ON THE CRITERION OF STOCHASTIC STRUCTURE FORMATION IN RANDOM MEDIA

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... Chaos is the place which serves to contain all things; for if this had not subsisted neither earth nor water nor the rest of the elements, nor the Universe as a whole, could have been constructed. ...

Sextus Empiricus, *Against the Physics, against the Ethicists*, R. G. Bury, p. 217, Harvard University Press, 1997.

Here you can see the words of Sextus Empiricus that all things have been constructed over the Chaos. There is a question: in what way is it possible to do?

The answer is that it is possible on the base of chaos properties, and my talk is about this.

Introduction

Parametrically excited dynamical systems are encountered in all branches of physics. Dynamical systems can be described by ordinary and partial differential equations.

Two features are characteristic of such parametric excitation in dynamic systems described by partial differential equations:

1. On the one hand, at the initial stages of dynamic system evolution, such a parametric excitation is accompanied by the increase of all statistical characteristics of the problem solution (such as moment and correlation functions of any order) with time;

2. On the other hand, *separate field realizations* can show the stochastic nonstationary phenomenon of *clustering* in phase and physical spaces.

Clustering* of a field is identified as the emergence of compact areas with large values of this field against the residual background of areas where these values are fairly low. *Naturally, statistical averaging completely destroys all data on clustering.

The notion of clustering by itself is related to the spatial behavior of a dynamic system *in separate realizations*! Consideration of clustering in terms of traditional statistical characteristics such as moment and correlation functions of arbitrary order is meaningless! Clustering either exists or not exists.

In itself, the physical phenomenon of structure formation in stochastic parametrically excited dynamic systems is well known in physics [1-3].

The examples are the *Anderson localization* for wave eigenfunctions of the stationary one-dimensional Schrödinger equation with a random potential and the *dynamic localization* of wave field intensity in a wave problem of propagation in randomly layered medium (Helmholtz stochastic equation).

Moreover, in a number of cases, *clustering of both passive scalar tracer* (density field) and *vector tracer* (magnetic field energy) can occur in problems on turbulent transfer in the scope of *kinematic approximation*!

The basic stochastic equations for the density field $\rho(\boldsymbol{r},t)$ and the nondivergent magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$ at the kinematic stage with homogeneous initial conditions are the scalar continuity equation

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\boldsymbol{u}(\boldsymbol{r},t)\right)\rho(\boldsymbol{r},t)=0,\quad \rho(\boldsymbol{r},0)=\rho_0, \tag{1}$$

and the vector induction equation

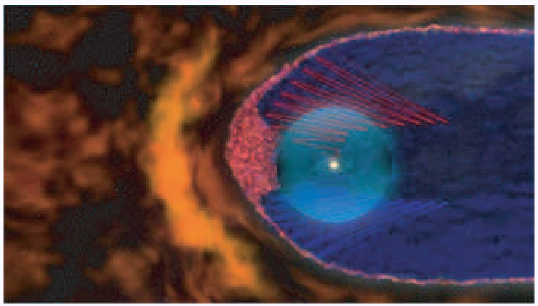
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\boldsymbol{u}(\boldsymbol{r},t)\right)\boldsymbol{H}(\boldsymbol{r},t)=\left(\boldsymbol{H}(\boldsymbol{r},t)\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)\boldsymbol{u}(\boldsymbol{r},t),\quad \boldsymbol{H}(\boldsymbol{r},0)=\boldsymbol{H}_0, \tag{2}$$

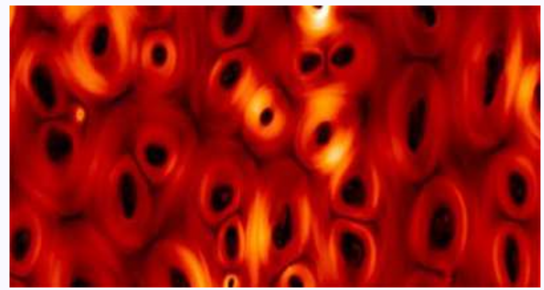
where $\boldsymbol{u}(\boldsymbol{r},t)$ is the field of turbulent velocities.

Dynamic system density (1) and magnetic field (2) are conservative, and both the total scalar mass $M=\int d\boldsymbol{r}\rho(\boldsymbol{r},t)$ and the magnetic flux $\int d\boldsymbol{r}\,\boldsymbol{H}(\boldsymbol{r},t)$ remain constant during the evolution. For the mean values the following equalities are a corollary of the conservatism of these dynamic systems $\langle\rho(\boldsymbol{r},t)\rangle=\rho_0, \langle\boldsymbol{H}(\boldsymbol{r},t)\rangle=\boldsymbol{H}_0$.

I illustrate structure formation in magnetic field by the extract from an internet-page: *What does puzzle astrophysicists so strongly?*

Contrary to hypotheses formed for fifty years, at the boundary of planetary system observers encountered a boiling foam of locally magnetized areas each of hundreds of millions kilometers in extent, which form a non-stationary cellular structure in which magnetic field lines are permanently breaking and recombining to form new areas—*magnetic "bubbles"* [4].



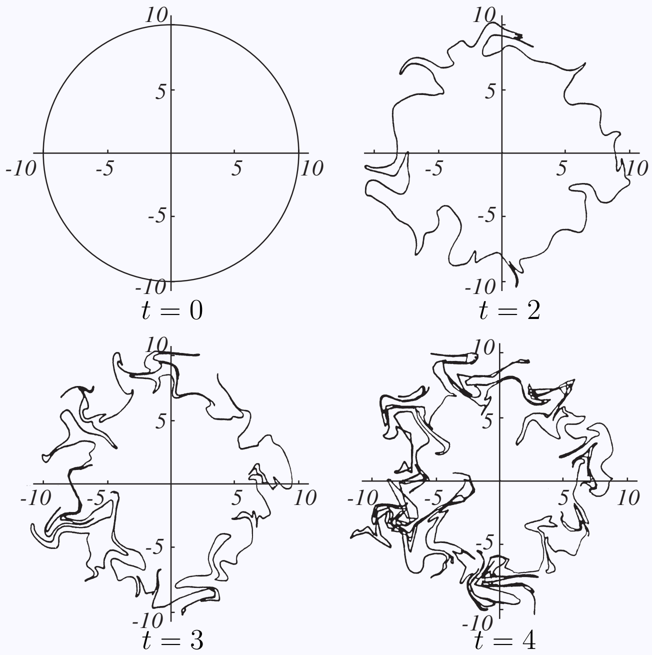


Note that the above partial differential equations are equivalent to the system of characteristic equations for particles, which are the simplest purely *kinematic equations*

$$\frac{d}{dt}\boldsymbol{r}(t) = \boldsymbol{u}\left(\boldsymbol{r}(t), t\right), \quad \boldsymbol{r}(0) = \boldsymbol{r}_0.$$

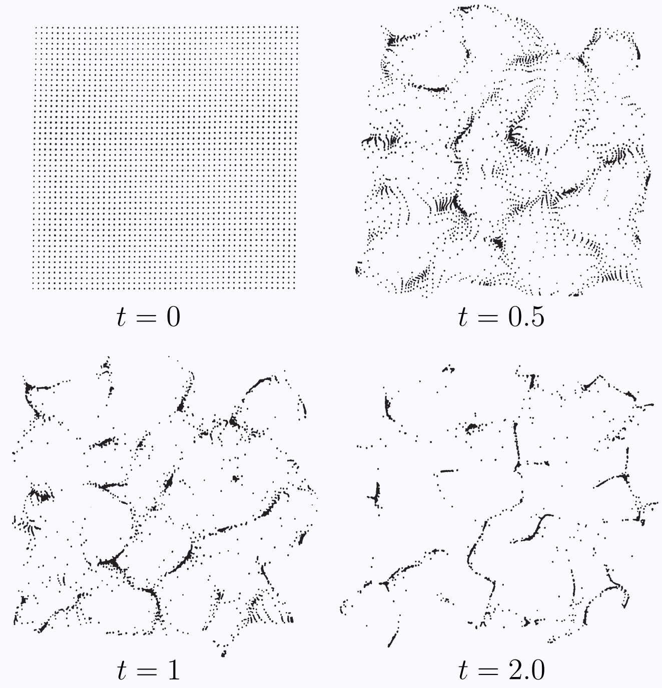
Numerical simulations show that the behavior of a system of particles essentially depends on whether the random field of velocities is *nondivergent* or *divergent.*

This figure shows a schematic of the evolution of the system of particles uniformly distributed within the circle for a particular realization of the nondivergent steady field $\boldsymbol{u}(\boldsymbol{r})$. In this case, the particles relatively uniformly fill the region within the deformed contour. The only feature consists in the *fractal-type* irregularity of the deformed contour.



On the contrary, in the case of the potential velocity field $\boldsymbol{u}(\boldsymbol{r})$, particles uniformly distributed in the square at the initial instant will *form clusters* during the temporal evolution. Results simulated for this case are shown in this figure. We emphasize that the formation of clusters in this case is purely a *kinematic effect.*

This feature of particle dynamics disappears on averaging over an ensemble of realizations of random velocity field.



Note that such type clustering in a system of particles was found as a result of numerical simulating the so-called *Eole experiment* with the use of the simplest equations of atmospheric dynamics.

In this global experiment, 500 constant-density balloons were launched in Argentina in 1970-1971; these balloons traveled at a height of about 12 km and spread along the whole of the southern hemisphere.

This figure shows the balloon distribution over the southern hemisphere for day 105 from the beginning of this process simulation; this distribution clearly shows that balloons are concentrated in groups, which just corresponds to *clustering*.

Now we consider the simplest model of the transport problems (1) and (2), where velocity field $\boldsymbol{u}(\boldsymbol{r}, t)$ has the form

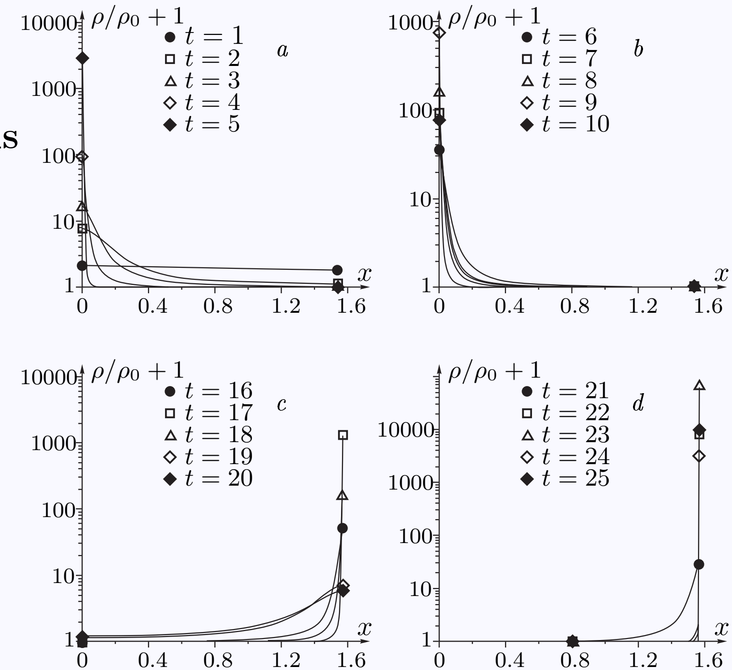
$$\boldsymbol{u}(\boldsymbol{r}, t) = \boldsymbol{v}(t) \sin(2kx),$$

to demonstrate the processes of density and magnetic fields clustering. In this model the function $\boldsymbol{v}(t)$ is the gaussian random vector process with correlation tensor $\langle v_i(t) v_j(t') \rangle = 2\sigma^2 \delta_{ij} \tau_0 \delta(t - t')$. Note that this form of velocity field corresponds to the first term of the expansion in harmonic components and is commonly used in numerical simulations.

For such model partial differential equations for density field and magnetic field have analytical solutions. The density field depends on x and solution has the form

$$\rho(x,t)/\rho_0 = \frac{1}{e^{T(t)}\cos^2(kx) + e^{-T(t)}\sin^2(kx)},$$

where $T(t) = 2k\int_0^t d\tau v_x(\tau)$ is Wiener random process. This figure shows successive patterns of concentration field rearrangement toward narrow neighborhoods of points $x \approx 0$ and $x \approx \pi/2$, i.e., the formation of clusters, in which relative density is as high as $10^3 - 10^4$, while relative density is practically zero in the whole other space. Note that the realization of the density field passes through the initial homogeneous state at the instants t such that $T(t) = 0$. The lifetimes of such clusters coincide on the order of magnitude with the time of cluster formation.

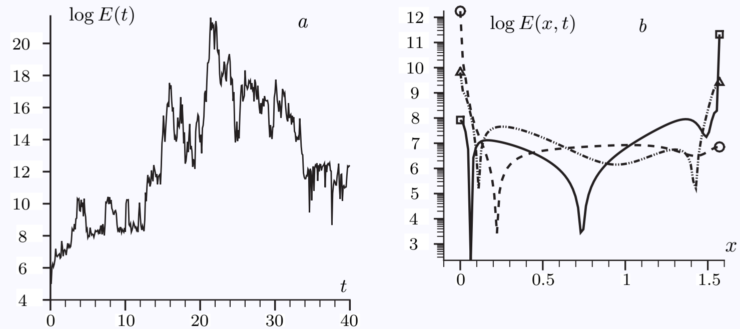


For this velocity model the x-component of the magnetic field remains constant $(H_x(x,t) = H_{x0})$, and magnetic field in the transverse (y,z)-plane $(\boldsymbol{H}_{\perp 0} = 0)$ is

$$\boldsymbol{H}_\perp(x,t) = 2kH_{x0}\int\limits_0^t d\tau \frac{\left[e^{T(\tau)}\cos^2(kx) - e^{-T(\tau)}\sin^2(kx)\right]}{\left[e^{T(\tau)}\cos^2(kx) + e^{-T(\tau)}\sin^2(kx)\right]^2} v_x(\tau)\boldsymbol{v}_\perp(\tau).$$

Here are realization of energy of the magnetic field generated in the transverse plane, $E(x,t) = \boldsymbol{H}_\perp^2(x,t)$.

The total energy concentrated in the segment $[0, \pi/2]$ increases rapidly with time ($\boldsymbol{a}$). A general space–time structure of the magnetic energy clustering is shown in ($\boldsymbol{b}$).



Another example of problems of the parametric excitation is the wave propagation problem in random media in terms of the complex *Leontovich parabolic equation* with a random function $\varepsilon(x, \boldsymbol{R})$ (x–axis is directed along the initial direction of wave propagation).

$$\frac{\partial}{\partial x}u(x,\boldsymbol{R}) = \frac{i}{2k}\Delta_{\boldsymbol{R}}u(x,\boldsymbol{R}) + \frac{ik}{2}\varepsilon(x,\boldsymbol{R})u(x,\boldsymbol{R}), \quad u(x,\boldsymbol{R}) = u_0, \tag{3}$$

Note that this equation is the *Schrödinger equation* with a random potential $\varepsilon(x,\boldsymbol{R})$, where coordinate x plays the role of time t.

If we introduce the amplitude and phase of the wave field by the formula $u(x,\boldsymbol{R}) = A(x,\boldsymbol{R})\exp\{iS(x,\boldsymbol{R})\}$, then we can derive the equation for wave field intensity $I(x,\boldsymbol{R}) = |u(x,\boldsymbol{R})|^2$

$$\frac{\partial}{\partial x} I(x,\boldsymbol{R}) + \frac{1}{k}\boldsymbol{\nabla}_{\boldsymbol{R}}\left\{\boldsymbol{\nabla}_{\boldsymbol{R}} S(x,\boldsymbol{R}) I(x,\boldsymbol{R})\right\} = 0, \qquad I(0,\boldsymbol{R}) = I_0, \tag{4}$$

which coincides in form with the equation for the tracer density field in a random potential flow, and, hence, the wave field intensity undergoes clustering which is manifested as appearance of *field caustic structure*. In this case all one-point statistical characteristics are independent of variable $\boldsymbol{R}$.

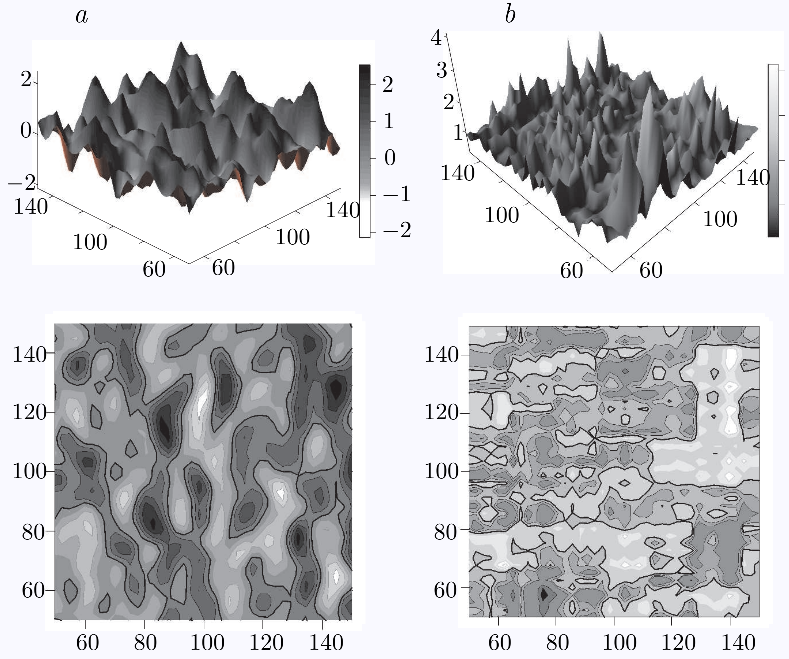
A similar situation should also be observed in the case of the monochromatic nonlinear *problem on wave self-interaction* in random inhomogeneous media described by the *nonlinear parabolic equation* (*nonlinear Schrödinger equation*)

$$\frac{\partial}{\partial x} u(x,\boldsymbol{R}) = \frac{i}{2k}\Delta_{\boldsymbol{R}} u(x,\boldsymbol{R}) + \frac{ik}{2}\varepsilon(x,\boldsymbol{R}; I(x,\boldsymbol{R}))u(x,\boldsymbol{R}), \qquad u(0,\boldsymbol{R}) = u_0(\boldsymbol{R}),$$

because equation (4) is independent of the shape of the function $\varepsilon(x,\boldsymbol{R}; I(x,\boldsymbol{R}))$.

Elements of the statistical topography of random fields

Randomness of medium parameters in dynamic systems gives rise to a stochastic behavior of physical fields. Indeed, individual samples of scalar two-dimensional fields $f(\boldsymbol{R}, t)$, where $\boldsymbol{R} = (x, y)$, resemble a rough mountainous terrain. The figure shows examples of realizations of (a) Gaussian and (b) lognormal random fields whose level curves are characterized by different statistical structures.



Phenomenon of clustering random fields can be detected and described only on the basis of the ideas of statistical topography. Similarly to common topography of mountain ranges, the statistical topography studies the systems of contours (level lines in the 2D case and surfaces of constant values in the 3D case) specified by the equality $f(\boldsymbol{r}, t) = f = \text{const}$.

For analyzing a system of contours (for simplicity, we will deal with the two-dimensional case and assume $\boldsymbol{r} = \boldsymbol{R}$), we introduce the singular *indicator function* $\varphi(\boldsymbol{R}, t; f) = \delta(f(\boldsymbol{R}, t) - f)$ concentrated on these contours. The convenience of this function consists, in particular, in the fact that it allows simple expressions for quantities such as the total area of regions where $f(\boldsymbol{R}, t) > f$ (i.e., within level lines $f(\boldsymbol{R}, t) = f$)

$$S(t; f) = \int \theta(f(\boldsymbol{R}, t) - f) d\boldsymbol{R} = \int\limits_{f}^{\infty} df' \int d\boldsymbol{R}\, \varphi(\boldsymbol{R}, t; f'),$$

and the total 'mass' of the field within these regions

$$M(t; f) = \int f(\boldsymbol{R}, t)\theta(f(\boldsymbol{R}, t) - f) d\boldsymbol{R} = \int\limits_{f}^{\infty} f' df' \int d\boldsymbol{R}\, \varphi(\boldsymbol{R}, t; f'),$$

where $\theta(f(\boldsymbol{R}, t) - f)$ is the Heaviside theta function.

The mean value of indicator function over an ensemble of realizations of random field $f(\boldsymbol{R}, t)$ determines the one-time (in time) and one-point (in space) probability density $P(\boldsymbol{R}, t; f) = \langle \delta\left(f(\boldsymbol{R}, t) - f\right) \rangle$.

Consequently, this probability density immediately determines ensemble-averaged values of the above expressions $S(t;f)$ and $M(t;f)$:

$$\langle S(t;f)\rangle = \int\limits_f^\infty df' \int d\boldsymbol{R}\, P(\boldsymbol{R},t;f'), \quad \langle M(t;f)\rangle = \int\limits_f^\infty f' df' \int d\boldsymbol{R}\, P(\boldsymbol{R},t;f').$$

Consider now the conditions of occurrence of stochastic structure formation. It is clear that, for a *positive field* $f(\boldsymbol{R},t)$, the condition of clustering with a probability of one, i.e., almost in all realizations, is formulated *in the general case* as simultaneous tendency of fulfillment of the following asymptotic equalities for $t \to \infty$

$$\langle S(t;f)\rangle \to 0, \quad \langle M(t;f)\rangle \to \int d\,\boldsymbol{R}\,\langle f(\boldsymbol{R},t)\rangle\,.$$

On the contrary, simultaneous tendency of fulfillment of the asymptotic equalities for $t \to \infty$

$$\langle S(t;f)\rangle \to \infty, \quad \langle M(t;f)\rangle \to \int d\,\boldsymbol{R}\,\langle f(\boldsymbol{R},t)\rangle$$

corresponds to the absence of structure formation.

In the case of a spatially homogeneous field $f(\boldsymbol{R},t)$, the corresponding probability density $P(\boldsymbol{R},t;f)$ is independent of $\boldsymbol{R}$. In this case, statistical averages of the above expressions (without integration over $\boldsymbol{R}$) will characterize the corresponding specific (per unit area) values of these quantities.

In this case, random field $f(\boldsymbol{R},t)$ is statistically equivalent to the random process whose statistical characteristics coincide with the spatial one-point characteristics of field $f(\boldsymbol{R},t)$.

So, the specific mean area $\langle S_{\mathbf{hom}}(t;f)\rangle$ over which the random field $f(\boldsymbol{R},t)$ exceeds a given level f, coincides with the probability of the event $f(\boldsymbol{R},t) > f$ at any spatial point, i.e., $\langle S_{\mathbf{hom}}(t;f)\rangle = \langle\theta(f(\boldsymbol{R},t)-f)\rangle = \mathsf{P}\{f(\boldsymbol{R},t) > f\}$ and therefore the mean specific area offers a geometric interpretation of the probability of the event $f(\boldsymbol{R},t) > f$, which is apparently independent of the point $\boldsymbol{R}$. Consequently, in the case of a *homogeneous* field, conditions of clustering are reduced to the tendency of asymptotic equalities for $t \to \infty$

$$\langle S_{\mathbf{hom}}(t;f)\rangle = \mathsf{P}\{f(\boldsymbol{r},t) > f\} \to 0, \quad \langle M_{\mathbf{hom}}(t;f)\rangle \to \langle f(t)\rangle\,.$$

Absence of clustering corresponds to the tendency of asymptotic equalities for $t \to \infty$

$$\langle S_{\mathbf{hom}}(t;f)\rangle = \mathsf{P}\{f(\boldsymbol{r},t) > f\} \to 1, \quad \langle M_{\mathbf{hom}}(t;f)\rangle \to \langle f(t)\rangle\,.$$

Thus, in spatially homogeneous problems, clustering is the physical phenomenon (realized with probability one, i.e., occurred in almost all realizations of a positive random field) generated by a rare event whose probability tends to zero. Namely availability of these rare events is the trigger that starts the process of structure formation, and a structure formation itself is the property of random medium.

In the conditions of developed clustering, the field is simply absent in the most part of space!

As for setup time of such spatial structure formation, it depends on limiting behavior of the right-hand expressions in all above asymptotic equalities.

It is clear that the above conditions of presence and absence of clustering field $f(\boldsymbol{R}, t)$ bear no relation to parametric growth in time of the field statistical characteristics such as moment and correlation functions of arbitrary order.

The above criterion of '*ideal*' clustering (analogously to *ideal hydrodynamic*) describes dynamics of cluster formation in the dynamic systems described in general by the first-order partial differential equations.

As for *actual* physical systems, various additional factors come to play with time; they are related to generation of random field spatial derivatives like spatial diffusion or diffraction, which *deform* the pattern of clustering, but not *dispose* it.

In particular, a possible situation can occur when the probability density rapidly approaches its steady-state regime $P(\boldsymbol{R}; f)$ for $t \to \infty$. In this case, functionals like $\langle S(f)\rangle = \int\limits_f^\infty df' \int d\boldsymbol{R}\, P(\boldsymbol{R}; f')$ and $\langle M(f)\rangle = \int\limits_f^\infty f'df' \int d\boldsymbol{R}\, P(\boldsymbol{R}; f')$ cease to describe further deformation of the clustering pattern, and we must study temporal evolution of functionals related to the spatial derivatives of field $f(\boldsymbol{R}, t)$, like the total length of contours and the number of contours.

a

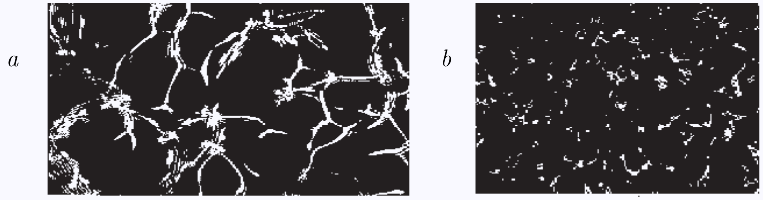
As an illustration of '*ideal*' and '*deformed*' clustering in nature we mention the lava lakes (a) [5] and (b) [6].

Another illustrations we have from the problem of the *waves propagation in random medium* (3). With increasing the distance statistical characteristics of wave intensity approach the saturated regime. In this region we have

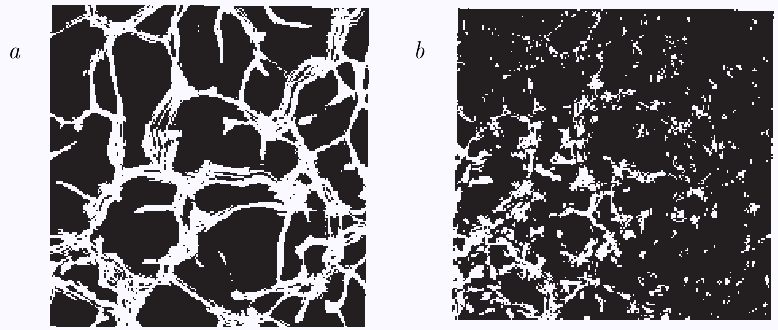
$$\langle I^n(x) \rangle = n!, \quad P(x, I) = e^{-I}.$$

In this case, the mean specific contour length and mean specific number of wave intensity contours continue to grow with distance; consequently, contour subdivision occurs, which was observed

in laboratory experiments



and in numerical simulations.



Lognormal positive random fields

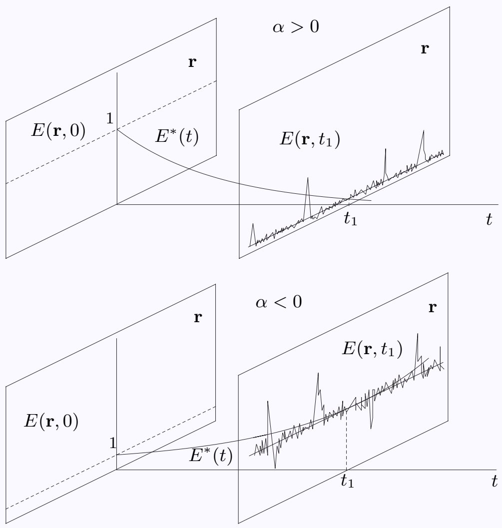
The pattern of *ideal clustering* is realized for positive random lognormal fields $E(\boldsymbol{r}, t)$ closely related to lognormal fields whose one-point probability density $P(\boldsymbol{r}, t; E)$ for homogeneous case $(E(\boldsymbol{r}, 0) = E_0)$ is independent of $\boldsymbol{r}$ and satisfies the equation

$$\frac{\partial}{\partial t} P(t; E) = \left\{ \alpha \frac{\partial}{\partial E} E + D \frac{\partial}{\partial E} E \frac{\partial}{\partial E} E \right\} P(t; E), \tag{5}$$

where coefficients α and D characterize diffusion in E-space. For definiteness, we will term field $E(\boldsymbol{r}, t)$ 'energy'.

Here, parameter α can be both positive and negative. Figure, schematically shows random realizations of energy for different signs of parameter α at arbitrary spatial point. The solution of this equation is given by

$$P(t; E) = \frac{1}{2E\sqrt{\pi D t}} \exp\left\{ -\frac{\ln^2 \left[E e^{\alpha t} / E_0 \right]}{4Dt} \right\}. \tag{6}$$



The corresponding asymptotic expressions ($t \to \infty$) for the specific values of the volume of large fluctuations and their total specific values of the energy become ($2D - \alpha > 0$)

$$\langle V_{\text{hom}}(t, E)\rangle = \mathsf{P}\{E(\boldsymbol{r}, t; \alpha) > E\} \approx \begin{cases} \dfrac{1}{\alpha}\sqrt{\dfrac{D}{\pi t}\left(\dfrac{E_0}{E}\right)^{\alpha/D}} e^{-\alpha^2 t/(4D)} \ (\alpha > 0), \\ 1 - \dfrac{1}{|\alpha|}\sqrt{\dfrac{D}{\pi t}\left(\dfrac{E}{E_0}\right)^{|\alpha|/D}} e^{-\alpha^2 t/(4D)} \ (\alpha < 0), \end{cases}$$

$$\langle E_{\text{hom}}(t, E)\rangle \approx E_0 e^{(D-\alpha)t}\left[1 - \frac{1}{(2D-\alpha)}\sqrt{\frac{D}{\pi t}\left(\frac{E}{E_0}\right)^{(2D-\alpha)/D}} e^{-(2D-\alpha)^2 t/(4D)}\right].$$

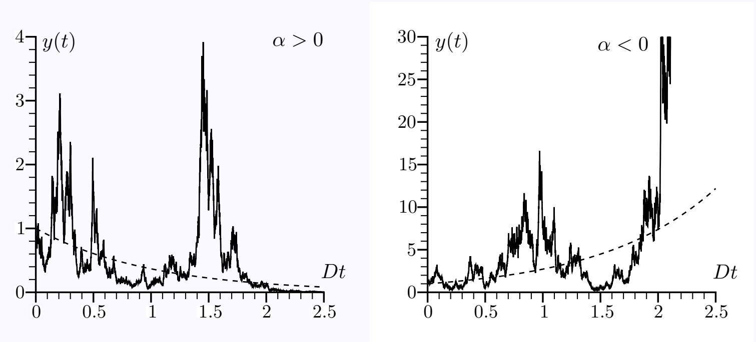
So clustering of random field of energy $E(\boldsymbol{r}, t; \alpha)$ will happen with a probability one (i.e., almost in each realization) under the condition $\alpha > 0$.

Note that the simplest Markovian lognormal process

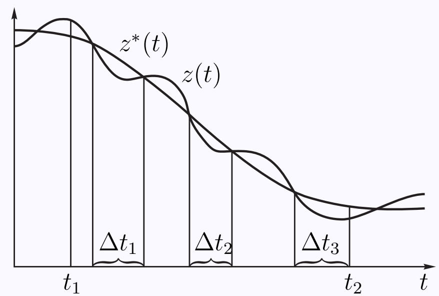
$$y(t; \alpha) = \exp\left\{-\alpha t + \int_0^t d\tau z(\tau)\right\}, \qquad (7)$$

where $z(t)$ is the Gaussian 'white noise' process with the parameters $\langle z(t)\rangle = 0$, $\langle z(t)z(t')\rangle = 2D\delta(t - t')$ is statistically equivalent to Eq. (5).

This figure displays realizations of process (7) for positive and negative parameter α and $D = |\alpha|$. The figure shows the presence of rare but strong fluctuations relative to the dashed curves towards both large values and zero. Such a property of random processes is called *intermittency*. This property is common to all random processes. The curve with respect to which the fluctuations are observed is referred to as the *typical realization curve*. The concept of *typical realization curve* of arbitrary random process $z(t)$ concerns the fundamental features of the behavior of a separate process realization.

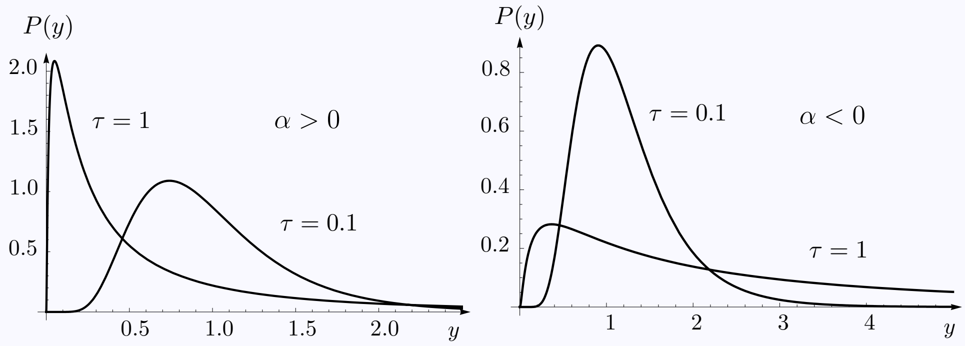


This means, on the one hand, that for any t the probabilities $\mathsf{P}\{z(t) > z^*(t)\} = \mathsf{P}\{z(t) < z^*(t)\} = 1/2$. On the other hand, this curve has a specific property that, for any time interval (t_1, t_2), the random process $z(t)$ 'winds' around the curve $z^*(t)$ such that the mean times are $\langle T_{z(t)>z^*(t)} \rangle = \langle T_{z(t)<z^*(t)} \rangle = \frac{1}{2}(t_2 - t_1)$. Curve $z^*(t)$ can significantly differ from any particular realization of process $z(t)$ and cannot describe possible magnitudes of spikes.



The typical realization curve for a Gaussian random process $z(t)$ coincides with the mean of the process $z(t)$, i.e., $z^*(t) = \langle z(t) \rangle$, while the typical realization curve for the *lognormal* random process $y(t;\alpha)$ is defined by the equality $y^*(t) = e^{\langle \ln y(t;\alpha) \rangle}$.

One can easily obtain that $\langle \ln y(t;\alpha) \rangle = -\alpha t$. Consequently, parameter α is the *Lyapunov characteristic index* of this lognormal process. So, the typical realization curve of this processes $y^*(t) = e^{-\alpha t}$, which are the exponentially decaying curve if $\alpha > 0$ and the exponentially increasing curve in the case of $\alpha < 0$.



These figures show the lognormal probability density functions for positive and negative parameters α ($|\alpha|/D = 1$) and dimensionless times $\tau = Dt = 0.1$ and 1. Structurally, these probability distribution functions are absolutely different. The only common feature of these distributions consists in the existence of long flat *tails* that appear in distributions at $\tau = 1$; these tails increase the role of high peaks of process $y(t;\alpha)$ in the formation of the one-time statistics.

In is clear that the following theorem holds:

Under parametric excitation, arbitrary positive conservative field shows the phenomenon of clustering with probability one.

Note that such conservative fields are the density field $\rho(\boldsymbol{r},t)$ with parameters $\alpha = D = \int\limits_0^\infty d\tau \left\langle \frac{\partial \boldsymbol{u}(\boldsymbol{r},t+\tau)}{\partial \boldsymbol{r}} \frac{\partial \boldsymbol{u}(\boldsymbol{r},t)}{\partial \boldsymbol{r}} \right\rangle$ and the wave intensity in problem of wave propagation in random media. The energy of magnetic field is not conservative field.

The probability density of magnetic energy $P(t;E)$ is lognormal with the parameter

$$\alpha = 2\frac{d-1}{d+2}\left(D^{\mathrm{p}} - D^{\mathrm{s}}\right),$$

where $D^{\mathrm{s}} = \frac{1}{d-1}\int\limits_0^\infty d\tau \left\langle \boldsymbol{\omega}(\boldsymbol{r},t+\tau)\boldsymbol{\omega}(\boldsymbol{r},t)\right\rangle$, $D^{\mathrm{p}} = \int\limits_0^\infty d\tau \left\langle \frac{\partial \boldsymbol{u}(\boldsymbol{r},t+\tau)}{\partial \boldsymbol{r}} \frac{\partial \boldsymbol{u}(\boldsymbol{r},t)}{\partial \boldsymbol{r}} \right\rangle$, d is dimension of space and $\boldsymbol{\omega}(\boldsymbol{r},t) = \boldsymbol{\nabla} \times \boldsymbol{u}(\boldsymbol{r},t)$ is the curl of the velocity field.

Now parameter α can be both positive and negative and the *typical realization curve* of random process $E(t)$ is the exponential function $E^*(t) = E_0 e^{-\alpha t}$.

For $\alpha > 0$ the typical realization curve exponentially decreases at every spatial point, which is indicative of cluster structure of energy field. Otherwise, for $\alpha < 0$, the typical realization curve exponentially increases with time, which is evidence of general increase of magnetic energy at every spatial point.

For this problem all moments of magnetic energy are functions exponentially increasing with time (for both positive $n > 0$ and negative $n < 0$ values of n) and parameters $D^{\rm p}$ and $D^{\rm s}$ *appear as additive terms in all statistical moment and correlation functions of magnetic field energy.*

The all regularities obtained for the above statistical quantities have identical structure for both noncompressible flow ($D^{\rm p} = 0$) and potential flow ($D^{\rm s} = 0$). Since *clustering is absent* in the first case and *present* in the second one, it becomes clear that the mentioned statistical characteristics *include no data about stochastic structure formation in separate realizations of magnetic field energy.*

In addition, the initial induction equation (2) for magnetic field holds in the framework of applicability of the kinematic approximation. In the presence of clustering, magnetic field is absent in the most portion of space, and its aftereffect on the velocity field is, naturally, insignificant.

On the contrary, in the absence of clustering, magnetic field is generated everywhere in space; in these conditions, the kinematic approximation can be expected to be valid only on a sufficiently short temporal interval, *and any discussion of the effect of the dynamic diffusion coefficient on the formation of magnetic field energy statistics during such intervals is, in my opinion, unfounded.*

Conclusion

To conclude with, I note that a point commonly accepted in many works suggests that, for an event to happen, it is required that this event was most probable.

For example, in recent work in *Physics – Uspekhi* (2010), Prof. G.R. Ivanitskii calculated certain probabilities and came out with a *hypothesis on origin of life* from the perspective of physics:

> ***'Life can be defined as resulting from a game involving interactions of matter one part of which acquires the ability to remember the success (or failure) probabilities from the previous rounds of the game, thereby increasing its chances for further survival in the next rounds. This part of matter is currently called living matter.'***

I cannot agree with the idea that *origin of life* is a game process. I believe that *origin of life* is an event happened with probability one.

THANK YOU VERY MUCH!

References

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