On Absence of Ultraviolet Divergences in Supergravity

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We review the recent progress in computations of the 3-and 4-loop in N=8 and 3-loop in N=4 supergravities, which during the last 5 years, unexpectedly for supergravity experts, were found to be UV finite.

Based on RK, 1103.4115, 1104.5480,
Carrasco, RK, Roiban, 1108.4390,
Chemissany, RK, Ortin, 1112.0332,
Broedel, Carrasco, Ferrara, RK, Roiban, 1202.0014
RK, 1202.4690,
RK, Ortin, 1205.4437

We discuss various available explanations of these computations, We argue that the perturbative finiteness of N>2 supergravity is plausible and discuss the future computations which may help to clarify this important issue.

Related talks by K. Stelle and P. Vanhove
Disagreements in interpretation of recent computations and predictions

The Null Results

1981, RK; Howe, Stelle, Townsend: N=8 d=4 supergravity is likely to diverge at 3-loops, R^4

Miracle #1 2007 N= 8 d=4 is UV free up to 3-loops

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban

2009, Bossard, Howe, Stelle: N=8 d=5 supergravity is likely to diverge at 4-loops, D^6R^4

Miracle #2 2009 N= 8 d=5 is UV free up to 4-loops

Bern, Carrasco, Dixon, Johansson, Roiban

2011, Bossard, Howe, Stelle, Vanhove, N=4 d=4 supergravity is likely to diverge at 3-loops, R^4

Miracle #3 20012 N= 4 d=4 is UV free up to 3-loops

Bern, Davies, Dennen, Huang : 3-loop d=4 computation in pure supergravity

#4 event in N=4 d=4 supergravity interacting with matter (miracle or not?)

2012, Tourkine, Vanhove

1. 1- and 2-loop string theory computation,
2. limit to d=4 QFT
3. used the 3-loop non-renormalization theorem

Realized later that according to supergravity counterterm predictions and old supergravity computations these models have 1-loop UV divergences in the matter sector, which are absent only in pure supergravity without matter

The most famous failed experiment in history, 1887

The Michelson-Morley experiment is a perfect example of a null experiment, one in which something that was expected to happen is not observed.

It was aimed at detecting the relative motion of matter relative to the stationary luminiferous aether.

The negative results are generally considered to be the first strong evidence against the then prevalent aether theory, and initiated a line of research that eventually led to special relativity, 1905, in which the classical aether concept has no role

It took 18 years between Michelson-Morley experiment and Special relativity

In d=4 supergravity we need more loop computations.

Old counterterm paradigm

Using the existence of the covariant on-shell superspace Brink, Howe, 1979 and the background field method in QFT one can use the tensor calculus and construct the invariant candidate counterterms RK; Howe, Lindstrom, 1981.

Such geometric counterterms are invariant under all known symmetries of the theory, including duality. They start at the N - loop level.

Tensor calculus = infinite proliferation of candidate counterterms

Linearized ones in N=8 start at the 3-loop level, R^4+... RK; Howe, Stelle, Townsend, 1981

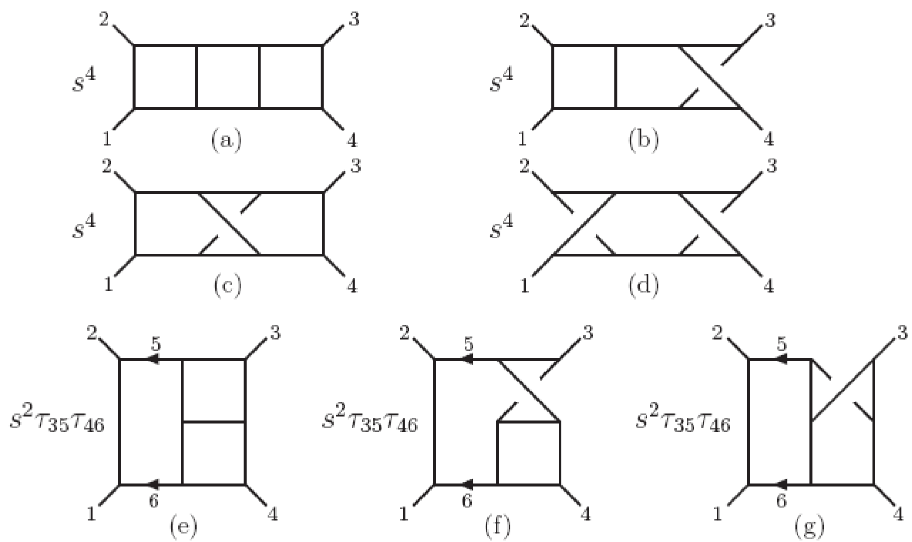
Miracle #1 2007 N= 8 is UV free at 3-loops

RK, 2009, no tensor calculus in N=8 light-cone superspace, no candidate counterterms, all-loop finiteness prediction

Complete Three-Loop N=8 Result

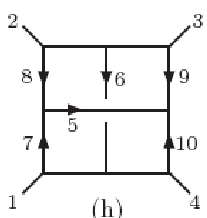
Bern, Carrasco, Dixon, Johansson, Kosower, Roiban (2007)

Obtained via on-shell unitarity method:

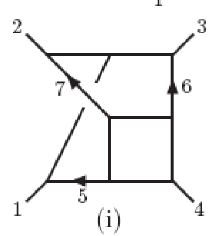


$$\tau_{ij} = 2k_i \cdot k_j$$

$$\begin{aligned}&(s(\tau_{26}+\tau_{36})+t(\tau_{15}+\tau_{25})+st)^2\\ &+(s^2(\tau_{26}+\tau_{36})-t^2(\tau_{15}+\tau_{25}))(\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})\\ &+s^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10})+t^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10})\\ &+u^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})\end{aligned}$$



$$\begin{aligned}&(s\tau_{45}-t\tau_{46})^2-\tau_{27}(s^2\tau_{45}+t^2\tau_{46})\\ &-\tau_{15}(s^2\tau_{47}+u^2\tau_{46})-\tau_{36}(t^2\tau_{47}+u^2\tau_{45})\\ &+l_5^2s^2t+l_6^2st^2-\tfrac{1}{3}l_7^2stu\end{aligned}$$



Three loops are not only ultraviolet finite they are "superfinite"— finite for *D* < 6.

4-graviton scattering is UV finite

Miracle #1

R^4

Bern, Carrasco, Johansson-duality

Gravity and Gauge Theory

Improved technics

kinematic numerator

color factor

gauge theory:

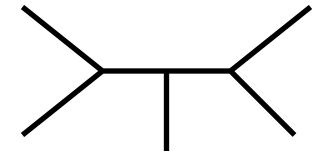
$$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

sum over diagrams with only 3 vertices

$$c_i \sim f^{a_1 a_2 b_1} f^{b_1 b_2 a_5} f^{b_2 a_4 a_5}$$

Assume we have:

$c_1 + c_2 + c_3 = 0$, $n_1 + n_2 + n_3 = 0$



Then: c_i) $\tilde{n}_i$ kinematic numerator of second gauge theory

gravity:

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

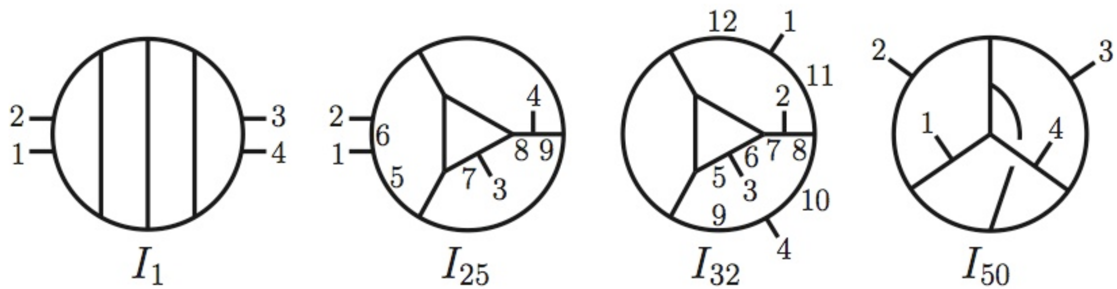
Gravity numerators are a double copy of gauge-theory ones!

This works for ordinary Einstein gravity and susy versions!

N=8 Four-Loop Amplitude Construction

Bern, Carrasco, Dixon, Johansson, Roiban

Get 50 distinct diagrams or integrals (ones with two- or three-point subdiagrams not needed).



$$M_4^{\text{4-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$$

Integral

symmetry factor

leg perms

UV finite for *D* < 11/2
It's very finite!

Originally took more than a year. Re-computed in 2012, fast and furious

Miracle #2

Explicit calculations: no UV divergences in N=8 at 4 loops

Bern, Carrasco, Dixon, Johansson, Roiban, 2009

Five-loop progress is continuing but no new results yet (as of May 2012).

Explanation? In d=4 QFT

- Light-cone superspace counterterms are not available at any loop order (prediction of UV finiteness). RK, 2009. This is still the case in 2012

- 3-loop finiteness follows from $E_{7(7)}$

Broedel, Dixon
Beisert, Elvang, Friedman, Kiermaier, Morales, Stieberger
Bossard, Howe, Stelle, 2009-2010

- String theory --> field theory, Green, Vanhove et al

If we trust continuous global $E_{7(7)}$ at the 3-loop quantum level what is the prediction at higher loops?

$E_{7(7)}$ revisited: RK, 2011

Noether-Gaillard-Zumino current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms

$E_{7(7)}$ revisited: Bossard-Nicolai, 2011

Yes, NGZ current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms. However, there is a procedure of deformation of the linear twisted self-duality constraint, which should be able to fix the problem. Examples of U(1) duality.

$E_{7(7)}$ revisited: Carrasco, RK, Roiban, 2011

Bossard-Nicolai deformation procedure needs a significant modification to explain the simplest case of Born-Infeld deformation of the Maxwell theory which conserves the NGZ current.

N=8 Born-Infeld supergravity?

See also the related duality talk by D. Sorokin

Recent work on New $E_{7(7)}$ invariants and Amplitudes with T. Ortin

We have found an obstruction to the Bossard-Nicolai procedure of deformation of the linear twisted self-duality constraint. The restoration of E77 current conservation broken by UV divergences is not possible in N=8 supergravity.

Therefore future loop computations will test our conjecture that the continuous duality symmetries may control perturbative quantum gravity.

Noether (1918), Gaillard and Zumino (1981)

- Emi Noether theorem “Invariante Variationsprobleme” published in Nachr. D. König. Gesellsch. D. Wiss. Zu Göttingen, Math-phys. Klasse 1918 (3): 235-257 : Any differentiable global symmetry of the action of a physical system has a corresponding conservation law
$$\mathcal{L} \to \mathcal{L} + \alpha\, \partial_\mu \mathcal{J}^\mu \,, \qquad \phi \to \phi + \alpha \Delta\phi$$
A conserved Noether current is $J_\mu^N \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - \mathcal{J}^\mu, \quad \partial_\mu J^{\mu\, N} = 0$

  and time-independent Noether charge $Q^N \equiv \int d^3x J^{0N}$

- Duality symmetry is a differentiable global symmetry of a system, but not of the total action, as discovered by Gaillard and Zumino in studies of supergravity.

- Noether theorem in the vector sector requires a generalization, which we call NGZ current conservation or equivalent to it NGZ identity.

Classical N extended supergravities satisfy NGZ identity $\tilde{G} = 2\frac{\delta S}{\delta F}$

$$\frac{\delta}{\delta F^\Lambda}\Big(S[F',\varphi'] - S[F,\varphi] - \frac{1}{4}\int (\tilde{F}CF + \tilde{G}BG)\Big) = 0$$

Counterterms are invariant under E & M duality

$$S^{ct}[F',\varphi'] = S^{ct}[F,\varphi]$$

The deformed action $S_{def} = S_{cl} + \lambda\, S^{ct}$

with deformed dual G-field $G_{def} = G_{cl} + 2\lambda \frac{\delta S^{ct}}{\delta F}$

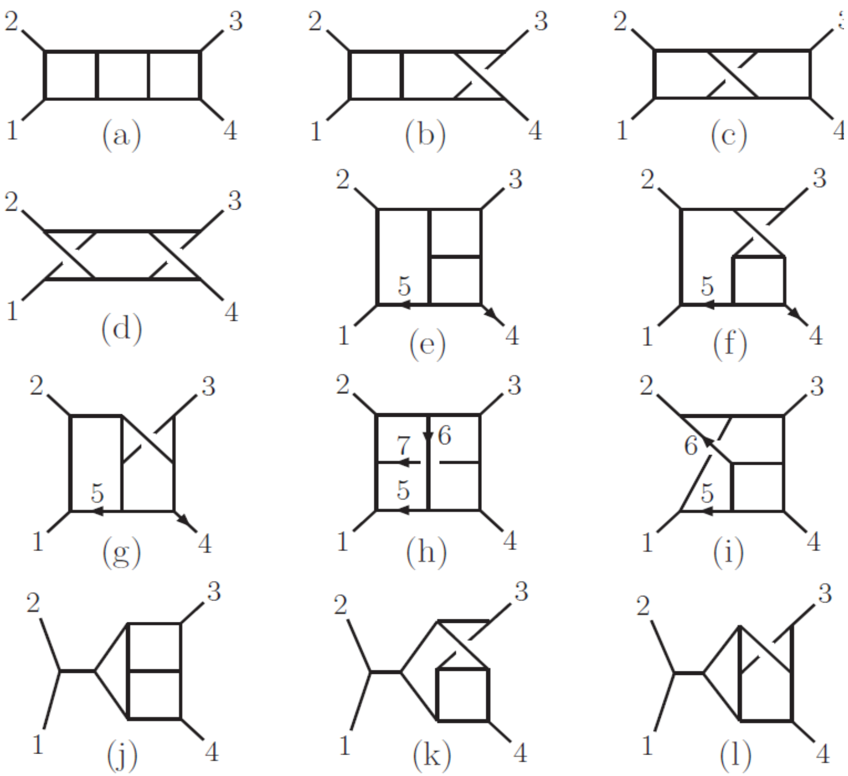
does not satisfy the NGZ identity and duality symmetry is broken.

This seems to be the only known reason why d=4 N=4 L=3 supergravity is UV finite, no other explanation are available

Three-loop construction of N=4 supergravity

Bern, Davies, Dennen, Huang

N = 4 sugra : (N = 4 sYM) x (N = 0 YM)



- For N = 4 sYM copy use known BCJ representation.

- For N = 0 YM copy use Feynman diagrams in Feynman gauge.
- 12 basic diagrams (include ghosts and contact contributions in these)

Numerator: $k^7 l^9$ + $k^8 l^8$ + finite

need to series expand in external momenta k

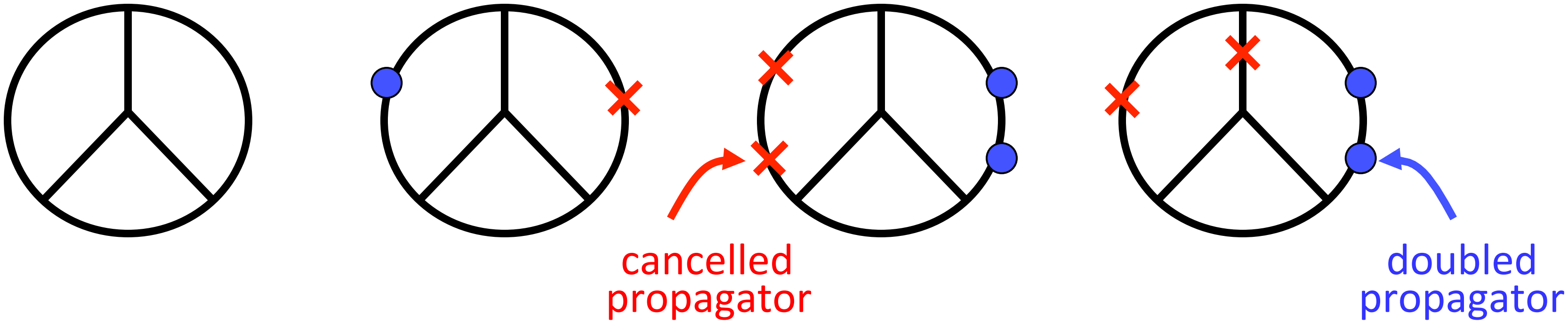
log divergent

Tens of thousands of high-rank tensor integrals combine into:

Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

Three loop results

- Expand in small external momenta.
- Get ~130 vacuum-like diagrams containing UV information.



Result: $\frac{c}{2} R^4 = \frac{c}{2} stu M_4^{\text{tree}}$

C=0

???

Feb. 15, Bern, Davies, Dennen, Huang 1202.3423

Feb.16 Tourkine,Vanhove, 1202.3692
In models with matter

Miracle #3, 2012 N = 4 supergravity is UV free at 3-loops

- We may apply the duality argument which was used for $E_{7(7)}$
- In N = 4 pure supergravity the duality group is $SL(2,\mathbb{R}) \times SO(6)$
- Is it possible to explain both N=8 and N=4 computations using a common language?
- Proposal RK 2011: revisit the old counterterm paradigm (reinforced by N=4 computation)

Electro-magnetic duality symmetry rotating the Bianchi into the vector field equations, is always broken when supersymmetric duality invariant quantum corrections are added to classical extended supergravity with type E7 groups.

E & M Duality and UV Properties of N Extended Supergravity

For N=8 supergravity the explanation

1. E77 is broken by 3-loop counterterm (most people) +
2. E77 current conservation is broken by 3-loop UV divergence +

For N=4 supergravity the explanation

1. SL(2,R)x SO(6) is broken by 3-loop counterterm is not valid −
2. SL(2,R)x SO(6) current conservation is broken by 3-loop UV divergence +

Disagreements in prediction for

7-loop UV divergence in N=8

But if indeed the UV divergence will start from 7 or 8 loops, we will be back to 1981, when such predictions were made for the first time

If some L-loop divergences will show up, the story will be forgotten soon and our original conclusion from 1981
7- or 8-loop and all higher loops UV divergences will be confirmed.

COUNTERTERMS IN EXTENDED

SUPERGRAVITIES

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To the memory of Felix Berezin

The geometrical invariants, integrals over the whole supermanifold, respecting all necessary symmetries of the theory, are shown to exist starting from the 8-th (4-th) loop approximation in the N= 8 (N=4) on-shell supergravity. 3-loop counterterms, which are integrals over some subsupermanifolds, are presented on linearised level in N=4 and N=8 theories. The corresponding 3-loop non-linear invariants are discussed.

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COUNTERTERMS IN EXTENDED

SUPERGRAVITIES

R.E.Kallosh

Moscow— September I980

Ref.TH.2953-CERN

HIGHER ORDER INVARIANTS IN EXTENDED SUPERGRAVITY

P. Howe
CERN -- Geneva

and

U. Lindström
I. T. P.
University of Stockholm,
Sweden

A B S T R A C T

On-shell linearized extended supergravity is presented in superspace for all N. The formalism is then used in the construction of higher-order invariants which may serve as counterterm Lagrangians. It is shown that three-loop counterterms exist for $N \leq 3$ and $(N-1)$ loop counterterms for $N \geq 4$. In the full non-linear theory, the presence of a global non-compact symmetry group for $N \geq 4$ does not allow a simple extension of the $(N-1)$ loop term, but N loop counterterms may be constructed.

Ref.TH.2953-CERN
23 September 1980

However, if the UV finiteness will persist in higher loops, one would like to view this as an opportunity to test some new ideas.

1. Light-cone superspace prediction

2. Duality Noether current conservation prediction

3. Hidden Symmetries of Supergravity

Even if the theory of extended supergravity is UV finite, perturbatively, so what?

Why is it useful for solving long term Quantum Gravity problem and for unifying all fundamental interactions?

The hope is
if we will understand the reason for observed UV finiteness
if it will persist in higher loops,

we may discover the unexpected.

Hidden Superconformal Symmetry of Supergravity

Conjecture: the Einstein (super)gravity may be a consistent gauge-fixed version of the (super)conformal theory where there are no dimensionful parameters and M_{PL} is the gauge-fixed value of the conformal compensator

Maybe, N=4 Poincare supergravity is just a unitary gauge of an N=4 superconformal model where the nice UV behavior is hidden and valid only for the on shell amplitudes, but some other gauges are available where the theory is renormalizable/finite but the unitarity is difficult to prove?

R_ξ gauge for gravity ??? As in non-abelian gauge symmetric SM of particle physics?

Work in progress.

More computations will be required before we know more about Quantum Gravity