SPIN DEPENDENT ADDITION TO THE MASS OF RELATIVISTIC ELECTRON IN QED WITH EXTERNAL ELECTRIC FIELD

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I. Preamble

II. Experimental background

III. Method (*in brief*)

IV. Conclusions

$$\Delta m = \Delta m_0 + \Delta m_s$$

Invariants:

$$F = \frac{(eF_{\mu\nu})^2}{4m^4} \equiv \beta^2, \qquad G = \frac{(eF_{\mu\nu}e\tilde{F}_{\mu\nu})}{4m^4}$$

$$\chi = \frac{\sqrt{(eF_{\mu\nu}\bar{p}_\nu)^2}}{m^3}, \ \tilde{\gamma} = \frac{\bar{p} \cdot e\tilde{F} \cdot \bar{s}}{2m^3}, \ \tilde{\delta} = \frac{(\bar{p} \cdot eF \cdot \bar{s})}{2m^3}$$

$$\chi^{mag} = \frac{p_\perp}{m}\frac{H}{H_c} \leftrightarrow \chi^{el} = \gamma_\perp \frac{E}{E_c} \equiv \gamma_\perp \beta \leftrightarrow \chi^{cr} = \frac{p_-}{m}\frac{F}{F_c}$$

$$F_c = \frac{m^2}{e} \equiv \frac{m^2 c^3}{e\hbar}$$

$$\Delta m_s = -\frac{\alpha m \tilde{\gamma}}{\pi}[\mathcal{F}_0(\chi)+r\mathcal{F}_1(\chi)+r^2\mathcal{F}_2(\chi)+\cdots],\ r=\gamma_{\perp}^{-2}$$

Universality (Nikishov and Ritus, 1964):

Suggested criterion:

$$\mathcal{E} \ll \mathcal{E}_c,\ \mathcal{H} \ll \mathcal{H}_c \qquad (a)$$

$$\beta \equiv \frac{\mathcal{E}}{\mathcal{E}_c} \ll \chi \qquad (b)$$

Under this conditions probabilities for variety of radiation processes in 1-particle sector coincide

One should expect that correspondences between probabilities of the external field processes good seen “at the level of $\mathcal{F}_0(\chi)$ ” would not be so good “at the level of $\mathcal{F}_1(\chi),\ \mathcal{F}_2(\chi)\ldots$ ”

Experimental background

Source:
V.A. Bordovitsyn et al., **УФН, 165 (1995);**
S.R. Mane et al., **Rep. Prog. Phys. 68 (2005)**
J. Esberg, U.I. Uggerhøj, **J. Phys: Conf. Ser. 198 (2009)**

Macroscopic spin effects:

- ***Radiative polarization***
- ***spin light***

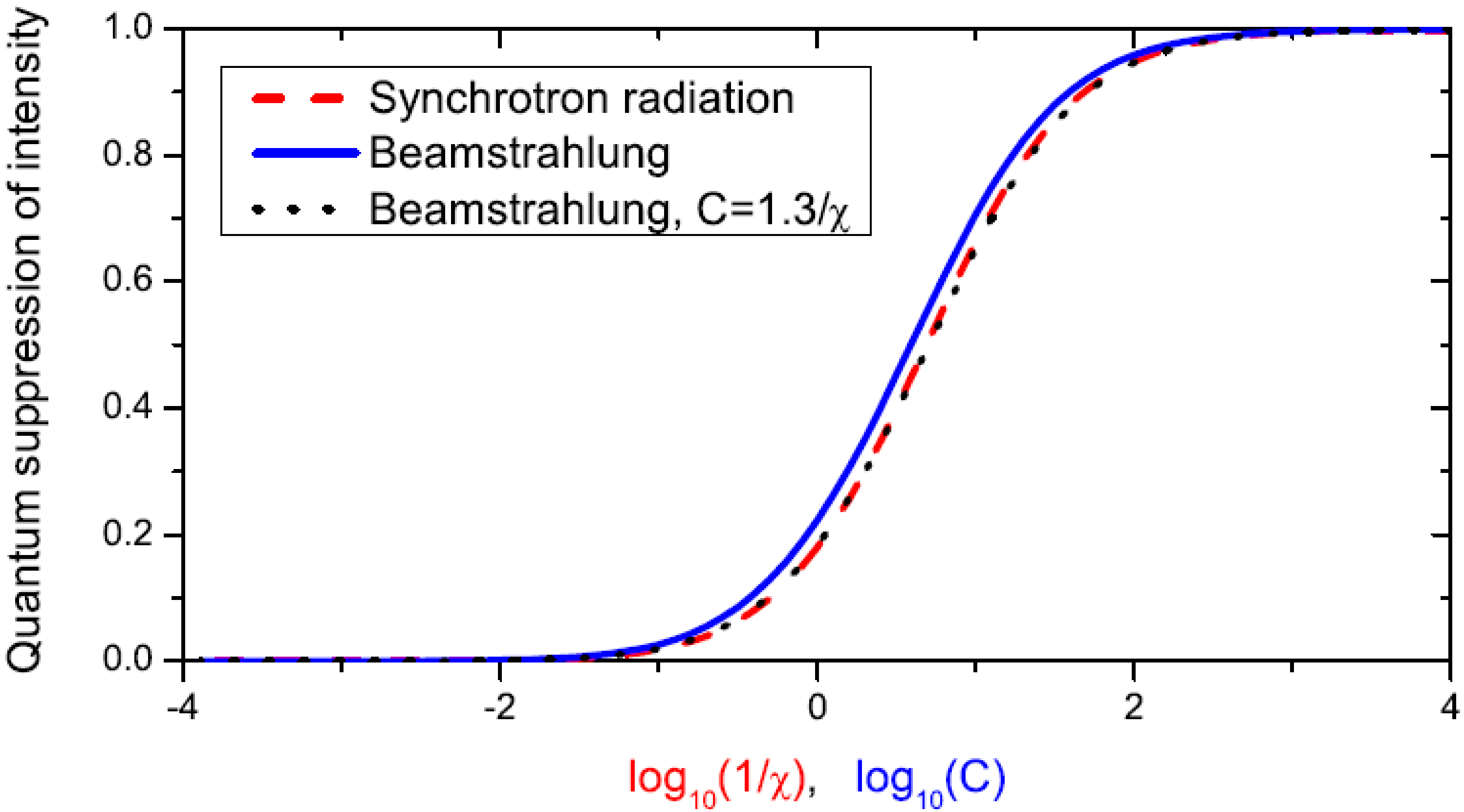
Beamstrahlung: energy of incident electron – few GeV; $1 \leq \chi \leq 10$

Channeling: crystalline electric fields; E~ few 10^{11} V/cm; $\gamma \sim 10^5 - 10^6$; $1 \leq \chi \leq 4$

General arguments: the existence of scale parameter of common meaning

Radiation processes in Beamstrahlung and Channelling have a good ‘conceptual frame’ due to QED of strong fields (Esberg & Uggerhǿj)

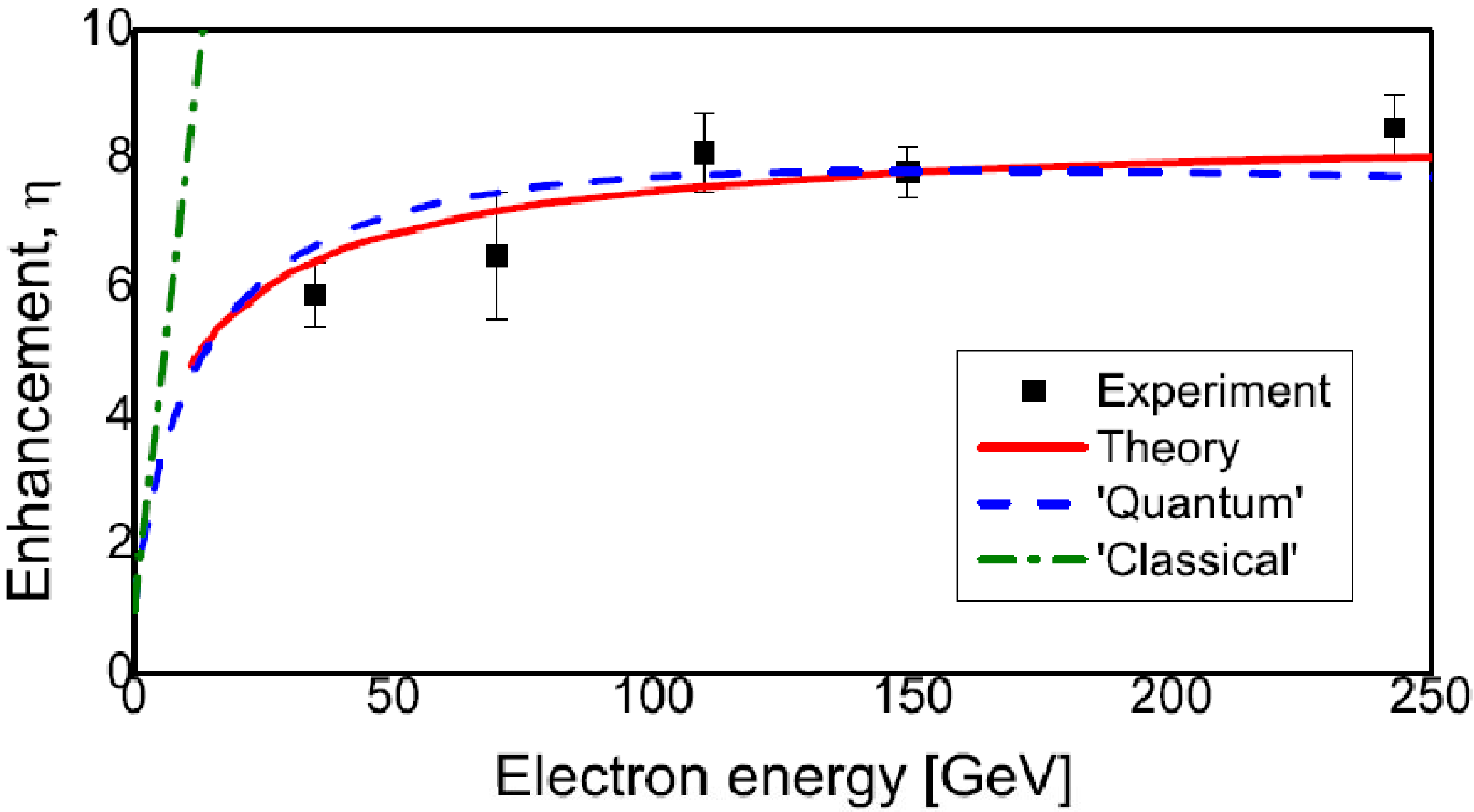
J. Esberg & U. I. Uggerhøj J. Phys. Conf. Ser. 198 (2009)



Scaling parameter for beamstrahlung:

$$C = \frac{m^2 c^3 R L}{4 N \gamma^2 e^2 \hbar} = \frac{\mathcal{E}_c}{\gamma \mathcal{E}} \leftrightarrow \frac{1}{\chi}$$

J. Esberg & U. I. Uggerhøj J. Phys. Conf. Ser. 198 (2009)



Method

$$\Delta m_s = -\frac{\alpha m\tilde{\gamma}}{\pi}\, I(\chi, r)$$

Ritus, 1978

$$I(\chi,r) = \frac{i}{\beta}\int_0^\infty dx \int_0^\infty du \frac{u}{(u+1)^2}\frac{2x\,\mathrm{cth}(x)-1}{(x\,\mathrm{cth}(x)+u)^2 - x^2}\,\mathrm{e}^{-iS/\beta r}$$

$$I(\chi,r) = \frac{i(1-r)}{\beta}\int_0^\infty dx \int_0^\infty du \frac{u}{(u+1)^4}\exp\left(-iS/\beta r\right) + r - r\int_0^\infty \frac{dx}{x}\int_0^\infty du \frac{2u}{(u+1)^3}\left(\exp\left(-iS/\beta r\right) - \exp\left(-ix/\beta\right)\right),$$

$$S \equiv S(x,u) = x - (1-r)\frac{xu}{u+1} - \mathrm{arcth}\left(\mathrm{cth}\, x + \frac{u}{x}\right)$$

$$S(x,u) = r\frac{ux}{u+1} + \frac{u}{3}\left(\frac{x}{u+1}\right)^3 + u a_5(u)\left(\frac{x}{u+1}\right)^5 + u a_7(u)\left(\frac{x}{u+1}\right)^7 + \cdots$$

$$a_5(u) = -\frac{1}{45}(u^2 + 8u - 2)\,,$$

$$a_7(u) = \frac{1}{945}(2u^4 + 24u^3 + 90u^2 - 64u + 3)\,,$$

Sabstitution: $x = \sqrt{r}(u+1)t$ ->Untwining of *x* (or *t*) and u

$$\sqrt{r}\int_0^\infty dx\int_0^\infty du\frac{u}{(u+1)^3}\,\mathrm{e}^{-i\frac{u}{\chi}\left(t+\frac{1}{3}t^3\right)}\left[1-i\frac{u}{\chi}a_5(u)rt^5-i\frac{u}{\chi}a_7(u)r^2t^7+\frac{1}{2}r^2\left(-\frac{iu}{\chi}a_5(u)t^5\right)^2+\cdots\right].$$

$$\Delta m_s=-\frac{\alpha m\tilde{\gamma}}{\pi}[\mathcal{F}_0(\chi)+r\mathcal{F}_1(\chi)+r^2\mathcal{F}_2(\chi)+\cdots],\ r=\gamma_\perp^{-2}$$

$$\mathcal{F}_0(\chi)=\frac{i}{\chi}\int_0^\infty dt\int_0^\infty du\frac{u}{(u+1)^3}\,\mathrm{e}^{-i\frac{u}{\chi}\left(t+\frac{1}{3}t^3\right)}$$

Example

$$\mathcal{F}_0(\chi)=\frac{1}{8\pi i}\int_{-i\infty}^{i\infty}dk\left(\frac{-i\chi}{\sqrt{3}}\right)^{k+1}\Gamma(-k)\Gamma(k+3)\Gamma\left(-\frac{k+1}{2}\right)\Gamma\left(\frac{3k+5}{2}\right)$$

$$\mathcal{F}_0(\chi)|_{\chi\gg 1} = \frac{\pi\Gamma(\frac{1}{3})}{9\sqrt{3}(3\chi)^{2/3}}\left(1 + \frac{6\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})(3\chi)^{2/3}} - \frac{243\left(\ln(\chi/\gamma_E\sqrt{3}) + \frac{3}{4}\right)}{2\pi\sqrt{3}\Gamma(\frac{1}{3})(3\chi)^{4/3}} + \cdots\right) +$$

$$+ i\frac{\pi\Gamma(\frac{1}{3})}{9(3\chi)^{2/3}}\left(1 - \frac{6\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})(3\chi)^{2/3}} + \frac{81}{4\Gamma(\frac{1}{3})(3\chi)^{4/3}} + \cdots\right)$$

$$\mathcal{F}_1(\chi)|_{\chi\gg 1} = -\frac{1}{10} + \frac{11}{15}\ln\left(\chi/\gamma_E\sqrt{3}\right) + \frac{37}{135}\frac{\pi\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)}{(3\chi)^{2/3}} - \frac{25}{9}\frac{\pi\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)}{(3\chi)^{4/3}} + \cdots$$

$$+ i\left(\frac{37}{35}\frac{\pi\,\Gamma\left(\frac{1}{3}\right)}{(3\chi)^{2/3}} - \frac{5\pi\sqrt{3}}{6\chi} + \frac{25}{3}\frac{\pi\Gamma\left(\frac{2}{3}\right)}{(3\chi)^{4/3}} + \cdots\right) ,$$

$$\mathcal{F}_0(\chi)|_{\chi\ll 1} = \frac{1}{2} + 6\chi^2\left(\frac{37}{12} + \ln(\chi/\gamma_E\sqrt{3}) + \cdots\right) + i\frac{\pi\sqrt{3}}{4}\chi\left(1 - 4\sqrt{3}\chi + \frac{105}{2}\chi^2 + \cdots\right)$$

$$\mathcal{F}_1(\chi)|_{\chi\ll 1} = -\chi^2\left(\frac{151}{10} + \frac{14}{3}\ln\frac{\chi}{\gamma_E\sqrt{3}} + \cdots\right) - i\frac{7\pi\sqrt{3}}{4}\chi\times$$

$$\times\left(\frac{2}{15} - \frac{4\sqrt{3}}{9}\chi + \frac{175}{24}\chi^2 + \cdots\right) .$$

$$\mathcal{F}_2(\chi)|_{\chi \gg 1} \sim \chi^2 \cdot (const, \ln \chi)$$

$$\mathcal{F}_3(\chi)|_{\chi \gg 1} \sim \chi^4 \cdot (const, \ln \chi)$$

At $\chi \gg 1$ true parameter of expansion is $r\chi^2 \equiv \beta^2$

$$\mathcal{AMM}(\chi, r)|_{\beta \gg 1} \rightarrow 2 \cdot \frac{\alpha}{2\pi}$$

V.Ritus, 2001

Abstract:

A new expression is found for the spin-dependent contribution Δm_s to the self-energy of electron moving with a transverse momentum $p_\perp$ in an electric field. The structure of the asymptotic expansion of $\Delta m_s(r,\chi)$ as a function of two dynamical invariants $r = \gamma_\perp^{-2}$ and $\chi = \gamma_\perp|\mathcal{E}|/\mathcal{E}_c$ ($\gamma_\perp^2 \equiv 1 + p_\perp^2/m^2c^2$, $\mathcal{E}_c \equiv m^2c^3/|e|\hbar$) is clarified with the aid of this expression. $\Delta m_s(r,\chi)$ can be represented as a Taylor series w.r.t. r,

$$\Delta m_s = -\frac{\alpha m\widetilde{\gamma}}{\pi}[\mathcal{F}_0(\chi)+r\mathcal{F}_1(\chi)+r^2\mathcal{F}_2(\chi)+\cdots],$$

where coefficients $\mathcal{F}_0(\chi), \mathcal{F}_1(\chi)$, etc., come up as the Mellin–type integrals and the dynamical invariant $\widetilde{\gamma} = \bar{p}eF\bar{s}/2m^3$ is expressed through conserved components of momentum and spin. The major coefficient $\mathcal{F}_0(\chi)$ is universal and, in the case of corresponding interpretation of χ, describes well–known spin-dependent additions to the mass in three different cases of a constant external field (the limit $r \to 0$ supposing). The asymptotic properties of $\mathcal{F}_1(\chi)$ are studied in detail. The orders of magnitude for $\mathcal{F}_2(\chi)$, $\mathcal{F}_3(\chi)$ are also obtained. The comparison between those contributions have shown that in the quasiclassical region $\chi \ll 1$ the parameter of the above mentioned expansion is really r, whereas at $\chi \gg 1$ the true parameter is $r\chi^2 \equiv \beta^2$. In particular, the anomalous magnetic moment acquires, thanks to $\mathcal{F}_1$, a contribution logarithmically growing at $\chi \gg 1$. This does not violate the hierarchy of the terms of Taylor series being considered, provided that β remains smaller than unity.