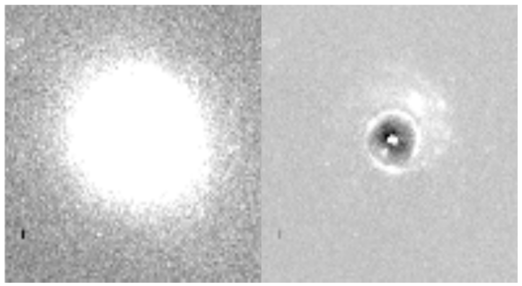
MOdified Newtonian Dynamics

application to gravitational lensing and galactic dynamics



SDSS J1538+5817

chung-ming ko
institute of astronomy and
department of physics
national central university
taiwan (R.O.C.)

GC2012, Lebedev Physical Institute, Moscow, Russia, 2012.05.29

a short discussion on

- acceleration discrepancy
- MOND
- strong lensing
- velocity dispersion

mass

- dynamical mass
  - observe motion then infer mass
  - direct (if we know the dynamics)
- luminous mass
  - observe luminosity then infer mass
  - indirect (involve many physics)
  - ideally, calibrate with dynamical mass (if we believe the dynamics)

what if they don’t agree?

- dynamical mass is often larger
  - excess acceleration unaccounted for
- some matters are not luminous
  - what are they? light is not a good tracer of mass? more physics is needed?
- gravitational law is not what we expected
  - modified gravity?
- law of motion is not what we expected
  - what to do?

minute discrepancy

- existence of Neptune
  - confirmation of dynamical mass by luminous mass (seeing is believing?)
  - successful story of missing mass
- extrasolar planets
  - believing even not seeing
- perihelion of Mercury
  - Einstein’s general relativity
  - successful story for modified gravity

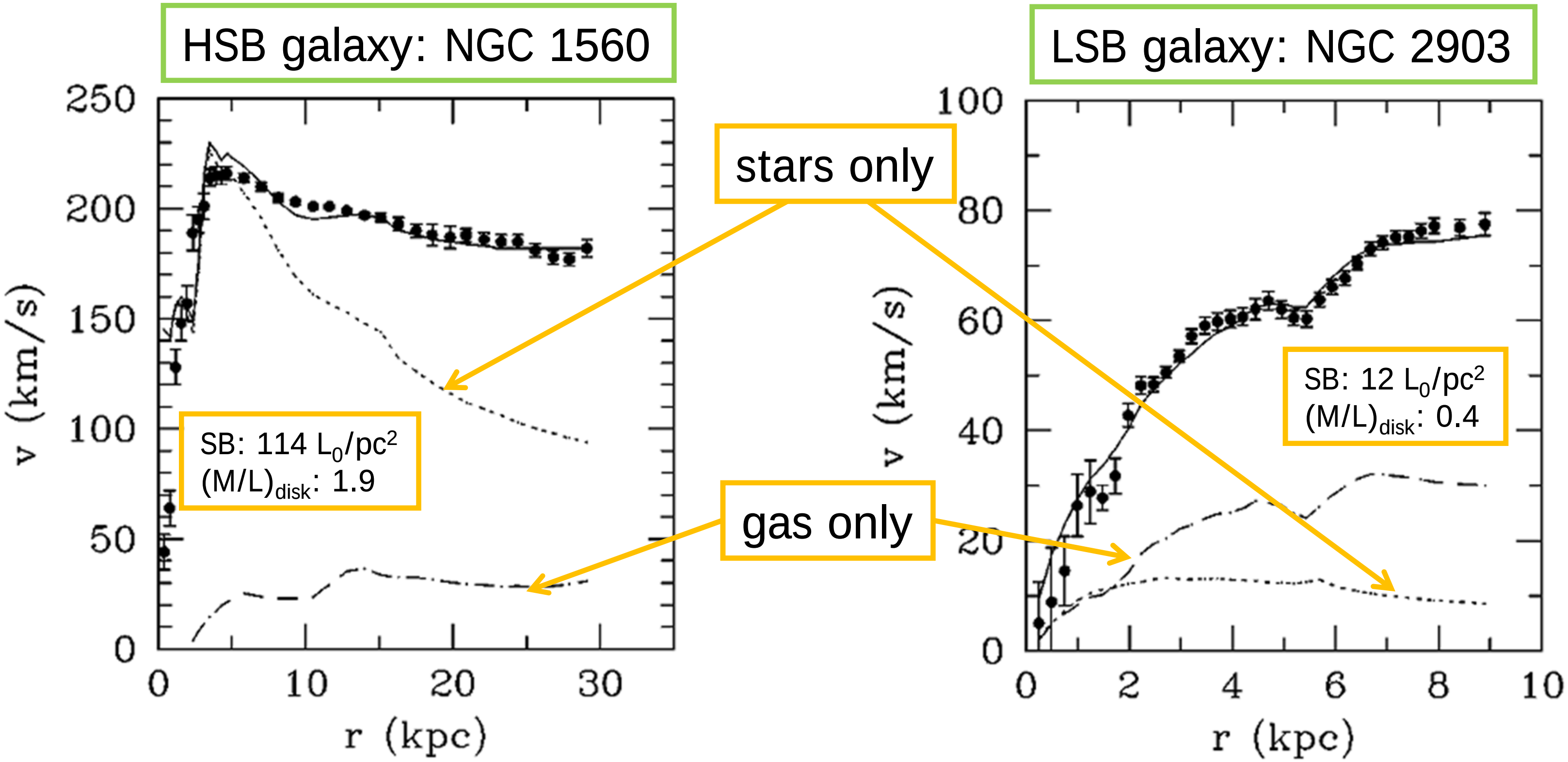
large discrepancy (O(1) or more)

- Oort (1932): acceleration of stars perpendicular to Galactic disk
- Zwicky (1933): radial velocity of galaxies in Coma cluster
- Babcock (1939), Mayall (1951): rotation curve of M31
- Kahn & Woltjer (1959): M31 approaches Milky Way against expansion of universe

large discrepancy (O(1) or more)

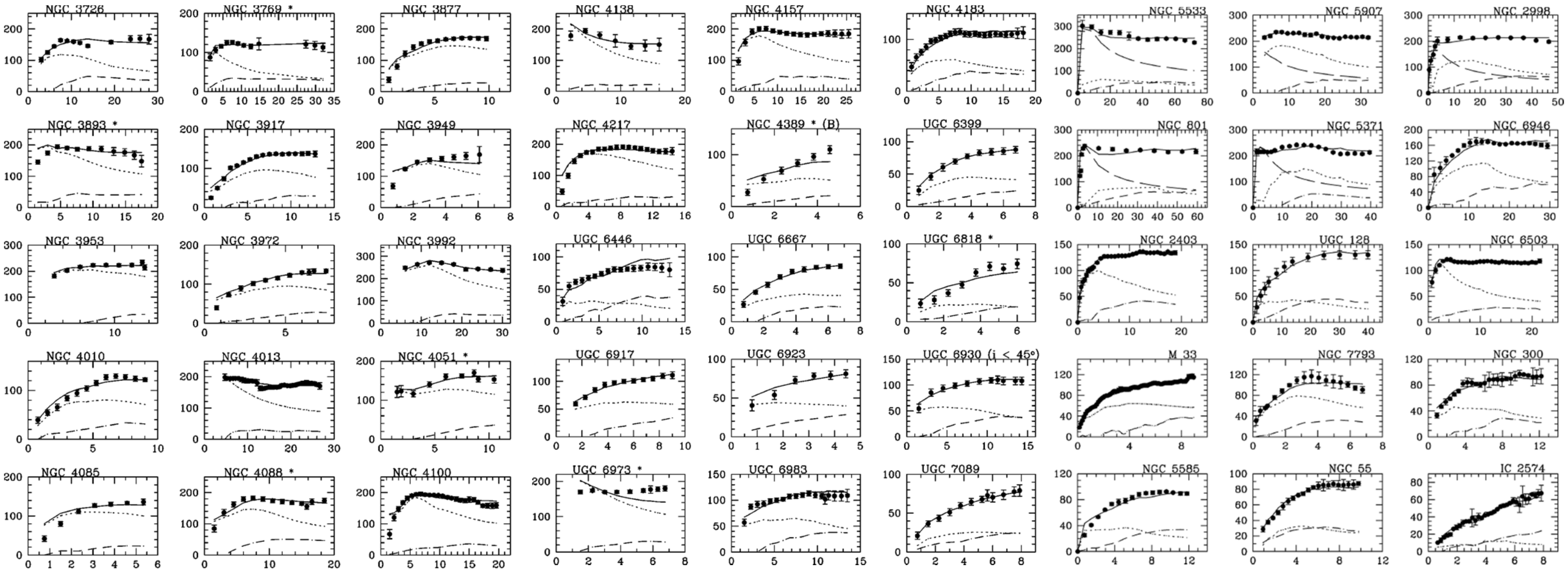
- Rogstad & Shostak (1972): rotation curve beyond optical disk of spirals from 21 cm
- Rubin et al. (1980): rotation curve of optical disk of spirals
- ...
- proper motion of stars near galactic centre (Eckart & Genzel 1997, Ghez et al. 1998)
- ...

rotation curve of spirals



Sanders & McGaugh (2002)

many more rotation curves



- both dark matter and MOdified Newtonian Dynamics (MOND) can explain the mass (or acceleration) discrepancy in many situations
- perhaps MOND is better than dark better in galaxy scales while dark matter is better at larger scales

acceleration scale?

- it seems that the mass discrepancy occurs when acceleration is smaller than a certain value, and not according to some length scale or mass scale
- of the order of cH_0 (a coincidence?)

MOND

- Milgrom (1983): small acceleration regime, Newton's law of motion is modified

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = \mathbf{a}_{\mathrm{N}}$$

acceleration

force

acceleration scale

$$\tilde{\mu}(x) \rightarrow \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases}$$

interpolation function

MOND

- view as modified gravity in gravitational systems (Bekenstein & Milgrom 1984)

$$\nabla \cdot \left[\tilde{\mu}\left(|\mathbf{g}|/a_0\right)\mathbf{g}\right] = \nabla \cdot \mathbf{g}_{\mathrm{N}} = -4\pi G\rho$$

Newtonian gravity

$$\tilde{\mu}\left(|\mathbf{g}|/a_0\right)\mathbf{g} - \mathbf{g}_{\mathrm{N}} = \nabla \times \mathbf{h}$$

$$\nabla \times \mathbf{h} = \mathcal{O}(1/r^3)$$

for bound density distribution

= 0 for spherical, cylindrical, planar systems

MOND

- if can be inverted

$$\mathbf{g} = \tilde{\nu}\left(|\mathbf{g}_\mathrm{N}|/a_0\right)\mathbf{g}_\mathrm{N}$$

acceleration

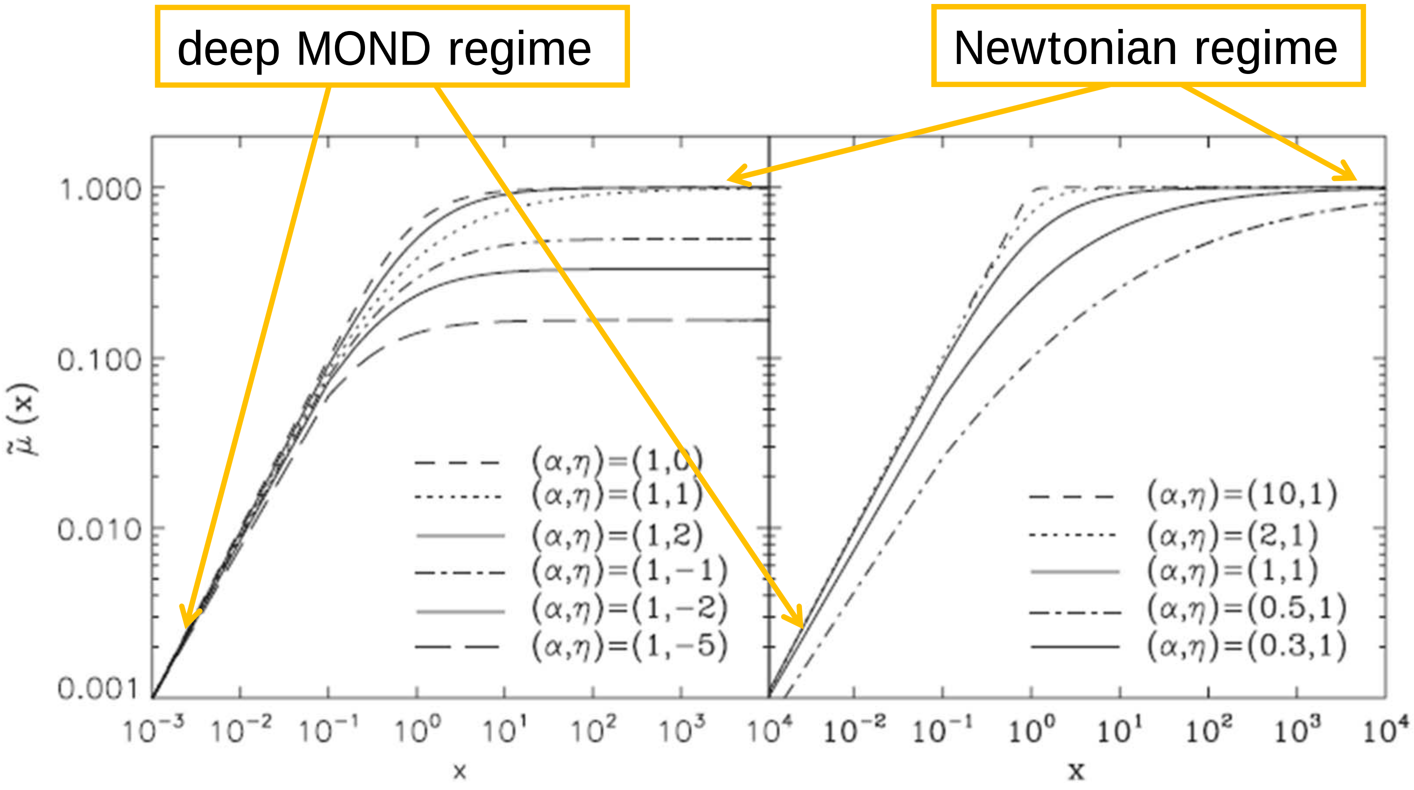
Newtonian gravitational force

$$\tilde{\nu}(x_\mathrm{N}) \rightarrow \begin{cases} 1 & \text{for } x_\mathrm{N} \gg 1 \\ 1/\sqrt{x_\mathrm{N}} & \text{for } x_\mathrm{N} \ll 1 \end{cases}$$

inverse interpolation function

$$\tilde{\nu}(x_\mathrm{N}) = \left[1 + \tfrac{1}{2}\left(\sqrt{4x_\mathrm{N}^{-\alpha} + \eta^2} - \eta\right)\right]^{1/\alpha}$$

canonical form



Chiu et al. (2011)

canonical interpolation function

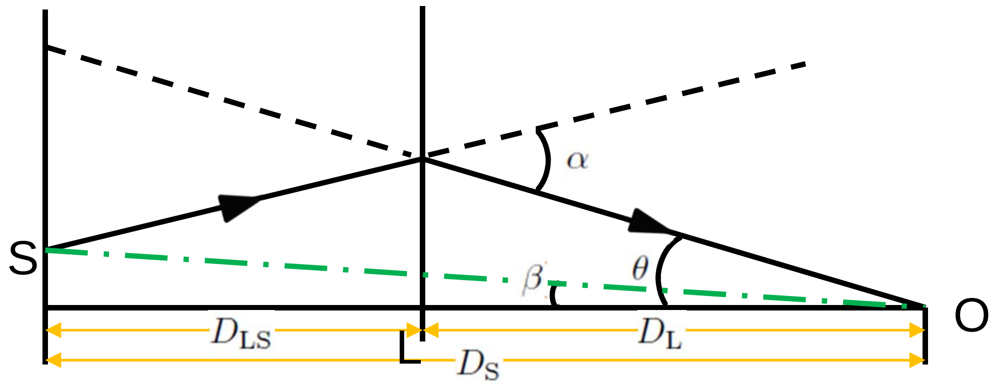
$$\tilde{\mu}(x) = \left[1 - \frac{2}{(1+\eta x^{\alpha}) + \sqrt{(1-\eta x^{\alpha})^2 + 4x^{\alpha}}}\right]^{1/\alpha}$$

relativistic theory

- Bekenstein (2004): Tensor-Vector-Scalar
- Milgrom (2009): BiMOND
- MOND as the nonrelativistic limit
- capable of addressing problems such as cosmology and gravitational lensing in the context of MOND
  - cosmology (e.g., Skordis 2006, Skordis et al. 2006, Dodelson & Liguori, 2006)
  - gravitational lensing ...

gravitational lensing

- TeVeS or BiMOND
- weak field, small angle deflection, spherical lens



  - same as GR except Newtonian potential is replaced by MONDian potential (Chiu et al. 2006)

image position

source position

$$\theta_\pm \mp \beta = \theta_\mathrm{E}^2 \chi(\theta_\pm) = \theta_\mathrm{E}^2 \int_0^\infty \frac{\theta_\pm \tilde{g}(r_\pm)}{r_\pm} \mathrm{d}\zeta$$

image position

$$r^2 = D_\mathrm{L}^2 \theta^2 + \zeta^2$$

$$\tilde{g} = \left[\tilde{g}_\mathrm{N}^\alpha + \tfrac{1}{2}\left(\sqrt{4(\tilde{g}_\mathrm{N}/\theta_0^2)^\alpha + \eta^2 \tilde{g}_\mathrm{N}^{2\alpha}} - \eta \tilde{g}_\mathrm{N}^\alpha\right)\right]^{1/\alpha}$$

$$\frac{g}{\tilde{g}} = \frac{g_\mathrm{N}}{\tilde{g}_\mathrm{N}} = \frac{G\mathcal{M}}{D_\mathrm{L}^2}$$

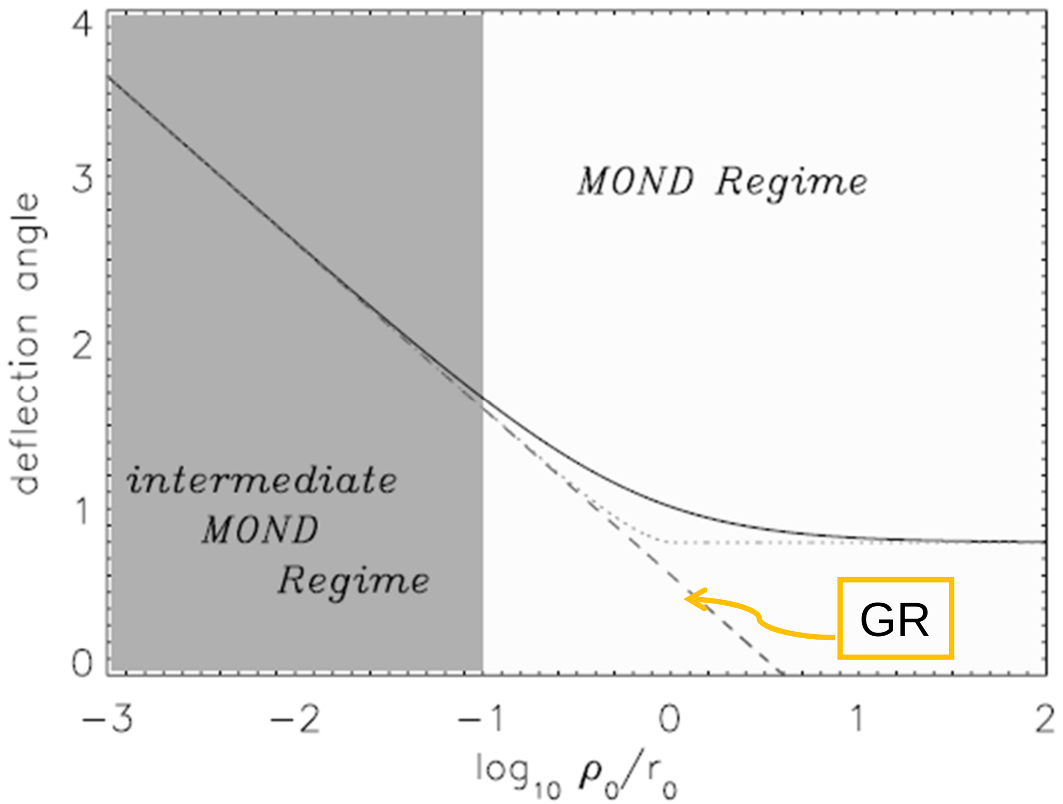
$$\theta_\mathrm{E}^2 = \frac{4G\mathcal{M}D_\mathrm{LS}}{c^2 D_\mathrm{L} D_\mathrm{S}}$$

$$\frac{1}{\theta_\mathrm{E}^2} = \frac{[\chi(\theta_+) + \chi(\theta_-)]}{(\theta_+ + \theta_-)}$$

a relation between mass, acceleration scale and Hubble constant

$$\theta_0^2 = \frac{G\mathcal{M}}{a_0 D_\mathrm{L}^2}$$

angle of deflection of a point mass lens



Chiu et al. (2006)

- CASTLES catalogue (Rusin et al. 2003)
  - quasar lensing, elliptical lens, lens resolved, double images, mass estimated by population synthesis
  - 10 candidates
  - Hernquist model for the lens
  - choose an interpolation function
  - assume a_0 and H_0 then solve for mass

mass of lens

mass consistent with population synthesis

Lens	M_L(TeVeS)$/10^{10}M_\odot$ $\frac{\langle\nabla\Phi_N\rangle}{a_0}$	Bekenstein	Simple	Standard	M_L (GR)	M_* [28] Chabrier	Salpeter
Q0142 − 100	6.9	11.3 (19.19)	14.2 (24.16)	16.6 (28.29)	19.1 (32.45)	$20.9^{30.8}_{13.0}$	$18.3^{32.2}_{13.2}$
HS0818 + 1227	6.8	18.8 (29.38)	24.0 (37.45)	28.6 (44.65)	32.7 (51.05)	$16.2^{21.2}_{12.6}$	$20.8^{28.1}_{13.4}$
FBQ0951 + 2635	11.2	1.56 (2.19)	1.93 (2.71)	2.16 (3.02)	2.36 (3.30)	$1.1^{2.1}_{0.5}$	$1.5^{3.0}_{0.8}$
BRI0952 − 0115	6.6	2.09 (2.58)	2.70 (3.33)	3.28 (4.04)	3.74 (4.61)	$3.5^{4.0}_{2.7}$	$4.4^{5.2}_{3.5}$
Q1017 − 207	6.8	2.63 (6.33)	3.36 (8.09)	4.02 (9.67)	4.60 (11.06)	$4.3^{13.0}_{1.4}$	$6.4^{19.0}_{2.3}$
HE1104 − 1805	6.6	48.1 (63.47)	62.1 (81.90)	75.3 (99.37)	85.9 (113.41)	$22.8^{51.2}_{12.7}$	$36.6^{63.7}_{23.1}$
LBQ1009 − 025	6.7	8.02 (11.23)	10.1 (14.15)	11.8 (16.53)	13.5 (18.92)	$5.5^{7.9}_{4.2}$	$7.4^{9.8}_{5.0}$
B1030 + 071	9.4	10.5 (17.88)	12.9 (22.27)	14.7 (25.06)	16.5 (28.09)	$10.6^{15.3}_{6.5}$	$14.5^{21.3}_{8.3}$
SBS1520 + 530	7.6	12.6 (17.64)	16.0 (22.41)	18.8 (26.38)	21.2 (29.67)	$18.5^{30.9}_{11.2}$	$21.8^{34.1}_{11.9}$
HE2149 − 274	7.1	7.34 (14.17)	9.31 (17.98)	11.0 (21.30)	12.5 (24.24)	$4.6^{6.7}_{3.6}$	$6.9^{8.9}_{5.0}$

Chiu et al. (2011)

- no dark matter is needed, as expected

time delay

- time delay between the two images formed by a spherical lens

$$\frac{1}{\theta_{\rm E}^2} = \frac{(1+z_{\rm L})}{c\delta T}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\left\{\tfrac{1}{2}\left(\theta_+ + \theta_-\right)\left[\chi(\theta_-) - \chi(\theta_+)\right] + \varepsilon(\theta_+) - \varepsilon(\theta_-)\right\}$$

$$\varepsilon(\theta) = \frac{c^2}{G\mathcal{M}}\int_{-D_{\rm LS}}^{D'_{\rm L}} \Phi \, {\rm d}\zeta$$

redshift of lens

time delay

MONDian potential

a relation between mass, acceleration scale and Hubble constant

- time delay systems (Danuta & Jens 2010, McGreer et al. 2010)
  - quasar lensing, elliptical lens, lens resolved, double images
  - 4 candidates
  - Hernquist model for the lens
  - choose an interpolation function
  - assume a_0 then solve for mass and H_0

Hubble constant from time delay

Bekenstein form

Name	Mass GR	Mass MOND	H_0 GR	H_0 MOND	x_-	x_+
HE 2149-2745	$23.2^{25.8}_{20.5}$	$16.1^{17.5}_{14.6}$	$72.2^{81.7}_{64.7}$	$57.6^{66.0}_{51.0}$	11.3	2.3
FBQ J0951+263	$2.9^{3.3}_{2.6}$	$2.3^{2.5}_{2.1}$	$93.2^{106.5}_{82.9}$	$79.3^{91.5}_{69.8}$	28.5	4.0
SBS 0909+532	$77.2^{78.9}_{58.3}$	$55.8^{56.8}_{44.3}$	$84.5^{111.9}_{82.7}$	$70.2^{95.0}_{68.5}$	8.7	5.9
SDSS J0946+1835	$95.3^{99.8}_{90.7}$	$64.1^{66.4}_{61.8}$	$84.9^{89.1}_{81.0}$	$67.0^{70.7}_{63.6}$	7.8	2.8

least uncertainty

Tian et al. (2012)

- H_0 large uncertainty but consistent with current value

galactic dynamics

- Jeans equation
- spherical, isotropic
- light traces mass distribution
  - deduce mass density distribution from brightness distribution
  - hence gravity distribution

velocity dispersion

- 3D velocity dispersion

$$\sigma_r^2(r) = \frac{G\mathcal{M}}{D_\mathrm{L}^2 \rho(r)} \int_r^\infty \rho(r')\tilde{g}(r')\,\mathrm{d}r'$$

- projected velocity dispersion

$$\frac{1}{\theta_\mathrm{E}^2} = \frac{c^2 D_\mathrm{S}}{2 D_\mathrm{L} D_\mathrm{LS} \sigma_I^2(R) I(R)} \int_R^\infty \frac{r\,\mathrm{d}r}{\Upsilon\sqrt{r^2 - R^2}} \int_r^\infty \rho(r')\tilde{g}(r')\,\mathrm{d}r'$$

a relation between mass, acceleration scale and Hubble constant

$$I(R) = 2\int_R^\infty \frac{\rho(r)}{\Upsilon}\,\mathrm{d}s$$

surface brightness

mass-to-light ratio

velocity dispersion

- cumulative projected velocity dispersion

$$\frac{1}{\theta_{\mathrm{E}}^2} = \frac{1}{\sigma_S^2(R)S(R)} \int_0^R \sigma_I^2(R')I(R')2\pi R'^2 \,\mathrm{d}R'$$

a relation between mass, acceleration scale and Hubble constant

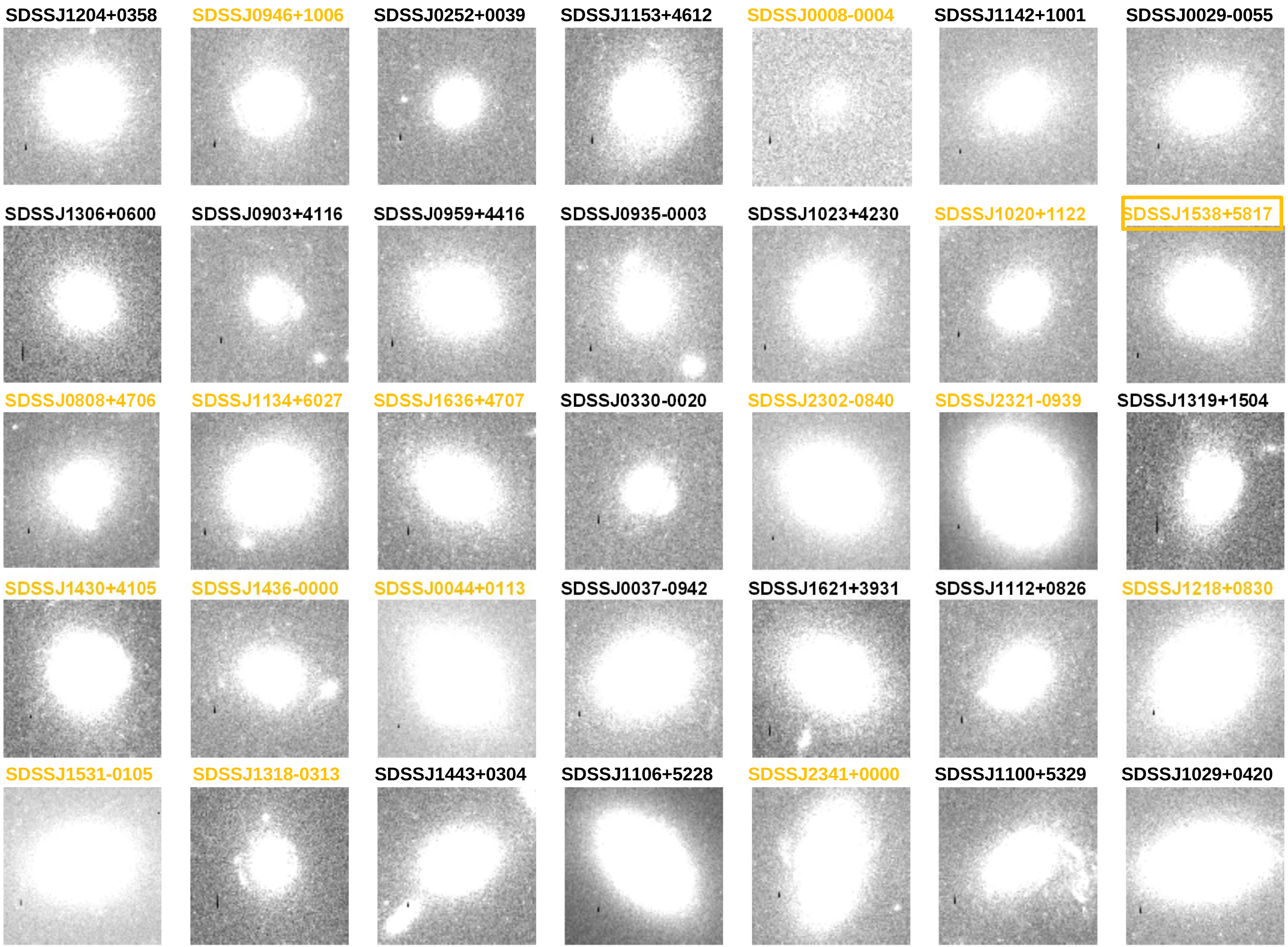
$$S(R) = \int_0^R I(R')2\pi R'^2 \,\mathrm{d}R'$$

cumulative surface brightness

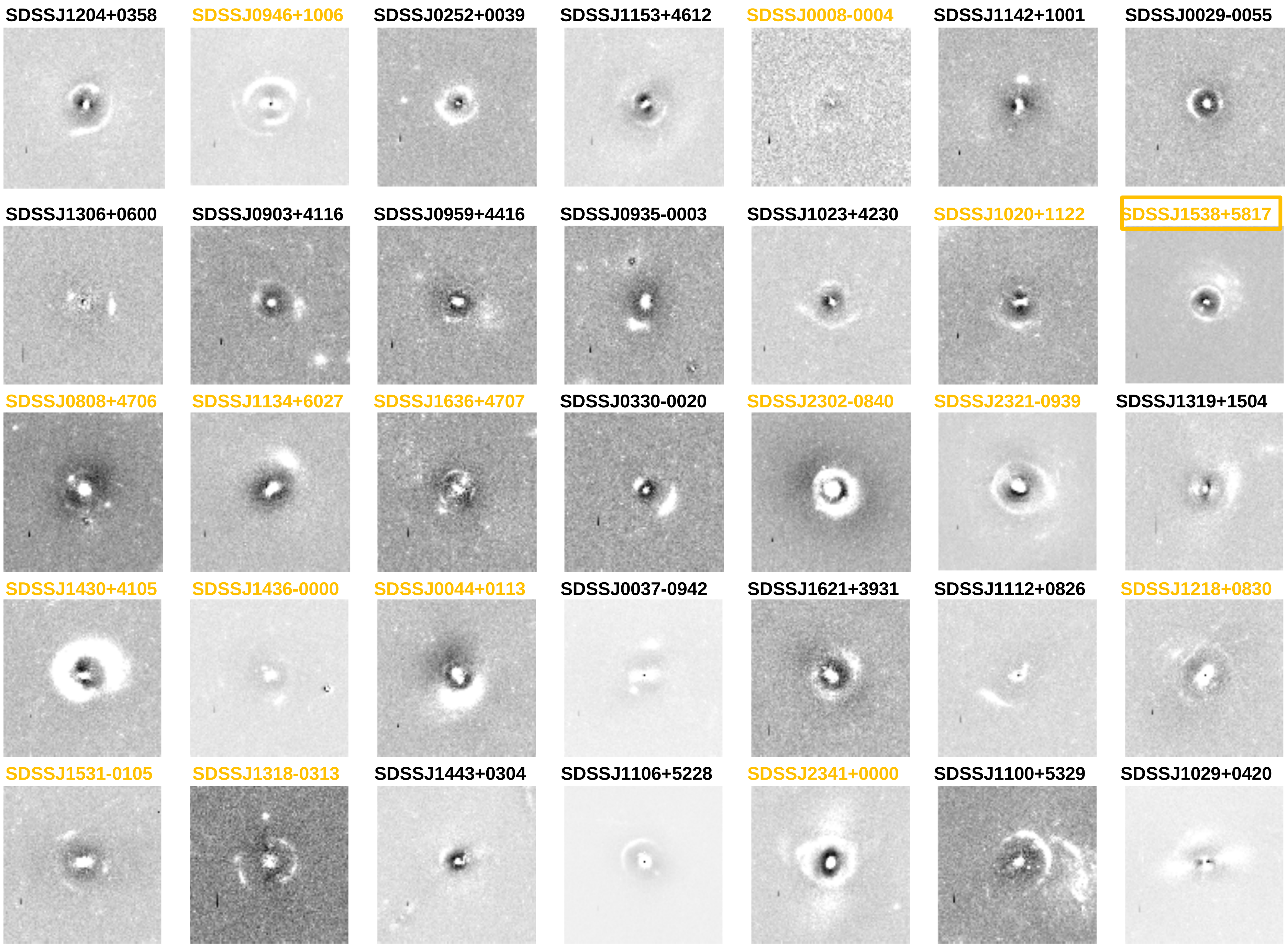
acceleration scale a_0

- a_0 is the most important number in MOND
  - in the past a_0 is obtained from spirals, such as rotation curves (e.g., Sanders & McGaugh 2002), baryonic Tully-Fisher relation (McGaugh 2011, 2012)
  - $a_0 \approx 1.2\times10^{-10}$ m/s^2
  - first attempt to find a_0 from ellipticals
  - image from gravitational lensing and velocity dispersion of lens

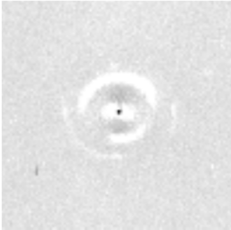
- SLACS (Auger et al. 2009)
  - galaxy-galaxy lensing, elliptical lens, lens resolved, double images
  - 35 candidates (1 with double sources)
  - Hernquist model for the lens
  - choose an interpolation function
  - assume H_0 then solve for mass and a_0



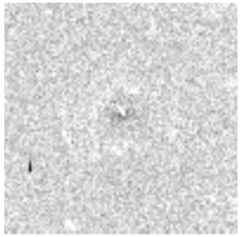
SDSSJ1204+0358
SDSSJ0946+1006
SDSSJ0252+0039
SDSSJ1153+4612
SDSSJ0008-0004
SDSSJ1142+1001
SDSSJ0029-0055
SDSSJ1306+0600
SDSSJ0903+4116
SDSSJ0959+4416
SDSSJ0935-0003
SDSSJ1023+4230
SDSSJ1020+1122
SDSSJ1538+5817
SDSSJ0808+4706
SDSSJ1134+6027
SDSSJ1636+4707
SDSSJ0330-0020
SDSSJ2302-0840
SDSSJ2321-0939
SDSSJ1319+1504
SDSSJ1430+4105
SDSSJ1436-0000
SDSSJ0044+0113
SDSSJ0037-0942
SDSSJ1621+3931
SDSSJ1112+0826
SDSSJ1218+0830
SDSSJ1531-0105
SDSSJ1318-0313
SDSSJ1443+0304
SDSSJ1106+5228
SDSSJ2341+0000
SDSSJ1100+5329
SDSSJ1029+0420


SDSSJ1204+0358
SDSSJ0946+1006
SDSSJ0252+0039
SDSSJ1153+4612
SDSSJ0008-0004
SDSSJ1142+1001
SDSSJ0029-0055
SDSSJ1306+0600
SDSSJ0903+4116
SDSSJ0959+4416
SDSSJ0935-0003
SDSSJ1023+4230
SDSSJ1020+1122
SDSSJ1538+5817
SDSSJ0808+4706
SDSSJ1134+6027
SDSSJ1636+4707
SDSSJ0330-0020
SDSSJ2302-0840
SDSSJ2321-0939
SDSSJ1319+1504
SDSSJ1430+4105
SDSSJ1436-0000
SDSSJ0044+0113
SDSSJ0037-0942
SDSSJ1621+3931
SDSSJ1112+0826
SDSSJ1218+0830
SDSSJ1531-0105
SDSSJ1318-0313
SDSSJ1443+0304
SDSSJ1106+5228
SDSSJ2341+0000
SDSSJ1100+5329
SDSSJ1029+0420

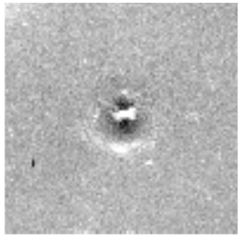
SDSSJ0946+1006



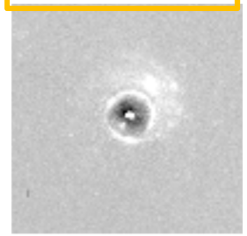
SDSSJ0008-0004



SDSSJ1020+1122

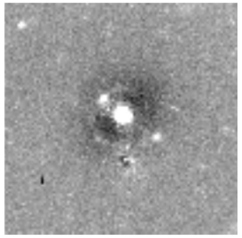


SDSSJ1538+5817

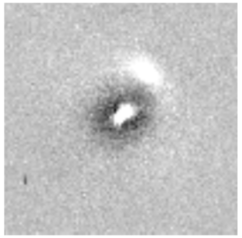


- only 17 of them give positive a_0

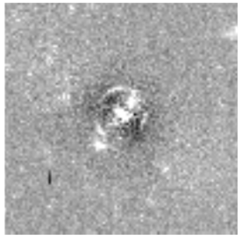
SDSSJ0808+4706



SDSSJ1134+6027

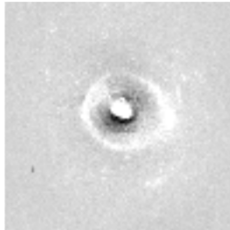


SDSSJ1636+4707

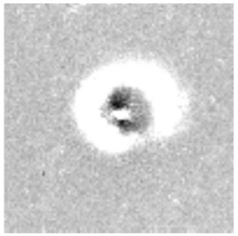


SDSSJ2302-0840

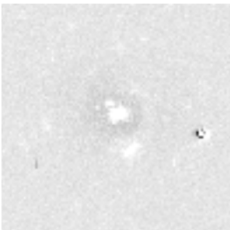
SDSSJ2321-0939



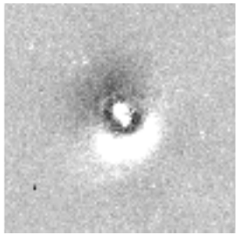
SDSSJ1430+4105



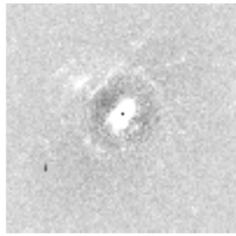
SDSSJ1436-0000



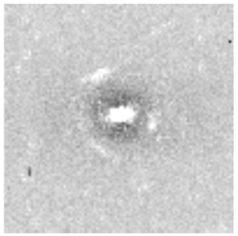
SDSSJ0044+0113



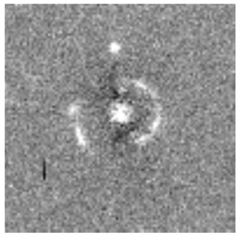
SDSSJ1218+0830



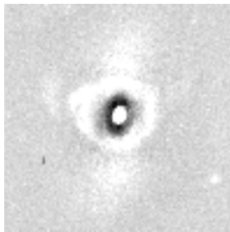
SDSSJ1531-0105



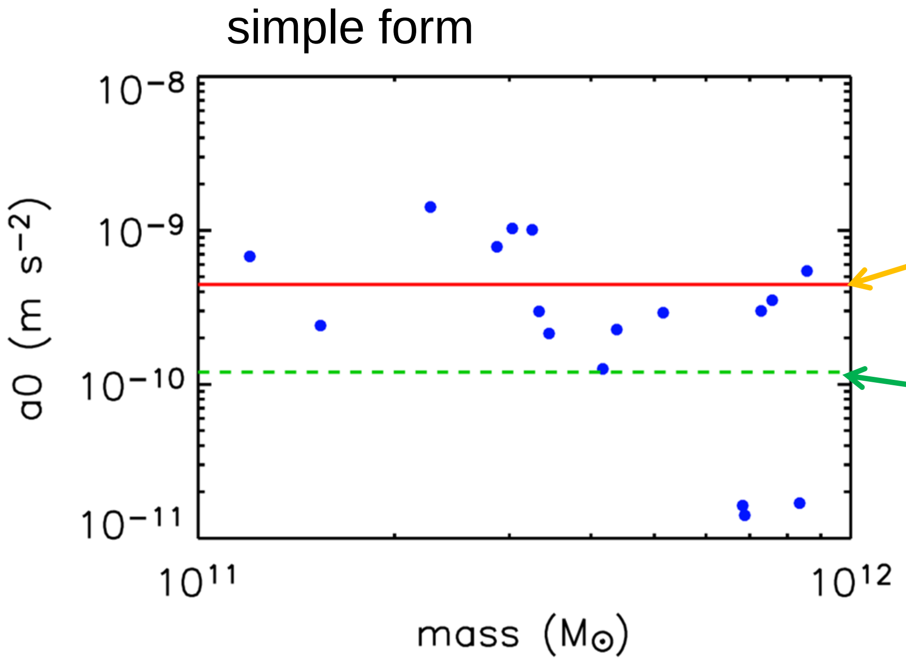
SDSSJ1318-0313



SDSSJ2341+0000



acceleration scale from ellipticals



preliminary

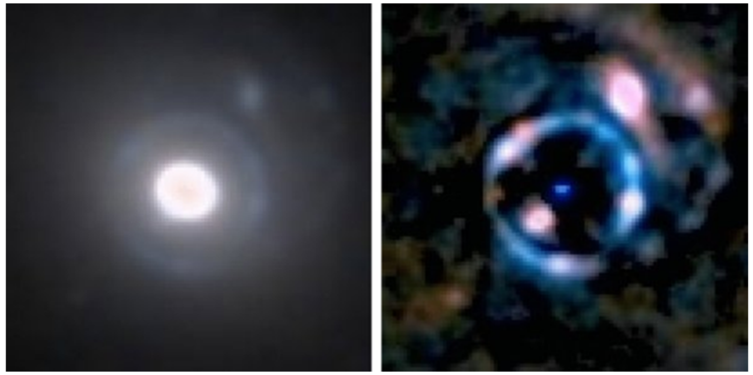
4.46×10^{-10} m/s^2

1.2×10^{-10} m/s^2 from spirals

- average a_0 is around 4.46×10^{-10} m/s^2 (preliminary)

golden opportunity?

Grillo et al. (2010)



- SDSS J1538+5817
  - a lensing system with double sources
  - dispersion velocity
  - three combinations to get mass and a_0
  - velocity + pair: $a_0 \approx 6.79\times10^{-10}$ m/s^2
  - velocity + ring: $a_0 \approx 2.42\times10^{-10}$ m/s^2
  - pair + ring: no positive solution for a_0

- MOND is in trouble???
  - method is sensitive to measurement error
  - better understanding and treatment of error may do the tricks
  - stay tuned

remarks

- excess acceleration in galaxy scale can be explained by dark matter or MOND
- both have their own free parameter or free function
  - MOND’s free function is more constrained and perhaps is easier to falsify

- if it turns out that MOND is only an empirical shortcut to explain observation, the underlying theory should come up with a natural explanation of a_0 (preferentially close to cH_0)

gravitational redshift

- gravitational redshift effect from clusters of galaxies is embedded in their velocity dispersion data (Cappi 1995)
- stacking kinematic data of a large number of clusters (several thousands or more) can pull out the gravitational redshift effect (Wojtak et al. 2011)

relative redshift

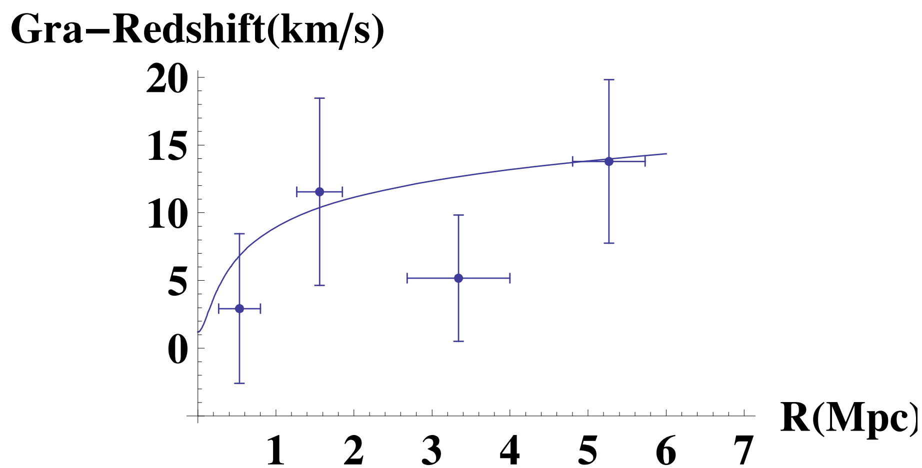
- 3D relative gravitational redshift

$$\Xi_r(r) = \Phi(r) - \Phi(0) = \frac{G\mathcal{M}}{c^2 D_{\mathrm{L}}^2} \int_0^r \tilde{g}(r')\,\mathrm{d}r'$$

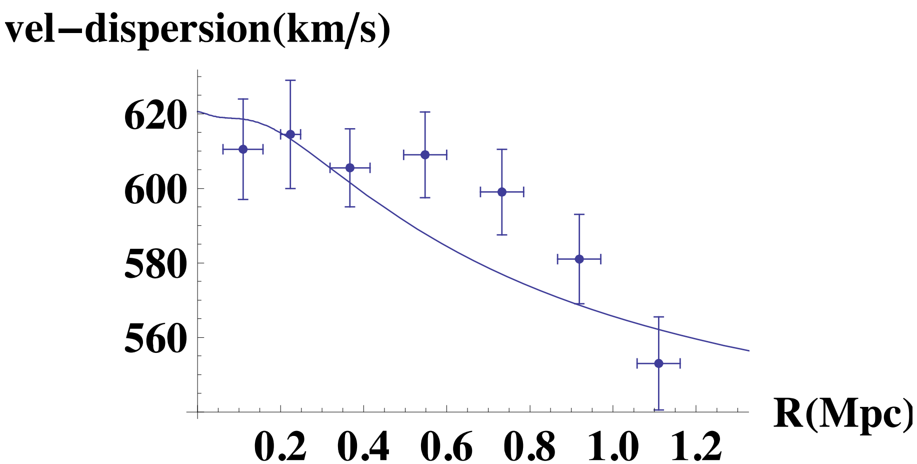
- projected gravitational redshift

$$\frac{1}{\theta_{\mathrm{E}}^2} = \frac{D_{\mathrm{S}}}{2D_{\mathrm{L}}D_{\mathrm{LS}}\Xi_I(R)I(R)} \int_R^\infty \frac{\rho(r)r\,\mathrm{d}r}{\Upsilon\sqrt{r^2 - R^2}} \int_0^r \tilde{g}(r')\,\mathrm{d}r'$$

a relation between mass, acceleration scale and Hubble constant

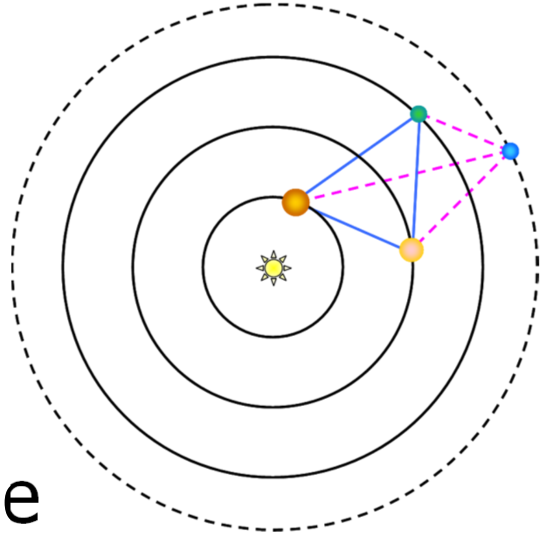


not very satisfactory

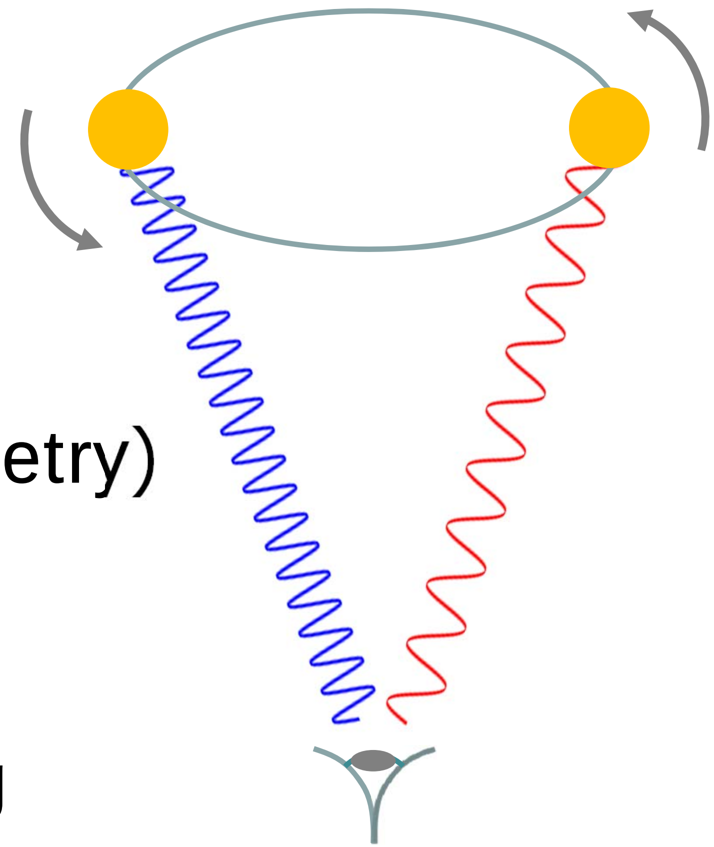


under construction

- existence of Neptune
  – Neptune was proposed to explain some irregularity of the orbit of Uranus (Le Verrier 1945,1946, Adams 1945, 1946)
  – Neptune was then discovered (Galle 1946)
  – confirmation of dynamical mass by luminous mass (seeing is believing?)
  – successful story of missing mass



- extrasolar planets
  - wobbling of stars
    (radial velocity or astrometry)
  - microlensing
  - transit timing variations
  - believing even not seeing



tally (2012.05.12)

Jean Schneider
exoplanet.eu/catalog.php

method	planets	planetary systems	multi-planet systems
wobbling (radial velocity and astrometry)	702	560	94
transit	231	197	30
microlensing	15	14	1
imaging	31	27	2
timing	17	12	4
total	765	613	101

note: transit confirmed by wobbling

- perihelion of mercury
  - precession of perihelion
  - interaction with known planets and shape of the sun can account for most but not all
  - unseen planet is proposed, but not found
  - Einstein’s general relativity
  - successful story for modified gravity