Anomalous diffusion of the cosmic rays in the Galaxy

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Normal diffusion model

If the propagation process is determined by scattering at magnetic field inhomogeneities which have small-scale characters and can be considered as a homogeneous Poisson ensemble, it can be described by a normal diffusion.

Normal diffusion equation

$$\frac{\partial N(\vec{r},t,E)}{\partial t} = D(E)\Delta N(\vec{r},t,E) + S(\vec{r},t,E).$$

$N(\vec{r},t,E)$ is the density of particles with energy E at location $\vec{r}$ and time t;
$D(E)$ is the diffusion coefficient;
$S(\vec{r},t,E)$ is the distribution density of a galactic sources.

Width of the diffusion packet

$$\Delta(t) = \langle r^2(t)\rangle^{1/2} \propto t^{1/2}$$

Gaussian propagator

During a last few decades many evidences, both from theory and observations, of the existence of multiscale structures in the Galaxy have been found. Filaments, shells, clouds are entities widely spread in the ISM. A rich variety of structures can be related to the fundamental property of turbulence called intermittency.
In such a fractal-like ISM we certainly do not expect the normal diffusion to hold.

Anomalous diffusion

Generalization of this equation leads to anomalous diffusion (AD). In this case

$$\Delta(t) \propto t^{p/2},$$

where the exponent p differs from 1, a value that corresponds to the normal diffusion.

$p > 1$ — superdiffusion regime,
$p < 1$ — subdiffusion regime.

AD has a non-Gaussian propagator.

Anomalous diffusion

In our recent papers[1–4] we proposed an AD model for solution of the «knee» problem in primary cosmic-rays spectrum. The anomaly results from large free paths (Levy flights) of particles between galactic inhomogeneities, and from the fact that a particle stays in a trap for a long time:

$$\int\limits_{|x|>r} p(x)dx \propto Ax^{-\alpha}, \quad 0 < \alpha < 2, \tag{1}$$

$$\int\limits_{t}^{\infty} q(\tau)d\tau \propto Bt^{-\beta}, \quad 0 < \beta < 1, \tag{2}$$

[1] Lagutin A.A., Nikulin Yu.A., Uchaikin V.V. *Nucl. Phys. B*, 2001.
[2] Lagutin A. A., Uchaikin V. V. *NIM B*, 2003.
[3] Erlykin A. D., Lagutin A. A., Wolfendale A.W. *Astropart. Phys.*, 2003.
[4] Lagutin A. A., Tyumentsev A. G., *Izv. Altai. Gos. Univ.*, 2004 (in Russian).

Fractional diffusion equation

Without energy losses ($\alpha \in (0, 2]$, $\beta \in (0, 1]$)

$$\frac{\partial N}{\partial t} = -D(E, \alpha, \beta)\mathrm{D}_{0+}^{1-\beta}(-\Delta)^{\alpha/2} N(\vec{r}, t, E) + S(\vec{r}, t, E), \quad (3)$$

$(-\Delta)^{\alpha/2}$ is the fractional Laplacian (Riss operator), D_{0+}^{β} is the Riemann-Liouville fractional derivative.

With energy losses ($\alpha \in (0, 2]$, $\beta = 1$)

$$\frac{\partial N}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2} N(\vec{r}, t, E) + \\ + \frac{\partial (B(E) N(\vec{r}, t, E))}{\partial E} + S(\vec{r}, t, E), \quad (4)$$

$B(E) = -dE/dt$ is the mean rate of continuous energy losses.

Anomalous diffusivity $D(E, \alpha, \beta)$

$$D(E, \alpha, \beta) = D_0(\alpha, \beta) E^{\delta}.$$

Fractional derivatives

Riss operator

$$(-\Delta)^{\alpha/2} f(x) = \frac{1}{d_{m,l}(\alpha)} \int\limits_{\mathrm{R}^m} \frac{\Delta_y^l f(x)}{|y|^{m+\alpha}} dy, \tag{5}$$

where $d_{m,l}(\alpha)$ — normalizing factor,

$$\Delta_y^l f(x) = \sum_{k=0}^{l} (-1)^k \binom{l}{k} f(x - ky). \tag{6}$$

Riemann-Liouville operator $0 < \beta < 1$

$${}_0\mathrm{D}_t^\beta \Phi = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial t} \int\limits_0^t \frac{\Phi(x,t)}{(t-\tau)^\beta} d\tau,$$

Samco S. G., Kilbas A. A., Marichev O. I. *Fractional integrals and derivatives — Theory and Applications*, NY, Gordon and Breach, 1993.

Fractional derivatives

Riss operator

$$\int\limits_{\mathrm{R}^m} \mathrm{e}^{ikx}(-\Delta)^{\alpha/2} f(x)\,dx = |k|^\alpha \tilde{f}(k), \tag{7}$$

Riemann-Liouville operator

$$\int\limits_0^\infty e^{-\lambda t}\,{}_0\mathrm{D}_t^\beta f(t)dt = \lambda^\beta \tilde{f}(\lambda). \tag{8}$$

Samco S. G., Kilbas A. A., Marichev O. I. *Fractional integrals and derivatives — Theory and Applications*, NY, Gordon and Breach, 1993.

Solution for point instantaneous source

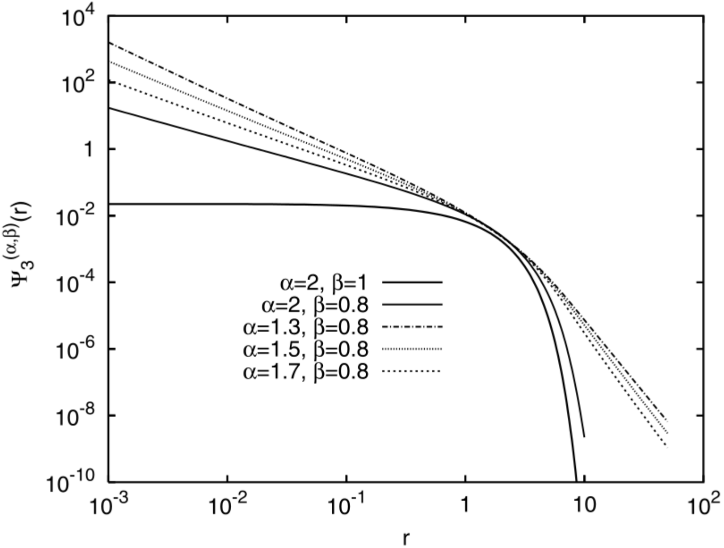
$$S(\vec{r}, t, E) = S_{\mathsf{in}} E^{-\gamma} \delta(\vec{r}) \delta(t)$$

$$N(\vec{r}, t, E) = S_{\mathsf{in}} E^{-\gamma} (D(E, \alpha, \beta) t^{\beta})^{-3/\alpha} \times \\ \times \Psi_3^{(\alpha,\beta)}(|\vec{r}|(D(E, \alpha, \beta) t^{\beta})^{-1/\alpha}), \quad (9)$$

$\Psi_3^{(\alpha,\beta)}(r)$ — is the density of fractional stable distribution.

Uchaikin V. V. Physics-Uspekhi, 2003.

Three-dimensional density of fractional stable distribution $\Psi_3^{(\alpha,\beta)}(r)$ for different values of (α, β)



Knee in the cosmic rays spectrum

Using the representation $N = N_0 E^{-\eta}$ and the asymptotic behaviour of the scaling function $\Psi_3^{(\alpha,\beta)}(r)(\alpha < 2, \beta < 1)$ one can evaluate the variation of spectral exponent.

$$\eta = \gamma - \delta, \quad E \ll E_k,$$

$$\eta = \gamma + \delta, \quad E \gg E_k,$$

E_k — knee energy

$$\Delta\eta = 2\delta$$

$$\left.\begin{array}{l} \eta|_{E \ll E_k} \sim 2.62 \\ \eta|_{E \gg E_k} \sim 3.16 \end{array}\right. \Rightarrow \begin{array}{l} \gamma \approx 2.85 \\ \delta \approx 0.3 \end{array}$$

Cosmic rays spectrum

$$J(\vec{r},t,E) = J_G(\vec{r},E) + J_L(\vec{r},t,E) =$$

$$= \frac{v}{4\pi}\left(N(\vec{r},E) + \sum_{\substack{r_j \le 1\text{kpc} \\ t_j < 10^6\text{yr}}} N(\vec{r}_j,t_j,E)\right).$$

J_G is the contribution of multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources.
J_L is the component determined by the nearby young sources ($r < 1$ kpc, $t < 10^6$ yr).

Cosmic rays spectrum

$$J(\vec{r},t,E) = \frac{v}{4\pi}N(\vec{r},E) + \frac{v}{4\pi}\frac{S_{\mathsf{im}}E^{-\gamma}}{D(E,\alpha,\beta)^{3/\alpha}}\times$$
$$\times \sum_{\substack{r_j \le 1\mathsf{kpc} \\ t_j < 10^6\mathsf{yr}}} \int\limits_{\max[0,t_j-T]}^{t_j} d\tau\tau^{-3\beta/\alpha}\Psi_3^{(\alpha,\beta)}\left(|\vec{r}_j|(D(E,\alpha,\beta)\tau^{\beta})^{-1/\alpha}\right) \quad (10)$$

$$N(\vec{r},E) \sim E^{-\gamma-\delta/\beta} \quad (11)$$

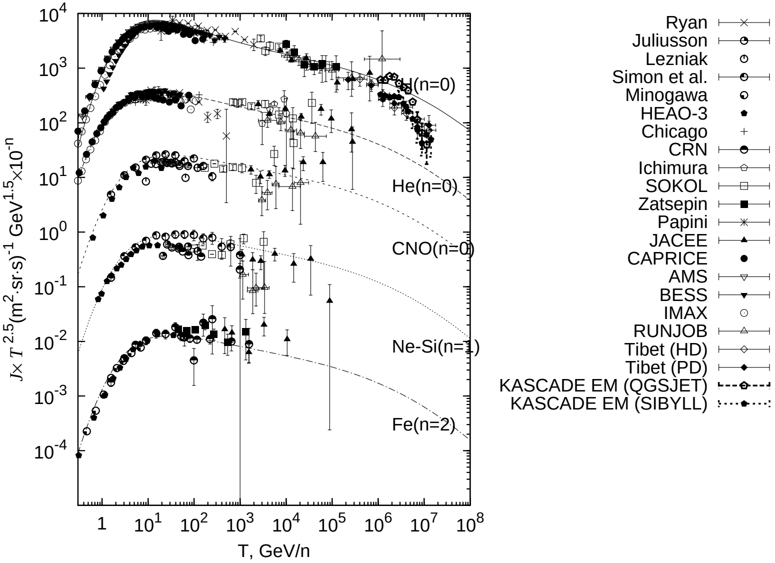
Main parameters of the AD model

$$\gamma = 2.85$$

$$\delta = 0.27$$

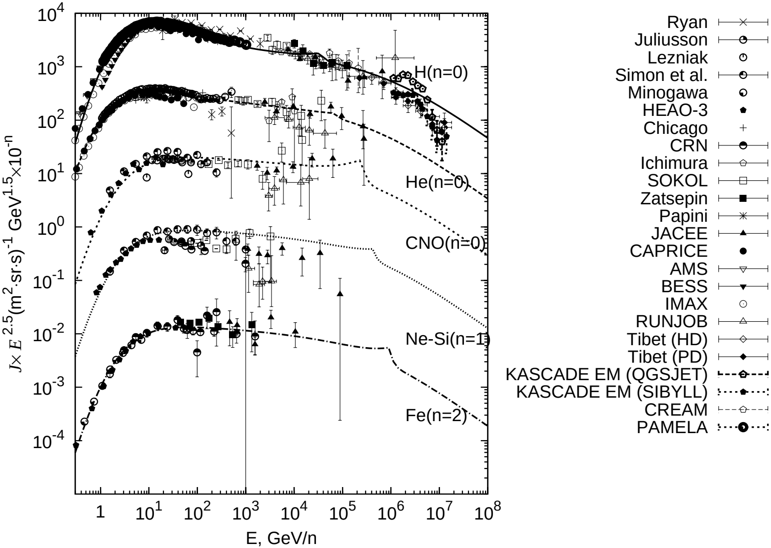
$$\left.\begin{array}{l}\alpha = 1.1 \\ \beta = 0.8\end{array}\right. \Rightarrow \quad D_0 = 10^{-4}\mathsf{pc}^{1.1}/\mathsf{yr}^{0.8}$$

Cosmic rays spectrum in the AD model

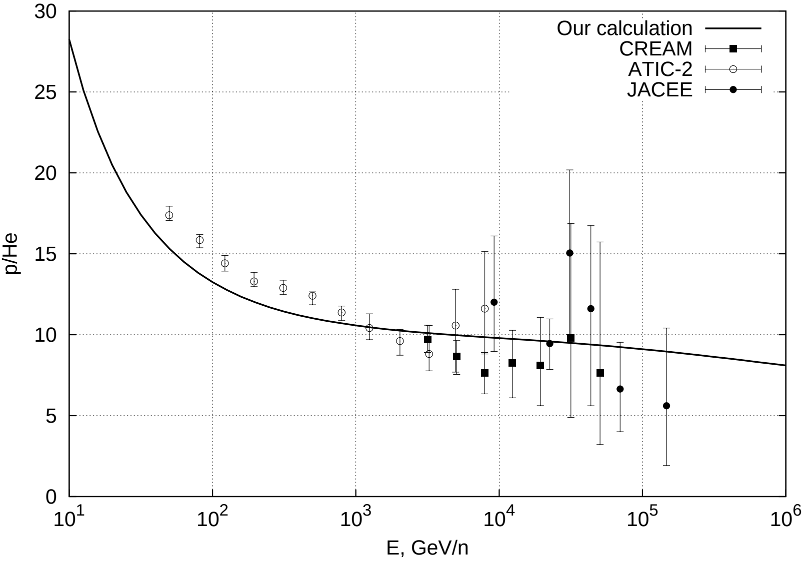


Cosmic rays spectrum in the AD model with two types of sources

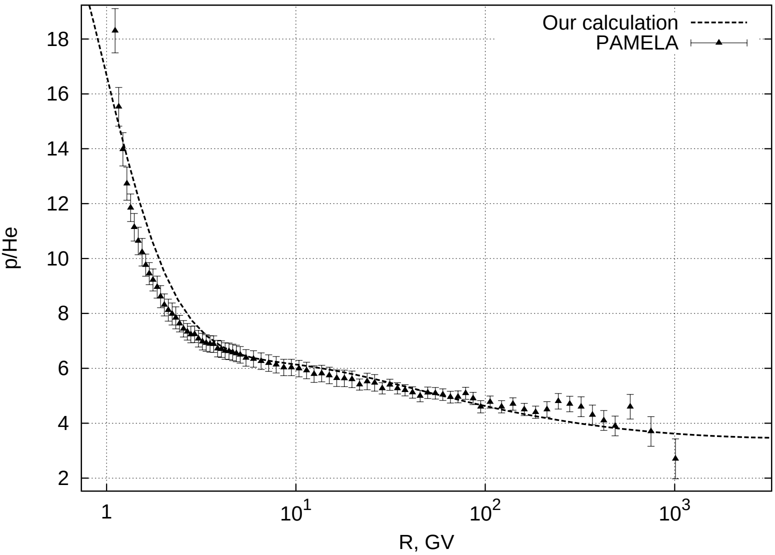
$$S(E) = S_0 E^{-2.85} + S_1 E^{-2} H(E_{\max} - E)$$



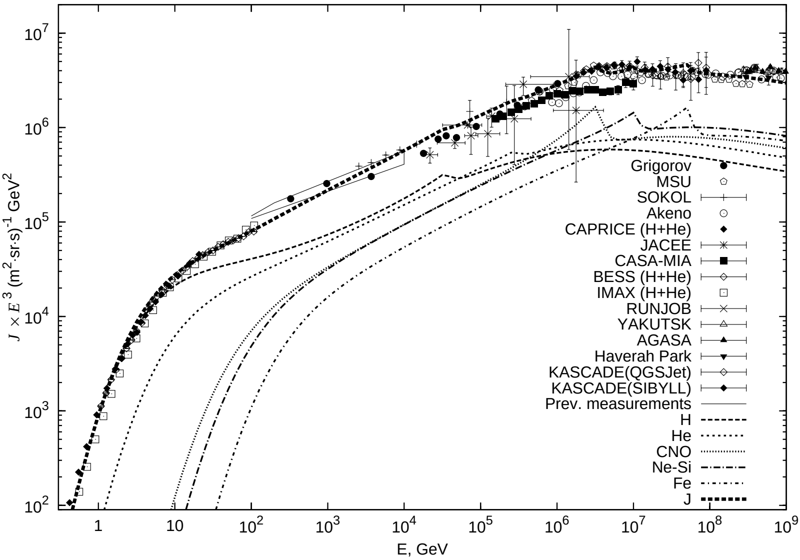
Proton to helium flux ratio as a function of energy in GeV per nucleon



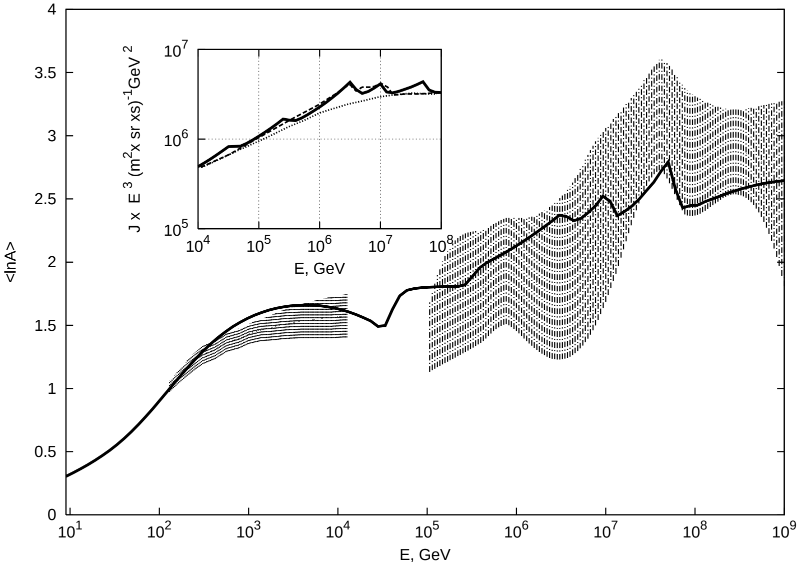
Proton to helium flux ratio as a function of rigidity



All particle spectrum in the AD model



Mean logarithmic cosmic ray mass vs. primary particle energy



Spectrum of electrons in the AD model

Energy losses

$$-\frac{dE}{dt} = B(E) = b_0 + b_1 E + b_2 E^2$$

$$J(\vec{r}, t, E) = \frac{v}{4\pi} N(\vec{r}, E) + \frac{S_{\mathsf{im}} v}{4\pi} \times$$

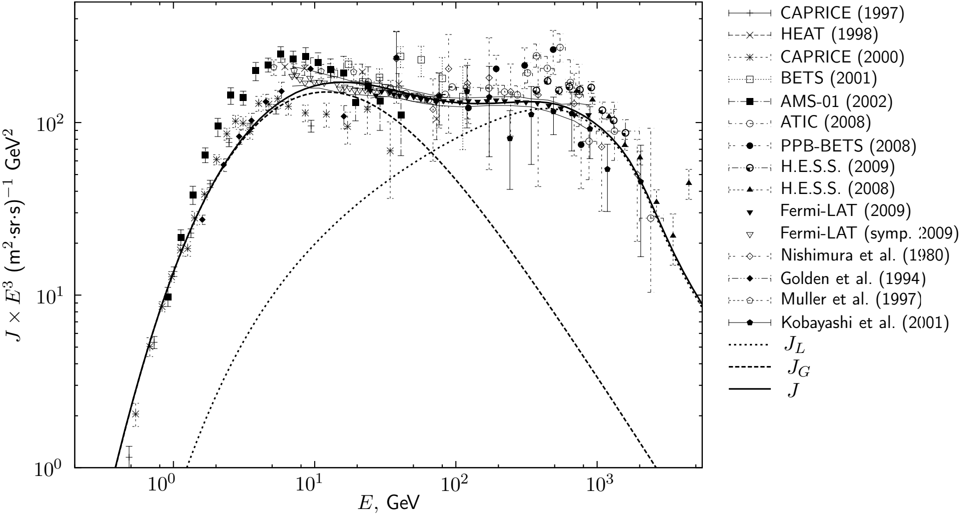
$$\times \sum_{\substack{r_j \le 1\mathsf{kpc} \\ t_j < 10^6 \mathsf{yr}}} \int\limits_{\max[0, t_j - T]}^{\min[t_j, 1/b_2(E+E_2)]} d\tau E_0(\tau)^{-\gamma} \lambda(\tau, E)^{-3/\alpha} \times$$

$$\times \left(1 - b_2 \tau (E + E_2)\right)^{-2} g_3^{(\alpha)} \left(|\vec{r}_j| \lambda(\tau, E)^{-1/\alpha} \right), \quad (12)$$

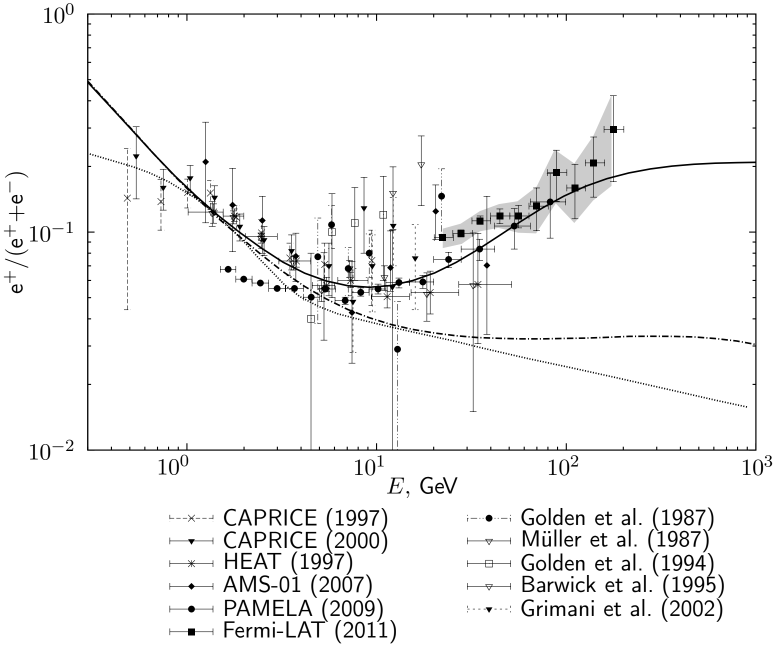
$g_3^{(\alpha)}(r)$ — is the probability density of three-dimentional spherically-symmetrical distribution.

$$E_0(t) = \frac{E + E_1}{1 - b_1 t (E + E_2)/(E_2 - E_1)} - E_1, \quad \lambda(t, E) = \int\limits_{E}^{E_0(t)} \frac{D(E', \alpha)}{B(E')} dE'$$

Spectrum of electrons in the AD model ($\alpha = 1.4$, $\beta = 1$)



Positron fraction in the AD model



Summary

- We considered the propagation of galactic cosmic rays in the fractal ISM. AD equation in terms of fractional derivates describing cosmic ray propagation taking into account anomalously long times in traps has been formulated. Asymptotical solutions of this equation covering both subdiffusive and superdiffusive regimes have been expressed in terms of stable distributions.
- Our results suggest that the knee in the primary cosmic ray spectrum is due to anomalously large free paths (Levy flights) of particles. The distribution of time in localized domains doesn't influence on the knee.
- Injection spectrum in the sources of the cosmic rays has spectral exponent $p \approx 2.85$.
- "Fine structure" of spectrum around the knee may arise due to presence of nearby supernova, accelerating particles up to energies $\sim 3 \cdot 10^4 Z$ GeV, if the energy output of such source is $\sim 2 \cdot 10^{48}$ erg/source.

Thank you for attention!