u coordinates

$$u => iu \qquad\qquad \partial_u => -i\partial_u$$

• and after compactification of the u coordinate a Fourier expansion of the gauge field is performed

$$h^{(s)}(u, x^\mu; a^\mu) = \sum_{m=-\infty}^{+\infty} e^{imu} h_m^{(s)}(x^\mu; a^\mu)$$

• In order to obtain gauge field in d+1 dimensional AdS space one should perform a truncation to the separate special mode with

$$m = s + d/2 - 2$$

$$\delta_{(0)} h^{(s)}(u, x; a^\mu) = se^{(d/2-s)u}(a^u \nabla_u + a^\mu D_\mu)e^{(s-d/2)u}\varepsilon^{(s-1)}(u, x; a^\mu)$$

$$= s[a^\mu \nabla_\mu + a^u(\partial_u - s - d/2 + 2)]\varepsilon^{(s-1)}(u, x; a^\mu)$$

$$m = s + d/2 - 2$$

$$\delta_{(0)} h_{(s+d/2-2)}^{(s)}(x; a^\mu) = s(a^\mu \nabla_\mu)\varepsilon_{(s+d/2-2)}^{(s-1)}(x; a^\mu)$$

- which we cannot say about the gauge transformation of the complex conjugate mode and gauge parameter $h^{(s)}_{(-m)}(x;a^\mu)$, $\varepsilon^{(s-1)}_{(-m)}(x;a^\mu)$.

- and therefore the natural restriction $h^{(s)*}_m = h^{(s)}_m$ to only one mode in the reduced action leads to a **contradiction** with the **gauge transformation rule**.

- In other words we cannot satisfy the first order differential equation

$$(\partial_u - s - d/2 + 2)\varepsilon^{(s-1)}(u,x;a^\mu) = 0$$

  expanding the real parameter $\varepsilon^{(s-1)}(u,x;a^\mu)$ in cos(mu) and sin(mu)

- This would force us to switch on another component of the initial d+2 dimensional tensor

$$h^{(s)}(u,x;a^u,a^\mu) = \phi^{(s-1)}_u(u,x;a^\mu)a^u = \phi^{(s-1)}(u,x)_{u\mu_1\cdots\mu_{s-1}} a^u a^{\mu_1}\ldots a^{\mu_{s-1}}$$

- But in that case we arrive in d+1 dimensional AdS at the set of four unconstrained fields and two unconstrained parameters instead of one double traceless gauge field and one traceless parameter. To get correctly the action for one massless field , one obtain a consistent truncation to one double traceless field, which is more or less clear in the quadratic free case but remains completely obscure in the case of an interaction.

Modified Radial Reduction

To overcome this difficulty we propose:

1) Restrict all double traceless higher spin gauge fields to corresponding double traceless tensors in space of dimension one less

$$h^{(s)}(X^A;a^A) \equiv a^{A_1} a^{A_2} \dots a^{A_s} h^{(s)}_{A_1 A_2 \dots A_s}(u,x^\mu) => h^{(s)}(u,x^\mu;a^\mu)$$

$$h^{(s)}(u,x^\mu;a^\mu) \equiv h^{(s)}(X^A;a^A)\big|_{a^u=0} = a^{\mu_1} a^{\mu_2} \dots a^{\mu_s} h^{(s)}_{\mu_1 \mu_2 \dots \mu_s}(u,x^\mu)$$

2) And second: instead of Weyl rescaling and the Kaluza-Klein mode extraction, just a simple restriction of the "u" dependence for the gauge field and the same for the gauge parameter:

$$h^{(s)}(u,x^\mu;a^\mu) = e^{2(s-1)u} h^{(s)}(x^\mu;a^\mu) \qquad \varepsilon^{(s-1)}(u,x;a^\mu) = e^{2(s-1)u} \varepsilon^{(s-1)}(x^\mu;a^\mu)$$

$$\delta_{(0)} h^{(s)}(u,x;a^\mu) = s[a^\mu \nabla_\mu + a^u(\partial_u - 2(s-1))]\varepsilon^{(s-1)}(u,x;a^\mu)$$

$$\delta_{(0)} h^{(s)}(x;a^\mu) = s a^\mu \nabla_\mu \varepsilon^{(s-1)}(x;a^\mu)$$

Modified Radial Reduction

Price to pay

$$\mathcal{F}^{(s)}(X^A;a^A) = e^{2(s-2)u}\,\mathcal{F}^{(s)}(x;a^\mu)$$

$$\mathcal{F}^{(s)}(X^A;a^A) = \Box_{d+2}\,h^{(s)}(X^A;a^A) - (a^A\tilde\nabla_A)[(\tilde\nabla^A\partial_{aA})h^{(s)}(X^A;a^A) - \tfrac{1}{2}(a^A\tilde\nabla_A)\Box_{aA}\,h^{(s)}(X^A;a^A)]$$

$$\mathcal{F}^{(s)}(x;a^\mu) = \Box_{d+1}\,h^{(s)}(x^\mu;a^\mu) - (a^\mu\nabla_\mu)[(\nabla^\nu\partial_{a^\nu})h^{(s)}(x;a^\mu) - \tfrac{1}{2}(a^\nu\nabla_\nu)\Box_{a^\mu}\,h^{(s)}(x;a^\mu)]$$

$$-[s^2 + s(d-5) - 2(d-2)]h^{(s)}(x^\mu;a^\mu)) - g_{\mu\nu}a^\mu a^\nu h^{(s)}(x^\mu;a^\mu)$$

$$(\nabla^\nu\partial_{a^\nu})\mathcal{F}^{(s)}(x;a^\mu) - \tfrac{1}{2}(a^\nu\nabla_\nu)\Box_{a^\mu}\,\mathcal{F}^{(s)}(x;a^\mu) = 0$$

Modified Radial Reduction

$$S_0[h^{(s)}(X^A; a^A)] = \left[\int du\, e^{(d+2s-4)u} \right] \times S_0[h^{(s)}(x^\mu; a^\mu)]$$

$$S_0[h^{(s)}(X^A; a^A)] = \int d^{d+2} X [-\frac{1}{2} h^{(s)}(X^A; a^A) *_{a^A} \mathcal{F}^{(s)}(X^A; a^A)$$

$$+ \frac{1}{8s(s-1)} \Box_{a^A} h^{(s)}(X^A; a^A) *_{a^A} \Box_{a^A} \mathcal{F}^{(s)}(X^A; a^A)]$$

$$S_0[h^{(s)}(x^\mu; a^\mu)] = \int d^{d+1} x \sqrt{g} [-\frac{1}{2} h^{(s)}(x; a^\mu) *_{a^\mu} \mathcal{F}^{(s)}(x; a^\mu)$$

$$+ \frac{1}{8s(s-1)} \Box_{a^\mu} h^{(s)}(x; a^\mu) *_{a^\mu} \Box_{a^\mu} \mathcal{F}^{(s)}(x; a^\mu)]$$

Cubic interactions for arbitrary spins: Leading terms

$$\mathcal{L}_I^{(0,0)}(h^{(s_1)}(a), h^{(s_2)}(b), h^{(s_3)}(c)) = \sum_{n_i} C_{n_1,n_2,n_3}^{s_1,s_2,s_3} \int dz_1 dz_2 dz_3 \delta(z_3 - z)\delta(z_2 - z)\delta(z_1 - z)$$

$$\hat{T}(Q_{12}, Q_{23}, Q_{31} \mid n_1, n_2, n_3) h^{(s_1)}(z_1; a) h^{(s_2)}(z_2; b) h^{(s_3)}(z_3; c)$$

where

$$\hat{T}(Q_{12}, Q_{23}, Q_{31} \mid n_1, n_2, n_3) = (\partial_a \partial_b)^{Q_{12}} (\partial_b \partial_c)^{Q_{23}} (\partial_c \partial_a)^{Q_{31}} (\partial_a \nabla_2)^{n_1} (\partial_b \nabla_3)^{n_2} (\partial_c \nabla_1)^{n_3}$$

Number of derivatives $\quad \Delta \quad$ ⟹ $\quad n_1 + n_2 + n_3 = \Delta$

$$n_1 + Q_{12} + Q_{31} = s_1 \qquad Q_{12} = n_3 - \nu_3 \qquad n_i \geq \nu_i$$

$$n_2 + Q_{23} + Q_{12} = s_2 \qquad Q_{23} = n_1 - \nu_1$$

$$\nu_i = 1/2(\Delta + s_i - s_j - s_k)$$

$$n_3 + Q_{31} + Q_{23} = s_3 \qquad Q_{31} = n_2 - \nu_2$$

I,j,k are all different

(RM,W.Rühl, K. Mkrtchyan, 2010)

$$n_1 + n_2 + n_3 = \Delta.$$

$$\Delta_{min} = \max[s_i + s_j - s_k] = s_1 + s_2 - s_3$$

$$\Delta_{min} < \Delta < \Delta_{max}$$

$$\Delta_{max} = s_1 + s_2 + s_3$$

$$C^{s_1,s_2,s_3}_{Q_{12},Q_{23},Q_{31}} = const \begin{pmatrix} 1/2(\sum s_i - \Delta) \\ Q_{12}, Q_{23}, Q_{31} \end{pmatrix}$$

Cubic interactions of HS fields and Modified RR

$$\mathcal{L}_{Int}^{main}(h^{(s_1)}(X,a^A),h^{(s_2)}(X,b^A),h^{(s_3)}(X,c^A)) = \sum_{Q_{ij}} C_{Q_{12},Q_{23},Q_{31}}^{s_1,s_2,s_3} \int d^{d+2}X$$

$$*_{c^A}^{Q_{31}+n_3} K^{(s_1)}(Q_{31},n_3;c^A,a^A;X) *_{a^A}^{Q_{12}+n_1} K^{(s_2)}(Q_{12},n_1;a^A,b^A;X) *_{b^A}^{Q_{23}+n_2} K^{(s_3)}(Q_{23},n_2;b^A,c^A;X)$$

where

$$K^{(s_1)}(Q_{12},n_1;a^A,b^A;X) = (a^A\partial_{b^A})^{Q_{12}}(a^B\tilde{\nabla}_B)^{n_1} h^{(s_1)}(X;b^C)$$

So we see that we can express our cubic interaction as a cube of a bitensor function

$$K^{(s_i)}(Q_{12}, n_1; a^A, b^A; X) = e^{2(s_i - 1)u} K^{(s_i)}_{AdS}(Q_{12}, n_1; a^\mu, b^\mu; x)$$

$$\int d^{d+2} X = \int du \, e^{(d+2)u} \sqrt{g}$$

$$*^{Q_{12}+n_1}_{\tilde{a}} = e^{-2(Q_{12}+n_1)u} \prod_{k=1}^{Q_{12}+n_1} (\overleftarrow{\partial}_a{}^{u_k} \overrightarrow{\partial}_a{}^{u_k} + \overleftarrow{\partial}_a{}^{\mu_k} g^{\mu_k \nu_k} \overrightarrow{\partial}_a{}^{\nu_k})$$

$$e^{[2(s_1-1)+2(s_2-1)+2(s_3-1)]u}$$

$$*^{Q_{23}+n_2}_{\tilde{b}} = e^{-2(Q_{23}+n_2)u} \prod_{k=1}^{Q_{23}+n_2} (\overleftarrow{\partial}_b{}^{u_k} \overrightarrow{\partial}_b{}^{u_k} + \overleftarrow{\partial}_b{}^{\mu_k} g^{\mu_k \nu_k} \overrightarrow{\partial}_b{}^{\nu_k})$$

$$\partial_u => 2(s_i - 1)$$

$$*^{Q_{31}+n_3}_{\tilde{c}} = e^{-2(Q_{31}+n_3)u} \prod_{k=1}^{Q_{31}+n_3} (\overleftarrow{\partial}_c{}^{u_k} \overrightarrow{\partial}_c{}^{u_k} + \overleftarrow{\partial}_c{}^{\mu_k} g^{\mu_k \nu_k} \overrightarrow{\partial}_c{}^{\nu_k})$$

$$e^{[d-4-2(\sum Q_{ij} + \sum n_i - \sum s_i)]u} \implies e^{[\sum s_i - \Delta + d - 4]u}$$

$$\sum s_i - \Delta_{min} + d - 4 = d + 2s_3 - 4,$$

$$s_1 \geq s_2 \geq s_3$$

Cubic interactions for arbitrary spins: Leading terms

For Selfinteraction $\qquad s_1 = s_2 = s_3 \qquad$ **and minimal numbers of derivatives**

$$S_0[h^{(s)}(X^A; a^A)] = \left[\int du\, e^{(d+2s-4)u} \right] \times S_0[h^{(s)}(x^\mu; a^\mu)]$$

$$S_{Int}[h^{(s)}(X^A; a^A)] = \left[\int du\, e^{(d+2s-4)u} \right] \times S_{Int}[h^{(s)}(x^\mu; a^\mu)]$$

supplemented with

$$\delta_{(0)} h^{(s)}(X^A; a^A) = e^{2(s-1)u} \delta_{(0)} h^{(s)}(x^\mu; a^\mu)$$

$$\delta_{(1)} h^{(s)}(X^A; a^A) = \varepsilon^{A_1 \ldots A_s} \nabla_{A_1} \ldots \nabla_{A_s} h^{(s)}(X^A; a^A) + \ldots$$

$$= e^{2(s-1)u} \delta_{(1)} h^{(s)}(x^\mu; a^\mu)$$

Operator algebra and way to AdS

The main object of investigation is the bitensorial function

$$K^{(s)}(Q, n; \tilde{a}, \tilde{b}; X) = (\tilde{a}\partial_{\tilde{b}})^Q (\tilde{a}\tilde{\nabla})^n h^{(s)}(X; \tilde{b})$$

$$h^{(s)}(X; \tilde{b})\big|_{b^u=0} = h^{(s)}(u, x^\mu; b^\mu) = e^{2(s-1)u} h^{(s)}(x^\mu; b^\mu)$$

$$(\tilde{a}\tilde{\nabla})^n = (a^0 \nabla_u + a^\mu D_\mu)^n$$

$$D_\mu = \nabla_\mu - a^0 \partial_{a^\mu} - a_\mu \partial_{a^0} - b^0 \partial_{b^\mu} - b_\mu \partial_{b^0}$$

$$\nabla_u = \partial_u - a^0 \partial_{a^0} - a^\mu \partial_{a^\mu} - b^0 \partial_{b^0} - b^\mu \partial_{b^\mu}$$

Operator algebra and way to AdS

The operator of interest is

$$(\tilde{a}\tilde{\nabla})^n = ((a, \hat{\nabla}) - R)^n$$

where

$$\hat{\nabla}_\mu = \nabla_\mu^{AdS}(g) - R_\mu^b$$

$$R = a^0[2(N_{a^\mu} + N_{a^0}) - s + 2] + (a^2 - (a^0)^2)\partial_{a^0}$$

$$N_{a^\mu} = a^\mu \partial_{a^\mu}, \qquad N_{a^0} = a^0 \partial_{a^0}$$

$$R_\mu^b = b^0 \partial_{b^\mu} + b_\mu \partial_{b^0}$$

We should evaluate
on the ground state

$$|0> = e^{2(s-1)u} h^{(s)}(x^\mu; b^\mu)$$

where

$$a^\mu \partial_{a^\mu} |0> = a^0 \partial_{a^0} |0> = 0$$

$$R |0> = (2 - s)a^0 |0>$$

Operator algebra and way to AdS

$$[(a, \hat{\nabla}) - R]^n \, | 0 > = \sum_{p=0}^{n} (-1)^p (a, \hat{\nabla})^{n-p} \sum_{n-p \geq i_p \geq i_{p-1} \geq i_{p-2} \ldots \geq i_1 \geq 0} \phi_{i_p} \phi_{i_{p-1}} \ldots \phi_{i_2} \phi_{i_1} \, | 0 >$$

Where ϕ_{i_k} is a very simple "creation" operators

$$\phi_{i_k} = a^0 [2(i_k + k) - s] + [a^2 - (a^0)^2] \partial_{a^0}$$
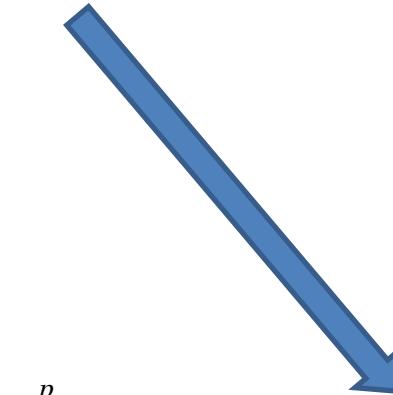
$$\sum_{k=0}^{[\frac{p}{2}]} \{n - p\}_{p-2k, 2k} (a^0)^{p-2k} (a^2)^k$$

The last stage

$$(a, \hat{\nabla})^p$$

Where

$$\hat{\nabla}_\mu = \nabla_\mu^{AdS}(g) - R_\mu^b$$

$$R_\mu^b = b^0 \partial_{b^\mu} + b_\mu \partial_{b^0}$$

Operator algebra and way to AdS

$$(a, \hat{\nabla})^p = [(a, \nabla^{AdS}) - (L^+ + L^-)]^p = \sum_{n=0}^{p} (-1)^n \binom{p}{n} (a, \nabla^{AdS})^{p-n} (L^+ + L^-)^n$$

where L^+, L^- generate a Lie algebra

$$L^+ = b^0 (a, \partial_b), \quad L^- = (a, b) \partial_{b^0}$$

$$[L^+, L^-] = H = a^2 b^0 \partial_{b^0} - (a, b)(a, \partial_b)$$

$$[H, L^{\pm}] = \pm 2 a^2 L^{\pm}$$

Now evaluate
on the "ground states" spanned by the vectors

$$\Phi_n(a; b) = h^{(s)}_{\mu_1, \mu_2, \ldots \mu_s} a^{\mu_1} a^{\mu_2} \ldots a^{\mu_n} b^{\mu_{n+1}} b^{\mu_{n+2}} \ldots b^{\mu_s}, \qquad L^- \Phi_n(a; b) = 0$$

These "ground states" crate representation of Lee algebra

Operator algebra and way to AdS

We start from

$$(L^+ + L^-)^n \Phi_0(b) = \sum_{p=1}^{n} (-1)^p (L^+)^{n-p} \sum_{p=i_p \geq i_{p-1} \geq i_{p-2} \ldots \geq i_2 \geq i_1 \geq 1} \psi_{i_p - p + 1} \psi_{i_{p-1} - p + 2} \psi_{i_{p-2} - p + 3} \ldots \psi_{i_2 - 1} \psi_{i_1} \Phi_0(b)$$

where
$$\psi_i = iH + [i]_2 a^2$$

The ansatz

$$H^n \Phi_0(b) = (-1)^n \{ [s]_n (a,b)^n \Phi_n(a;b) + \sum_{r=1}^{n-1} A_{n-r}^{(n)} [s]_{n-r} (a^2)^r (a,b)^{n-r} \Phi_{n-r}(a;b) \}$$

implies the recursion

$$A_{r-1}^{(n)} + r A_r^{(n)} = A_r^{(n+1)}$$

$$A_r^{(n)} = 0 \quad for \quad r > n$$

Conclusion

- We propose the new modification of Radial Reduction to obtain main terms of the cubic selfinteraction in AdS from flat space cubic interaction with the minimal number of derivatives
- This method yields an interaction Lagrangian which is infrared divergent by an infinite factor. The factorization allows us to renormalize the full interacting action by extracting the infinite overall factor.

What should be done

- ❏ Direct check for s=2 case (in progress)
- ❏ Include all terms of interaction in flat space