Higher-spin Lagrangians with transverse symmetry

Dario Francia

Centro E. Fermi & Scuola Normale Superiore

(with A. Campoleoni - to appear)

May 28 - June 2, 2012

CENTRO FERMI
Museo Storico della Fisica e
Centro Studi e Ricerche Enrico Fermi

SCUOLA
NORMALE
SUPERIORE

Object

We propose a class of Lagrangians for bosons of arbitrary spin based on

Maxwell's operator

$$(M\varphi)_{\mu_1 \cdots \mu_s} = \Box\, \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial^{\alpha} \varphi_{\alpha\, \mu_2 \cdots \mu_s)}$$

Their distinctive feature is *transverse gauge invariance:*

$$\delta\, \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \cdots \mu_s)}$$

s.t.

$$\partial^{\alpha} \Lambda_{\alpha\, \mu_2 \cdots \mu_{s-1}} = 0$$

Object

We propose a class of Lagrangians for bosons of arbitrary spin based on

Maxwell's operator

$$(M\varphi)_{\mu_1 \cdots \mu_s} = \Box \, \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial^{\alpha} \varphi_{\alpha \, \mu_2 \cdots \mu_s)}$$

Their distinctive feature is *transverse gauge invariance:*

$$\delta \, \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \cdots \mu_s)}$$

s.t.

$$\partial^{\alpha} \Lambda_{\alpha \, \mu_2 \cdots \mu_{s-1}} = 0$$

Motivations: simplicity

On-shell conditions for massless particles *(Fierz 1939)*

$$\delta\,\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\Lambda_{\mu_2\cdots\mu_s)}$$

$$\Box\,\varphi_{\mu_1\cdots\mu_s} = 0$$
$$\partial^{a}\varphi_{\alpha\,\mu_2\cdots\mu_s} = 0$$
$$\varphi^{\alpha}{}_{\alpha\,\mu_3\cdots\mu_s} = 0$$

$$\Box\,\Lambda_{\mu_1\cdots\mu_{s-1}} = 0$$
$$\partial^{a}\Lambda_{\alpha\,\mu_2\cdots\mu_{s-1}} = 0$$
$$\Lambda^{\alpha}{}_{\alpha\,\mu_3\cdots\mu_{s-1}} = 0$$

Motivations: simplicity

On-shell conditions for massless particles *(Fierz 1939)*

$$\delta\,\varphi_{\,\mu_1\cdots\,\mu_s} \;=\; \partial_{\,(\mu_1}\,\Lambda_{\,\mu_2\cdots\,\mu_s)}$$

to go off-shell...

$$\Box\,\varphi_{\,\mu_1\cdots\,\mu_s} \;=\; 0 \qquad\qquad \Box\,\Lambda_{\,\mu_1\cdots\,\mu_{s-1}} \;=\; 0$$

$$\partial^{\,a}\,\varphi_{\,\alpha\,\mu_2\cdots\,\mu_s} \;=\; 0 \qquad\qquad \partial^{\,a}\,\Lambda_{\,\alpha\,\mu_2\cdots\,\mu_{s-1}} \;=\; 0$$

$$\varphi^{\,\alpha}{}_{\alpha\,\mu_3\cdots\,\mu_s} \;=\; 0 \qquad\qquad \Lambda^{\,\alpha}{}_{\alpha\,\mu_3\cdots\,\mu_{s-1}} \;=\; 0$$

Motivations: simplicity

On-shell conditions for massless particles *(Fierz 1939)*

$$\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \cdots \mu_s)}$$

to go off-shell...

$$\cancel{\Box \varphi_{\mu_1 \cdots \mu_s} = 0} \qquad \cancel{\Box \Lambda_{\mu_1 \cdots \mu_{s-1}} = 0}$$

$$\partial^a \varphi_{\alpha \mu_2 \cdots \mu_s} = 0 \qquad \partial^a \Lambda_{\alpha \mu_2 \cdots \mu_{s-1}} = 0$$

$$\varphi^\alpha{}_{\alpha \mu_3 \cdots \mu_s} = 0 \qquad \Lambda^\alpha{}_{\alpha \mu_3 \cdots \mu_{s-1}} = 0$$

Motivations: simplicity

On-shell conditions for massless particles *(Fierz 1939)*

$$\delta\,\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\Lambda_{\mu_2\cdots\mu_s)}$$

to go off-shell...

$$\cancel{\Box\,\varphi_{\mu_1\cdots\mu_s} = 0}$$
$$\partial^{\,a}\varphi_{\alpha\,\mu_2\cdots\mu_s} = 0$$
$$\varphi^{\,\alpha}{}_{\alpha\,\mu_3\cdots\mu_s} = 0$$

$$\cancel{\Box\,\Lambda_{\mu_1\cdots\mu_{s-1}} = 0}$$
$$\partial^{\,a}\Lambda_{\alpha\,\mu_2\cdots\mu_{s-1}} = 0$$
$$\Lambda^{\,\alpha}{}_{\alpha\,\mu_3\cdots\mu_{s-1}} = 0$$

$\Box\,\delta\,\varphi_{\mu_1\cdots\mu_s}$ *can be compensated only by* $\delta\,\partial_{(\mu_1}\partial^{\,\alpha}\varphi_{\alpha\,\mu_2\cdots\mu_s)}$

Motivations: simplicity

On-shell conditions for massless particles *(Fierz 1939)*

$$\delta\,\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\Lambda_{\mu_2\cdots\mu_s)}$$

to go off-shell...

$$\cancel{\Box\,\varphi_{\mu_1\cdots\mu_s} = 0} \qquad \cancel{\Box\,\Lambda_{\mu_1\cdots\mu_{s-1}} = 0}$$

$$\cancel{\partial^{a}\varphi_{\alpha\,\mu_2\cdots\mu_s} = 0} \qquad \partial^{a}\Lambda_{\alpha\,\mu_2\cdots\mu_{s-1}} = 0$$

$$\varphi^{\alpha}{}_{\alpha\,\mu_3\cdots\mu_s} = 0 \qquad \Lambda^{\alpha}{}_{\alpha\,\mu_3\cdots\mu_{s-1}} = 0$$

$\Box\,\delta\,\varphi_{\mu_1\cdots\mu_s}$ *can be compensated only by* $\delta\,\partial_{(\mu_1}\partial^{\alpha}\varphi_{\alpha\,\mu_2\cdots\mu_s)}$

$$\Box\,\varphi_{\mu_1\cdots\mu_s} - \partial_{(\mu_1}\partial^{\alpha}\varphi_{\alpha\,\mu_2\cdots\mu_s)}$$

minimal building-block for any off-shell formulation

Motivations: comparing interacting hsp theories

- Higher-spin interactions are mainly understood for the class of *symmetric tensor;*

- we would like to get some insight into more general classes of particles, (tensors with *mixed symmetry*) e.g. to better compare with String Theory.

Motivations: comparing interacting hsp theories

- Higher-spin interactions are mainly understood for the class of *symmetric tensor;*

- we would like to get some insight into more general classes of particles, (tensors with *mixed symmetry*) e.g. to better compare with String Theory.

In (Anti)-de Sitter space-time free Lagrangians known only for special cases

Antecedents: Einstein 1919

Equations of motion for unimodular gravity:

$$\mathcal{R}_{\mu\nu} - \frac{1}{D} g_{\mu\nu} \mathcal{R}^{\alpha}{}_{\alpha} = T_{\mu\nu} - \frac{1}{D} g_{\mu\nu} T^{\alpha}{}_{\alpha}$$

- *alternative view of cosmological constant (via Bianchi identity)*
- *keeping g = const* $\Rightarrow$ *transverse gauge symmetry for the linear theory*
- *keeping* $h_{\mu\nu}$ *traceful* $\Rightarrow$ *scalar-tensor theory of gravity*

Einstein '19; van der Bij, van Dam, Ng '82, Buchmuller, Dragon '88, Unruh '88, Henneaux, Teitelboim '89, ...
Alvarez et al. '05 - '12

Antecedents: Einstein 1919

Equations of motion
for unimodular gravity:

$$\mathcal{R}_{\mu\nu} - \frac{1}{D} g_{\mu\nu} \mathcal{R}^{\alpha}{}_{\alpha} = T_{\mu\nu} - \frac{1}{D} g_{\mu\nu} T^{\alpha}{}_{\alpha}$$

- *alternative view of cosmological constant (via Bianchi identity)*
- *keeping g = const* $\Rightarrow$ *transverse gauge symmetry for the linear theory*
- *keeping* $h_{\mu\nu}$ *traceful* $\Rightarrow$ *scalar-tensor theory of gravity*

Einstein '19; van der Bij, van Dam, Ng '82, Buchmuller, Dragon '88, Unruh '88, Henneaux, Teitelboim '89, ...
Alvarez et al. '05 - '12

Antecedents: Einstein 1919

Equations of motion
for unimodular gravity:

$$\mathcal{R}_{\mu\nu} - \frac{1}{D} g_{\mu\nu} \mathcal{R}^{\alpha}{}_{\alpha} = T_{\mu\nu} - \frac{1}{D} g_{\mu\nu} T^{\alpha}{}_{\alpha}$$

- *alternative view of cosmological constant (via Bianchi identity)*
- *keeping g = const* $\Rightarrow$ *transverse gauge symmetry for the linear theory*
- *keeping* $h_{\mu\nu}$ *traceful* $\Rightarrow$ *scalar-tensor theory of gravity*

Einstein '19; van der Bij, van Dam, Ng '82, Buchmuller, Dragon '88, Unruh '88, Henneaux, Teitelboim '89, ...
Alvarez et al. '05 - '12

Antecedents: Einstein 1919

Equations of motion
for unimodular gravity:

$$\mathcal{R}_{\mu\nu} - \frac{1}{D} g_{\mu\nu} \mathcal{R}^{\alpha}{}_{\alpha} = T_{\mu\nu} - \frac{1}{D} g_{\mu\nu} T^{\alpha}{}_{\alpha}$$

- *alternative view of cosmological constant (via Bianchi identity)*
- *keeping g = const* $\Rightarrow$ *transverse gauge symmetry for the linear theory*
- *keeping* $h_{\mu\nu}$ *traceful* $\Rightarrow$ *scalar-tensor theory of gravity*

Einstein '19; van der Bij, van Dam, Ng '82, Buchmuller, Dragon '88, Unruh '88, Henneaux, Teitelboim '89, ...
Alvarez et al. '05 - '12

Plan

§ I. *Lagrangians*

§ II. *Spectrum*

§ III. *Comments & conclusions*

§ $\mathbb{I}$.

Lagrangians

Symmetric tensors, flat bkg

- Consider the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\,\varphi\, M\, \varphi$$

where the Maxwell operator is defined as:

$$M = \Box - \partial\,\partial\cdot$$

Under $\delta\,\varphi = \partial\,\Lambda$ one finds, up to total derivatives $\delta\,\mathcal{L} \sim \partial\cdot\partial\cdot\varphi\,\partial\cdot\Lambda$

simplest choice for gauge invariance:

$$\partial\cdot\Lambda \equiv 0$$

Symmetric tensors, flat bkg

- Consider the Lagrangian:

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\, M\,\varphi$$

where the Maxwell operator is defined as:

$$M \;=\; \Box \;-\; \partial\,\partial\cdot$$

Under $\delta\,\varphi = \partial\,\Lambda$ one finds, up to total derivatives $\delta\,\mathcal{L} \sim \partial\cdot\partial\cdot\varphi\,\partial\cdot\Lambda$

simplest choice for gauge invariance:

$$\partial\cdot\Lambda \;\equiv\; 0$$

→ *No need for trace constraints:*

- *traceless* φ *and* Λ : *irreducible spin s* (Skvortsov-Vasiliev '07)
- *traceful* φ *and* Λ : *reducible spin s:* s, s-2, s-4, . . .

Mixed-symmetry tensors, flat bkg

Notation:

avoid space-time indices as far as possible

here: only *family indices*

- $\varphi_{\mu_1 \ldots \mu_{s_1} ; \nu_1 \ldots \nu_{s_2} ; \cdots} \longrightarrow \varphi$

- $\partial_{(\mu_1^i} \varphi_{\ldots ; \mu_2^i \ldots \mu_{s_i+1}^i) ; \ldots} \longrightarrow \partial^i \varphi$

- $\partial^\lambda \varphi_{\ldots ; \lambda \mu_2^i \ldots \mu_{s_i}^i ; \ldots} \longrightarrow \partial_i \varphi$

- $\varphi_{\ldots ;}{}^{\lambda}{}_{\mu_2^i \ldots \mu_{s_i}^i ; \ldots ; \lambda \mu_2^j \ldots \mu_{s_j}^j ; \ldots} \longrightarrow T_{ij} \varphi$

φ can be GL(D)-reducible, GL(D)-irreducible or O(D)-irreducible

Mixed-symmetry tensors, flat bkg

According to our general scheme, we start with a simple trial Lagrangian

$$\mathcal{L} = \frac{1}{2}\varphi\,(\Box - \partial^{\,i}\,\partial_{\,i})\,\varphi \equiv \frac{1}{2}\varphi\,M\,\varphi$$

and compute the variation of the Maxwell operator:

$$\delta\,(\Box - \partial^{\,i}\,\partial_{\,i})\,\varphi = -\frac{1}{2}\partial^{\,i}\,\partial^{\,j}\,\partial_{\,(i}\,\Lambda_{\,j)}$$

Mixed-symmetry tensors, flat bkg

According to our general scheme, we start with a simple trial Lagrangian

$$\mathcal{L} = \frac{1}{2}\,\varphi\,(\Box - \partial^{\,i}\,\partial_{i})\,\varphi \equiv \frac{1}{2}\,\varphi\,M\,\varphi$$

and compute the variation of the Maxwell operator:

$$\delta\,(\Box - \partial^{\,i}\,\partial_{i})\,\varphi = -\frac{1}{2}\,\partial^{\,i}\,\partial^{\,j}\,\partial_{\,(i}\,\Lambda_{\,j)}$$

the simplest condition for invariance involves *symmetrised divergences* of the parameters

$\rightarrow$

$$\boxed{\partial_{\,(i}\,\Lambda_{\,j)} = 0}$$

<u>Computation of d.o.f. from generalised light-cone gauge fixing</u>

Mixed-symmetry tensors, flat bkg

let us compare with irreducible case:

Transverse-invariant *Labastida*

Mixed-symmetry tensors, flat bkg

let us compare with irreducible case:

Transverse-invariant *Labastida*

Lagrangians

$$\mathcal{L} = \frac{1}{2}\,\varphi\,M\,\varphi$$

(N families)

$$\mathcal{L} = \frac{1}{2}\,\varphi\,\{\mathcal{F} - \frac{1}{2}\,\eta^{ij}\,T_{ij}\,\mathcal{F} + \frac{1}{36}\,\eta^{ij}\,\eta^{kl}\,\big(2\,T_{ij}\,T_{kl} - T_{i\,(\,k}\,T_{l\,)\,j}\big)\,\mathcal{F}\}\,\varphi$$

(2 families (!))

Mixed-symmetry tensors, flat bkg

let us compare with irreducible case:

Transverse-invariant | *Labastida*

Lagrangians

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\,M\,\varphi$$

(N families)

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\,\{\mathcal{F} \,-\, \frac{1}{2}\,\eta^{ij}\,T_{ij}\,\mathcal{F} \,+\, \frac{1}{36}\,\eta^{ij}\,\eta^{kl}\,\big(\,2\,T_{ij}\,T_{kl} \,-\, T_{i\,(\,k}\,T_{l\,)\,j}\,\big)\,\mathcal{F}\}\,\varphi$$

(2 families (!))

Equations of motion

$$M\,\varphi \,=\, (\Box \,-\, \partial^{\,i}\,\partial_{\,i})\,\varphi$$

(Lagrangian equations)

$$\mathcal{F}\,\varphi \,=\, \{M \,+\, \partial^{\,i}\partial^{\,j}\,T_{ij}\,\}\varphi$$

(non-Lagrangian equations)

Mixed-symmetry tensors, flat bkg

let us compare with irreducible case:

Transverse-invariant | *Labastida*

<u>Lagrangians</u>

$$\mathcal{L} \, = \, \frac{1}{2} \, \varphi \, M \, \varphi$$

(N families)

$$\mathcal{L} \, = \, \frac{1}{2} \, \varphi \, \{ \mathcal{F} \, - \, \frac{1}{2} \, \eta^{ij} \, T_{ij} \, \mathcal{F} \, + \, \frac{1}{36} \, \eta^{ij} \, \eta^{kl} \, \left(2 \, T_{ij} \, T_{kl} \, - \, T_{i\,(\,k} \, T_{l\,)\,j} \right) \mathcal{F} \} \, \varphi$$

(2 families (!))

<u>Equations of motion</u>

$$M \, \varphi \, = \, (\Box \, - \, \partial^{\,i} \, \partial_{\,i}) \, \varphi$$

(Lagrangian equations)

$$\mathcal{F} \, \varphi \, = \, \{ M \, + \, \partial^{\,i} \partial^{\,j} \, T_{ij} \, \} \varphi$$

(non-Lagrangian equations)

<u>Constraints</u>

$$\partial_{\,(i} \, \Lambda_{\,j)} \, = \, 0$$

$$\begin{cases} T_{(\,ij} \, \Lambda_{\,k\,)} \, = \, 0 \\ T_{(\,ij} \, T_{kl\,)} \, \varphi \, = \, 0 \end{cases}$$

Symmetric tensors, AdS bkg

Our starting point

$$M\,\varphi \,\equiv\, (\,\Box\, -\, \nabla\,\nabla\cdot\,)\,\varphi\,, \qquad \text{with} \qquad \begin{cases} \delta\varphi \,=\, \nabla\Lambda\,, \\ \nabla\cdot\Lambda \,=\, 0 \end{cases}$$

as usual, gauge invariance requires an additional term:

$$M_L\,\varphi \,\equiv\, M\,\varphi \,-\, \frac{1}{L^{\,2}}\,\{\,[\,(s-2)(D+s-3)\,-\,s\,]\,\varphi\,-\,2\,g\,\varphi'\,\}$$

the Lagrangian is simply

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\,M_L\,\varphi\,,$$

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

let us consider the two-family case, for simplicity

$$\varphi_{\mu_1 \cdots \mu_s,\, \nu_1 \cdots \nu_r} \,\equiv\, \varphi_{\,\mu_s,\, \nu_r} \qquad\qquad \delta\, \varphi_{\,\mu_s,\, \nu_r} \,=\, \nabla_\mu\, \Lambda_{\,\mu_{s-1},\, \nu_r} \,+\, \nabla_\nu\, \lambda_{\,\mu_s,\, \nu_{r-1}}$$

$$(M\,\varphi)_{\,\mu_s,\, \nu_r} \equiv \Box \varphi_{\,\mu_s,\, \nu_r} - \nabla_\mu\, \partial^{\,\alpha}\, \varphi_{\,\alpha \mu_{s-1},\, \nu_r} - \nabla_\nu\, \partial^{\,\alpha}\, \varphi_{\,\mu_s,\, \alpha \nu_{r-1}}$$

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

let us consider the two-family case, for simplicity

$$\varphi_{\mu_1 \cdots \mu_s, \nu_1 \cdots \nu_r} \equiv \varphi_{\mu_s, \nu_r} \qquad \delta \varphi_{\mu_s, \nu_r} = \nabla_\mu \Lambda_{\mu_{s-1}, \nu_r} + \nabla_\nu \lambda_{\mu_s, \nu_{r-1}}$$

$$(M \varphi)_{\mu_s, \nu_r} \equiv \Box \varphi_{\mu_s, \nu_r} - \nabla_\mu \partial^\alpha \varphi_{\alpha \mu_{s-1}, \nu_r} - \nabla_\nu \partial^\alpha \varphi_{\mu_s, \alpha \nu_{r-1}}$$

the gauge variation of the ``Maxwell'' tensor is

$$\begin{aligned} \delta (M \varphi)_{\mu_s, \nu_r} = \frac{1}{L^2} \{ & [(s-1)(D+s-3) - (D+2s-3)] \nabla_\mu \Lambda_{\mu_{s-1}, \nu_r} \\ & + [(r-1)(D+r-3) - (D+2r-3)] \nabla_\nu \lambda_{\mu_s, \nu_{r-1}} \\ & + \text{exchanges betw. families} + \mathcal{O}(tr, \, div) \} \end{aligned}$$

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

let us consider the two-family case, for simplicity

$$\varphi_{\mu_1 \cdots \mu_s, \nu_1 \cdots \nu_r} \equiv \varphi_{\mu_s, \nu_r} \qquad \delta \varphi_{\mu_s, \nu_r} = \nabla_\mu \Lambda_{\mu_{s-1}, \nu_r} + \nabla_\nu \lambda_{\mu_s, \nu_{r-1}}$$

$$(M \varphi)_{\mu_s, \nu_r} \equiv \Box \varphi_{\mu_s, \nu_r} - \nabla_\mu \partial^\alpha \varphi_{\alpha \mu_{s-1}, \nu_r} - \nabla_\nu \partial^\alpha \varphi_{\mu_s, \alpha \nu_{r-1}}$$

the gauge variation of the ``Maxwell'' tensor is

$$\boxed{\begin{aligned} \delta (M \varphi)_{\mu_s, \nu_r} = \frac{1}{L^2} \{ & [(s-1)(D+s-3) - (D+2s-3)] \nabla_\mu \Lambda_{\mu_{s-1}, \nu_r} \\ & + [(r-1)(D+r-3) - (D+2r-3)] \nabla_\nu \lambda_{\mu_s, \nu_{r-1}} \\ & + \text{exchanges betw. families} + \mathcal{O}(tr,\, div) \} \end{aligned}}$$

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

let us consider the two-family case, for simplicity

$$\varphi_{\mu_1 \cdots \mu_s,\, \nu_1 \cdots \nu_r} \equiv \varphi_{\mu_s,\, \nu_r} \qquad \delta\, \varphi_{\mu_s,\, \nu_r} = \nabla_\mu\, \Lambda_{\mu_{s-1},\, \nu_r} + \nabla_\nu\, \lambda_{\mu_s,\, \nu_{r-1}}$$

$$(M\, \varphi)_{\mu_s,\, \nu_r} \equiv \Box \varphi_{\mu_s,\, \nu_r} - \nabla_\mu\, \partial^\alpha\, \varphi_{\alpha \mu_{s-1},\, \nu_r} - \nabla_\nu\, \partial^\alpha\, \varphi_{\mu_s,\, \alpha \nu_{r-1}}$$

the gauge variation of the ``Maxwell'' tensor is

$$\begin{aligned}\delta\,(M\,\varphi)_{\mu_s,\,\nu_r} = \frac{1}{L^2}\{ & [(s-1)(D+s-3)-(D+2s-3)]\nabla_\mu \Lambda_{\mu_{s-1},\nu_r} \\ & +[(r-1)(D+r-3)-(D+2r-3)]\,\nabla_\nu \lambda_{\mu_s,\nu_{r-1}} \\ & + \text{exchanges betw. families} + \mathcal{O}(tr,\, div)\}\end{aligned}$$

impossible in general to compensate both terms:

Mixed-symmetry tensors, AdS bkg

In the case of mixed-symmetry tensors on (A)dS there is a new phenomenon

let us consider the two-family case, for simplicity

$$\varphi_{\mu_1 \cdots \mu_s,\, \nu_1 \cdots \nu_r} \equiv \varphi_{\mu_s,\, \nu_r} \qquad \delta\, \varphi_{\mu_s,\, \nu_r} = \nabla_\mu\, \Lambda_{\mu_{s-1},\, \nu_r} + \nabla_\nu\, \lambda_{\mu_s,\, \nu_{r-1}}$$

$$(M\, \varphi)_{\mu_s,\, \nu_r} \equiv \Box \varphi_{\mu_s,\, \nu_r} - \nabla_\mu\, \partial^\alpha\, \varphi_{\alpha\mu_{s-1},\, \nu_r} - \nabla_\nu\, \partial^\alpha\, \varphi_{\mu_s,\, \alpha\nu_{r-1}}$$

the gauge variation of the ``Maxwell'' tensor is

$$\delta\, (M\, \varphi)_{\mu_s,\, \nu_r} = \frac{1}{L^2} \{ [(s-1)(D+s-3) - (D+2s-3)] \nabla_\mu \Lambda_{\mu_{s-1},\nu_r} \\ + [(r-1)(D+r-3) - (D+2r-3)]\, \nabla_\nu \lambda_{\mu_s,\nu_{r-1}} \\ + \text{exchanges betw. families} + \mathcal{O}(tr,\, div) \}$$

impossible in general to compensate both terms:

loses `half' of the gauge-symmetry → $\mathcal{B}MV$

Metsaev '95, '98 (talk SQS '97); Brink, Metsaev, Vasiliev '00

Alkalaev. Shaynkman, Vasiliev '05, Alkalaev, Grigoriev '09, '11 Zinoviev '02, '09, Boulanger, Iazeolla, Sundell '08, Skvortsov '09, Burdik, Reshetnyak '12, . . .

Mixed-symmetry tensors, AdS bkg

Keep only the BMV-parameter, and consider the variation of M for N families:

- $M\,\varphi \equiv \left(\Box - \nabla^i \nabla_i\right)\varphi$

- $$M\,\delta\varphi = -\frac{1}{L^2}\left\{(D-1)\,\nabla^i\Lambda_i - (D-N-3)\,\nabla^i S^j{}_i\,\Lambda_j - \nabla^i S^j{}_k S^k{}_i\,\Lambda_j\right\}$$
$$+\frac{1}{2}\,\nabla^i\nabla^j\nabla_{(\,i}\Lambda_{\,j)} + \frac{1}{L^2}\left\{2\,g^{ij}\,\nabla_{(\,i}\Lambda_{\,j)} + g^{ij}S^k{}_i\nabla_{[\,j}\Lambda_{\,k]} - 2\,\nabla^i g^{jk}\,T_{ij}\,\Lambda_k\right\}$$

Mixed-symmetry tensors, AdS bkg

Keep only the BMV-parameter, and consider the variation of M for N families:

- $M\,\varphi \equiv \left(\Box - \nabla^i\nabla_i\right)\varphi$

- $$M\,\delta\varphi = -\frac{1}{L^2}\left\{ (D-1)\,\nabla^i\Lambda_i - (D-N-3)\,\nabla^i S^j{}_i\,\Lambda_j - \nabla^i S^j{}_k S^k{}_i\,\Lambda_j \right\}$$
$$+\frac{1}{2}\,\nabla^i\nabla^j\nabla_{(\,i}\Lambda_{\,j)} + \frac{1}{L^2}\left\{ 2\,g^{ij}\,\nabla_{(\,i}\Lambda_{\,j)} + g^{ij}S^k{}_i\nabla_{[\,j}\Lambda_{\,k]} - 2\,\nabla^i g^{jk}\,T_{ij}\,\Lambda_k \right\}$$

At this stage we make some additional assumptions:

Mixed-symmetry tensors, AdS bkg

Keep only the BMV-parameter, and consider the variation of M for N families:

- $M\,\varphi \equiv \left(\Box - \nabla^i\nabla_i\right)\varphi$

- $$M\,\delta\varphi = -\frac{1}{L^2}\left\{(D-1)\,\nabla^i\Lambda_i - (D-N-3)\,\nabla^i S^j{}_i\,\Lambda_j - \nabla^i S^j{}_k S^k{}_i\,\Lambda_j\right\}$$
$$+\frac{1}{2}\,\nabla^i\nabla^j\nabla_{(\,i}\,\Lambda_{\,j)} + \frac{1}{L^2}\left\{2\,g^{ij}\,\nabla_{(\,i}\,\Lambda_{\,j)} + g^{ij}S^k{}_i\nabla_{[\,j}\,\Lambda_{\,k]} - 2\,\nabla^i g^{jk}\,T_{ij}\,\Lambda_k\right\}$$

At this stage we make some additional assumptions:

1) the field is *GL(D)-irreducible*: $S^i{}_j\,\varphi = 0 \quad \text{for} \quad i < j$

$\Rightarrow$

- the first line can be compensated by means of a ``mass'' term
- $\nabla_{(i}\,\Lambda_{\,j)} = 0 \quad \Rightarrow \quad \nabla_i\,\Lambda_{\,j} = 0$ *(second line almost = 0)*

Mixed-symmetry tensors, AdS bkg

Keep only the BMV-parameter, and consider the variation of M for N families:

- $M\,\varphi \,\equiv\, \left(\,\Box - \nabla^{i}\nabla_{i}\,\right)\varphi$

- $$M\,\delta\varphi = -\,\frac{1}{L^2}\left\{\,(D-1)\,\nabla^{i}\Lambda_{i} \,-\, (D-N-3)\,\nabla^{i}S^{j}{}_{i}\,\Lambda_{j} \,-\, \nabla^{i}S^{j}{}_{k}S^{k}{}_{i}\,\Lambda_{j}\,\right\}$$
$$+\frac{1}{2}\,\nabla^{i}\nabla^{j}\nabla_{(\,i}\Lambda_{\,j)} + \frac{1}{L^2}\left\{\,2\,g^{ij}\,\nabla_{(\,i}\Lambda_{\,j)} \,+\, g^{ij}S^{k}{}_{i}\nabla_{[\,j}\Lambda_{\,k]} \,-\, 2\,\nabla^{i}g^{jk}\,T_{ij}\,\Lambda_{k}\,\right\}$$

At this stage we make some additional assumptions:

1) the field is *GL(D)-irreducible*: $S^{i}{}_{j}\,\varphi \,=\, 0 \quad \text{for} \quad i<j$

$\Rightarrow$

- the first line can be compensated by means of a ``mass" term
- $\nabla_{(i}\Lambda_{\,j)} \,=\, 0 \qquad \Rightarrow \qquad \nabla_{i}\Lambda_{\,j} \,=\, 0$ *(second line almost = 0)*

2) the field is *traceless*: $T_{ij}\,\varphi \,=\, 0$

$\Rightarrow$

- the last term does not contribute to the Lagrangian

Mixed-symmetry tensors, AdS bkg

Our final result is

$$\boxed{\mathcal{L} = \frac{1}{2}\,\varphi\,\Big\{M - \frac{1}{L^2}\Big[(s_n - n - 1)(D + s_n - n - 2) - \sum_{k=1}^{N} s_k\Big]\Big\}\,\varphi}$$

a Lagrangian for N-family mixed-symmetry fields on AdS

§ II

Spectrum

Hamiltonian analysis for symmetric tensors

➙ *For* $\left\{ \begin{array}{l} \text{flat bkg, any symmetry} \\ \\ \text{AdS bkg, symmetric tensors} \end{array} \right.$ ➽ Only first-class constraints

count # independent components of Λ and $\dot{\Lambda}$

Hamiltonian analysis for symmetric tensors

➙ *For* $\left\{ \begin{array}{l} \text{flat bkg, any symmetry} \\ \text{AdS bkg, symmetric tensors} \end{array} \right.$ ➽ **Only first-class constraints**

count # independent components of Λ and $\dot{\Lambda}$

➙ Systematically: decompose $\Lambda_{\mu_1 \cdots \mu_{s-1}}$ in spatially-transverse parts and exploit the constraint; for a faster counting just observe:

$$\partial^{\alpha} \Lambda_{\alpha \mu_2 \cdots \mu_{s-1}} = 0 \quad \Rightarrow \quad \dot{\Lambda}_{0 \mu_2 \cdots \mu_{s-1}} = \vec{\nabla} \cdot \Lambda_{\mu_2 \cdots \mu_{s-1}}$$

Hamiltonian analysis for symmetric tensors

➙ *For* $\left\{ \begin{array}{l} \text{flat bkg, any symmetry} \\ \text{AdS bkg, symmetric tensors} \end{array} \right.$ ➽ **Only first-class constraints**

count # independent components of Λ and $\dot{\Lambda}$

➙ Systematically: decompose $\Lambda_{\mu_1 \cdots \mu_{s-1}}$ in spatially-transverse parts and exploit the constraint; for a faster counting just observe:

$$\partial^{\alpha} \Lambda_{\alpha\, \mu_2 \cdots \mu_{s-1}} = 0 \quad \Rightarrow \quad \dot{\Lambda}_{0\, \mu_2 \cdots \mu_{s-1}} = \vec{\nabla} \cdot \Lambda_{\mu_2 \cdots \mu_{s-1}}$$

➙ $\left\{ \begin{array}{l} \text{count } \textit{twice } \Lambda_{i_1 \cdots i_{s-1}} \text{: their time derivatives are independent} \\ \text{count } \textit{once } \Lambda_{0\, \mu_2 \cdots \mu_{s-1}} \text{: their time derivatives are not independent} \end{array} \right.$

Hamiltonian analysis for symmetric tensors

➙ *For* $\left\{\begin{array}{l}\text{flat bkg, any symmetry}\\ \text{AdS bkg, symmetric tensors}\end{array}\right.$ ➸ **Only first-class constraints**

count # independent components of Λ and $\dot{\Lambda}$

➙ Systematically: decompose $\Lambda_{\mu_1 \cdots \mu_{s-1}}$ in spatially-transverse parts and exploit the constraint; for a faster counting just observe:

$$\partial^{\alpha} \Lambda_{\alpha\, \mu_2 \cdots \mu_{s-1}} = 0 \quad \Rightarrow \quad \dot{\Lambda}_{0\, \mu_2 \cdots \mu_{s-1}} = \vec{\nabla} \cdot \Lambda_{\mu_2 \cdots \mu_{s-1}}$$

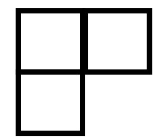
➙ $\left\{\begin{array}{l}\text{count } \textit{twice } \Lambda_{i_1 \cdots i_{s-1}} \text{: their time derivatives are independent}\\ \text{count } \textit{once } \Lambda_{0\, \mu_2 \cdots \mu_{s-1}} \text{: their time derivatives are not independent}\end{array}\right.$

e.g. spin 2:

$$\underbrace{\frac{D\,(D+1)}{2}}_{h_{\mu\nu}} - \underbrace{2\,(D-1)}_{\Lambda_i \,+\, \dot{\Lambda}_i} - \underbrace{1}_{\Lambda_0} = \underbrace{\frac{(D-1)\,(D-2)}{2}}_{graviton\ +\ scalar}$$

Mixed-symmetry on AdS:
BMV-pattern & gauge-per-gauge symmetry breaking

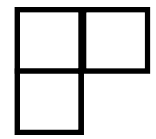
$$\varphi \,=\, \varphi_{\,\mu\,\mu,\,\nu}$$

For simplicity, consider a {2, 1} field  :

s.t.

$$\delta_0\,\varphi_{\,\mu\,\mu,\,\nu} \,=\, \nabla_{\mu}\,\Lambda_{\,\mu,\,\nu} \,+\, \nabla_{\nu}\,\lambda_{\,\mu\mu} \,-\, \frac{1}{2}\,\nabla_{\mu}\,\lambda_{\,\mu\,\nu}$$

Mixed-symmetry on AdS:
BMV-pattern & gauge-per-gauge symmetry breaking

For simplicity, consider a {2, 1} field  :

$$\varphi = \varphi_{\mu\,\mu,\,\nu}$$

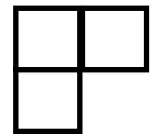
s.t.

$$\delta_0\,\varphi_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \frac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu}$$

➡ Besides the impossibility to keep both gauge invariances unbroken on AdS there is a deeper issue:

Mixed-symmetry on AdS:
BMV-pattern & gauge-per-gauge symmetry breaking

$$\varphi = \varphi_{\mu\mu,\nu}$$

For simplicity, consider a {2, 1} field  :

s.t.

$$\delta_0 \varphi_{\mu\mu,\nu} = \nabla_\mu \Lambda_{\mu,\nu} + \nabla_\nu \lambda_{\mu\mu} - \frac{1}{2} \nabla_\mu \lambda_{\mu\nu}$$

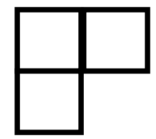
- Besides the impossibility to keep both gauge invariances unbroken on AdS there is a deeper issue:

gauge-per-gauge invariance-breaking

$$\begin{cases} \delta \Lambda_{\mu,\nu} = \nabla_\nu \theta_\mu - \nabla_\mu \theta_\nu \\ \delta \lambda_{\mu\mu} = -2 \nabla_\mu \theta_\mu \end{cases} \quad \Rightarrow \quad \delta \varphi_{\mu\mu,\nu} \sim [\nabla_\mu, \nabla_\nu] \theta_\mu$$

Mixed-symmetry on AdS:
BMV-pattern & gauge-per-gauge symmetry breaking

$$\varphi = \varphi_{\mu\,\mu,\,\nu}$$

For simplicity, consider a {2, 1} field  :

s.t.

$$\delta_0\, \varphi_{\mu\,\mu,\,\nu} = \nabla_\mu \Lambda_{\mu,\,\nu} + \nabla_\nu \lambda_{\mu\mu} - \frac{1}{2} \nabla_\mu \lambda_{\mu\,\nu}$$

- Besides the impossibility to keep both gauge invariances unbroken on AdS there is a deeper issue:

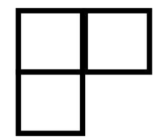
gauge-per-gauge invariance-breaking

$$\begin{cases} \delta\, \Lambda_{\mu,\,\nu} = \nabla_\nu\, \theta_\mu - \nabla_\mu\, \theta_\nu \\ \delta\, \lambda_{\mu\,\mu} = -\,2\, \nabla_\mu\, \theta_\mu \end{cases} \quad \rightarrowtail \quad \delta\, \varphi_{\mu\mu,\,\nu} \sim [\nabla_\mu, \nabla_\nu]\, \theta_\mu$$

- In order to ``neutralize'' the effect of θ_μ on the initial {2,1} field
we promote it to a gauge parameter for a new field:

Mixed-symmetry on AdS:
BMV-pattern & gauge-per-gauge symmetry breaking

$$\varphi = \varphi_{\mu\mu,\nu}$$

For simplicity, consider a {2, 1} field  :

s.t.

$$\delta_0 \varphi_{\mu\mu,\nu} = \nabla_\mu \Lambda_{\mu,\nu} + \nabla_\nu \lambda_{\mu\mu} - \frac{1}{2} \nabla_\mu \lambda_{\mu\nu}$$

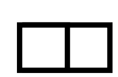
- Besides the impossibility to keep both gauge invariances unbroken on AdS there is a deeper issue:

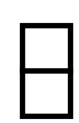
gauge-per-gauge invariance-breaking

$$\begin{cases} \delta \Lambda_{\mu,\nu} = \nabla_\nu \theta_\mu - \nabla_\mu \theta_\nu \\ \delta \lambda_{\mu\mu} = -2 \nabla_\mu \theta_\mu \end{cases} \quad \rightarrowtail \quad \delta \varphi_{\mu\mu,\nu} \sim [\nabla_\mu, \nabla_\nu] \theta_\mu$$

- In order to ``neutralize'' the effect of θ_μ on the initial {2,1} field
we promote it to a gauge parameter for a new field:

Two options for fields whose variation $\sim \nabla_\mu \theta_\nu$:

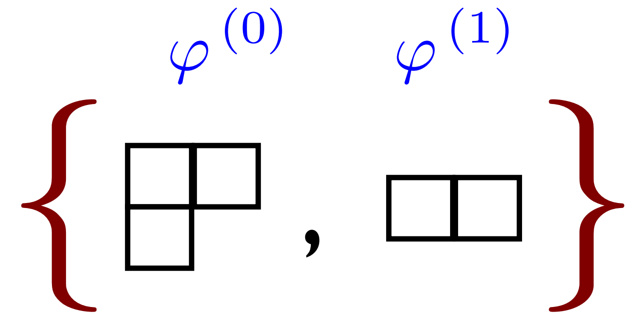
 unitary choice

 non-unitary choice

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the BMV multiplet s.t. :

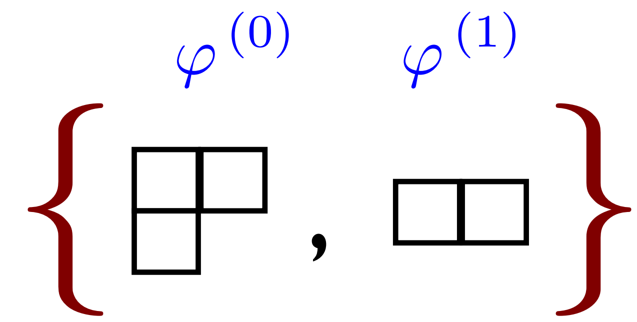
- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*



BMV multiplet

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the
BMV multiplet s.t. :



BMV multiplet

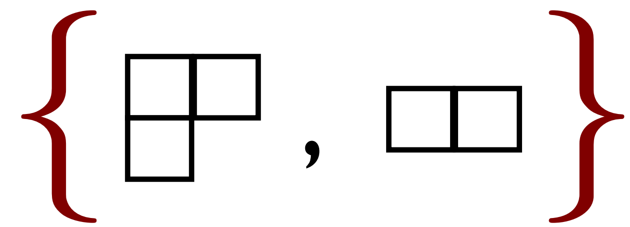
- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*

Lagrangian $\longrightarrow$ $$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{1} \varphi^{(i)}\left(M - \frac{m_i}{L^2}\right)\varphi^{(i)} + \frac{c}{L}\,\varphi^{(1)}\,\widehat{\nabla}\cdot\varphi^{(0)}$$

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the
BMV multiplet s.t. :

$\varphi^{(0)}$ $\varphi^{(1)}$



BMV multiplet

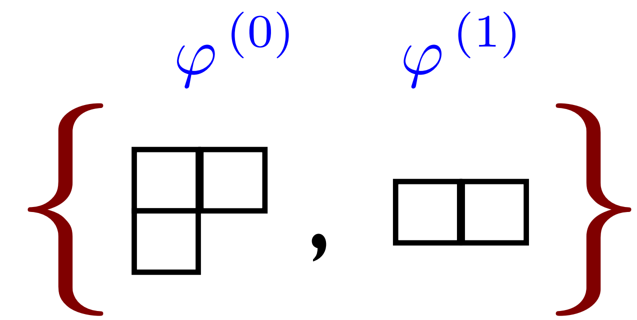
- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*

Lagrangian $\longrightarrow$ $$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{1} \varphi^{(i)} \left(M - \frac{m_i}{L^2} \right) \varphi^{(i)} + \frac{c}{L}\, \varphi^{(1)}\, \widehat{\nabla} \cdot \varphi^{(0)}$$

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\, \varphi^{(0)}_{\mu\mu,\,\nu} = \nabla_\mu \Lambda_{\mu,\,\nu} + \nabla_\nu \lambda_{\mu\mu} - \frac{1}{2} \nabla_\mu \lambda_{\mu\nu} + \frac{\beta}{L} [\nabla_\nu, \nabla\mu]\, \xi_\mu \\ \delta\, \varphi^{(1)}_{\mu\mu} = \nabla_\mu \xi_\mu + \frac{\alpha}{L} \lambda_{\mu\mu} \end{cases}$$

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the
BMV multiplet s.t. :



BMV multiplet

- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*

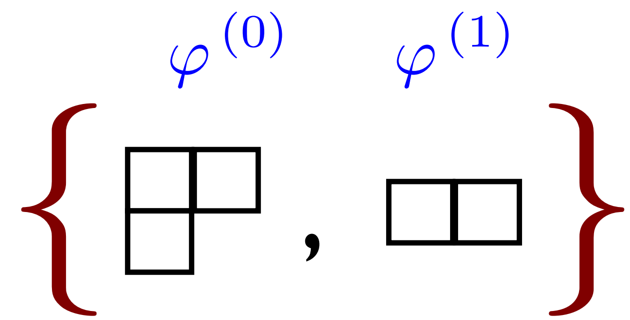
Lagrangian $\longrightarrow$ $$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{1} \varphi^{(i)}\left(M - \frac{m_i}{L^2}\right)\varphi^{(i)} + \frac{c}{L}\,\varphi^{(1)}\,\widehat{\nabla}\cdot\varphi^{(0)}$$

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \dfrac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu} + \dfrac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_\mu \\ \delta\,\varphi^{(1)}_{\mu\,\mu} = \nabla_\mu\,\xi_\mu + \dfrac{\alpha}{L}\,\lambda_{\mu\,\mu} \end{cases}$$

Gauge-per-gauge $\longrightarrow$ $$\delta\,\Lambda_{\mu,\,\nu} = \nabla_\nu\,\theta_\mu - \nabla_\mu\,\theta_\nu\ , \quad \delta\,\lambda_{\mu\,\mu} = -\,2\,\nabla_\mu\,\theta_\mu\ , \quad \delta\,\xi_\mu = \frac{b}{L}\theta_\mu$$

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the
BMV multiplet s.t. :



BMV multiplet

- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*

Lagrangian $\longrightarrow$

$$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{1} \varphi^{(i)}\left(M - \frac{m_i}{L^2}\right)\varphi^{(i)} + \frac{c}{L}\,\varphi^{(1)}\,\widehat{\nabla}\cdot\varphi^{(0)}$$

Gauge transformations $\longrightarrow$

$$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \frac{1}{2}\nabla_\mu\,\lambda_{\mu\,\nu} + \frac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_\mu \\ \delta\,\varphi^{(1)}_{\mu\,\mu} = \nabla_\mu\,\xi_\mu + \frac{\alpha}{L}\,\lambda_{\mu\,\mu} \end{cases}$$

Gauge-per-gauge $\longrightarrow$

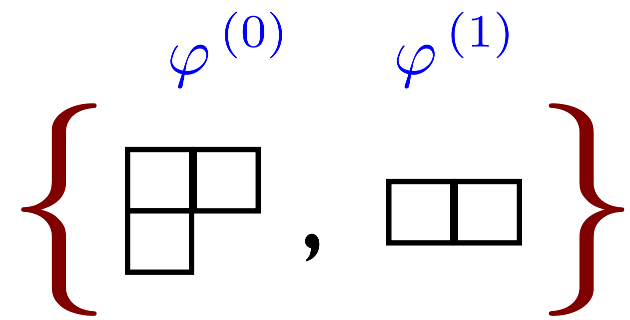
$$\delta\,\Lambda_{\mu,\,\nu} = \nabla_\nu\,\theta_\mu - \nabla_\mu\,\theta_\nu\ , \quad \delta\,\lambda_{\mu\,\mu} = -2\,\nabla_\mu\,\theta_\mu\ , \quad \delta\,\xi_\mu = \frac{b}{L}\theta_\mu$$

Constraints $\longrightarrow$

$$\begin{cases} \nabla^{\alpha}\,(\Lambda_{\alpha,\,\mu} - \frac{1}{2}\,\lambda_{\alpha\,\mu}) + \frac{k}{L}\,\xi_\mu = 0 \\ \nabla^{\alpha}\,\xi_\alpha = 0 \end{cases}$$

Mixed-symmetry on AdS: Stueckelberg Lagrangian

We want to find a Lagrangian for the
BMV multiplet s.t. :



BMV multiplet

- *it is a smooth deformation of the corresponding flat, transverse-invariant Lagrangians, including possible deformations of transversality constraints*
- *the overall gauge-invariance of the system is the same as its flat counterpart, including all gauge-per-gauge*

Lagrangian $\longrightarrow$ $$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{1} \varphi^{(i)}\left(M - \frac{m_i}{L^2}\right)\varphi^{(i)} + \frac{c}{L}\,\varphi^{(1)}\,\widehat{\nabla}\cdot\varphi^{(0)}$$

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \frac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu} + \frac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_\mu \\ \delta\,\varphi^{(1)}_{\mu\,\mu} = \nabla_\mu\,\xi_\mu + \frac{\alpha}{L}\,\lambda_{\mu\,\mu} \end{cases}$$

Gauge-per-gauge $\longrightarrow$ $$\delta\,\Lambda_{\mu,\,\nu} = \nabla_\nu\,\theta_\mu - \nabla_\mu\,\theta_\nu\,, \quad \delta\,\lambda_{\mu\,\mu} = -\,2\,\nabla_\mu\,\theta_\mu\,, \quad \delta\,\xi_\mu = \frac{b}{L}\theta_\mu$$

Constraints $\longrightarrow$ $$\begin{cases} \nabla^{\alpha}\,(\Lambda_{\alpha,\,\mu} - \frac{1}{2}\,\lambda_{\alpha\,\mu}) + \frac{k}{L}\,\xi_\mu = 0 \\ \nabla^{\alpha}\,\xi_\alpha = 0 \end{cases}$$

$$\delta\,\mathcal{L} = 0$$
$$\delta^{2}\,\varphi^{(i)} = 0$$

a unique solution exists, found for the two-family case {s, k}

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Mixed-symmetry on AdS: gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\,\mu,\,\nu} + \nabla_\nu\,\lambda_{\,\mu\mu} - \dfrac{1}{2}\,\nabla_\mu\,\lambda_{\,\mu\,\nu} + \dfrac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_{\,\mu} \\ \delta\,\varphi^{(1)}_{\mu\,\mu} = \nabla_\mu\,\xi_{\,\mu} + \dfrac{\alpha}{L}\,\lambda_{\,\mu\,\mu} \end{cases}$$

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

$$\text{Gauge transformations} \longrightarrow \begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \dfrac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu} + \dfrac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_\mu \\ \delta\,\cancel{\varphi^{(1)}_{\mu\,\mu}} = \nabla_\mu\,\xi_\mu + \dfrac{\alpha}{L}\,\lambda_{\mu\,\mu} \quad \text{up to GpG leaving it invariant} \end{cases}$$

Mixed-symmetry on AdS: gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\,\mu,\,\nu} + \nabla_\nu\,\lambda_{\,\mu\mu} - \dfrac{1}{2}\,\nabla_\mu\,\lambda_{\,\mu\,\nu} + \dfrac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_{\,\mu} \\ \delta\,\cancel{\varphi}^{(1)}_{\mu\,\mu} = \nabla_\mu\,\xi_{\,\mu} + \dfrac{\alpha}{L}\,\lambda_{\,\mu\,\mu} \end{cases}$$ up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$ $\delta\,\Lambda_{\,\mu,\,\nu} = \nabla_\nu\,\theta_{\,\mu} - \nabla_\mu\,\theta_{\,\nu}\ ,\quad \delta\,\lambda_{\,\mu\,\mu} = -\,2\,\nabla_\mu\,\theta_{\,\mu}\quad \delta\,\xi_{\,\mu} = \dfrac{b}{L}\theta_{\,\mu}$

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} \;=\; \nabla_\mu\,\Lambda_{\,\mu,\,\nu} \;+\; \nabla_\nu\,\lambda_{\,\mu\mu} \;-\; \dfrac{1}{2}\,\nabla_\mu\,\lambda_{\,\mu\,\nu} \;+\; \dfrac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_{\,\mu} \\ \cancel{\delta\,\varphi^{(1)}_{\mu\,\mu}} \;=\; \nabla_\mu\,\xi_{\,\mu} \;+\; \dfrac{\alpha}{L}\,\lambda_{\,\mu\,\mu} \end{cases}$$ up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$ $\delta\,\Lambda_{\,\mu,\,\nu} \;=\; \nabla_\nu\,\theta_{\,\mu} \;-\; \nabla_\mu\,\theta_{\,\nu}$, $\cancel{\delta\,\lambda_{\,\mu\,\mu}} \;=\; -\,2\,\nabla_\mu\,\theta_{\,\mu}$ $\cancel{\delta\,\xi_{\,\mu}} \;=\; \dfrac{b}{L}\theta_{\,\mu}$

via ``gauge-per-gauge'' fixing

Mixed-symmetry on AdS: gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$ $\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\,\mu,\,\nu} + \nabla_\nu\,\lambda_{\,\mu\mu} - \frac{1}{2}\,\nabla_\mu\,\lambda_{\,\mu\,\nu} + \frac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_{\,\mu} \\ \cancel{\delta\,\varphi^{(1)}_{\mu\,\mu}} = \nabla_\mu\,\xi_{\,\mu} + \frac{\alpha}{L}\,\lambda_{\,\mu\,\mu} \end{cases}$ up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$ $\delta\,\Lambda_{\,\mu,\,\nu} = \nabla_\nu\,\theta_{\,\mu} - \nabla_\mu\,\theta_{\,\nu}$, $\cancel{\delta\,\lambda_{\,\mu\,\mu}} = -\,2\,\nabla_\mu\,\theta_{\,\mu}$, $\cancel{\delta\,\xi_{\,\mu}} = \frac{b}{L}\theta_{\,\mu}$

via ``gauge-per-gauge'' fixing

Lagrangian $\longrightarrow$ $\mathcal{L} = \frac{1}{2}\,\varphi^{(0)}\left(M - \frac{m_0}{L^2}\right)\varphi^{(0)}$

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$
$$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \frac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu} + \frac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_\mu \\ \cancel{\delta\,\varphi^{(1)}_{\mu\,\mu}} = \nabla_\mu\,\xi_\mu + \frac{\alpha}{L}\,\lambda_{\mu\,\mu} \end{cases}$$
up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$ $\delta\,\Lambda_{\mu,\,\nu} = \nabla_\nu\,\theta_\mu - \nabla_\mu\,\theta_\nu$, $\cancel{\delta\,\lambda_{\mu\,\mu}} = -\,2\,\nabla_\mu\,\theta_\mu$, $\cancel{\delta\,\xi_\mu} = \frac{b}{L}\theta_\mu$

via ``gauge-per-gauge'' fixing

Lagrangian $\longrightarrow$ $\mathcal{L} = \frac{1}{2}\,\varphi^{(0)}\left(M - \frac{m_0}{L^2}\right)\varphi^{(0)}$

Constraints $\longrightarrow$
$$\begin{cases} \nabla^{\,\alpha}\,(\Lambda_{\,\alpha,\,\mu} - \frac{1}{2}\,\lambda_{\,\alpha\,\mu}) + \frac{k}{L}\,\xi_{\,\mu} = 0 \\ \nabla^{\,\alpha}\,\xi_{\,\alpha} = 0 \end{cases}$$

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$
$$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} \,=\, \nabla_\mu\,\Lambda_{\,\mu,\,\nu} \,+\, \nabla_\nu\,\lambda_{\,\mu\mu} \,-\, \frac{1}{2}\,\nabla_\mu\,\lambda_{\,\mu\,\nu} \,+\, \frac{\beta}{L}\,[\nabla_\nu, \nabla\mu]\,\xi_{\,\mu} \\ \cancel{\delta\,\varphi^{(1)}_{\mu\,\mu}} \,=\, \nabla_\mu\,\xi_{\,\mu} \,+\, \frac{\alpha}{L}\,\lambda_{\,\mu\,\mu} \end{cases}$$
up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$
$$\delta\,\Lambda_{\,\mu,\,\nu} \,=\, \nabla_\nu\,\theta_{\,\mu} \,-\, \nabla_\mu\,\theta_{\,\nu}\,, \quad \delta\,\cancel{\lambda_{\,\mu\,\mu}} \,=\, -\,2\,\nabla_\mu\,\theta_{\,\mu}\,, \quad \delta\,\cancel{\xi_{\,\mu}} \,=\, \frac{b}{L}\theta_{\,\mu}$$
via ``gauge-per-gauge'' fixing

Lagrangian $\longrightarrow$
$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi^{(0)}\left(M - \frac{m_0}{L^2}\right)\varphi^{(0)}$$

Constraints $\longrightarrow$
$$\begin{cases} \nabla^{\,\alpha}\,(\Lambda_{\,\alpha,\,\mu} \,-\, \frac{1}{2}\,\cancel{\lambda_{\,\alpha\,\mu}}) \,+\, \frac{k}{L}\,\cancel{\xi_{\,\mu}} \,=\, 0 \\ \nabla^{\,\alpha}\,\cancel{\xi_{\,\alpha}} \,=\, 0 \end{cases}$$

Mixed-symmetry on AdS:
gauge-fixing to transverse-invariant action

Last step: show that the Stueckelberg Lagrangian can be gauge fixed to the transverse-invariant one

Gauge transformations $\longrightarrow$ $$\begin{cases} \delta\,\varphi^{(0)}_{\mu\,\mu,\,\nu} = \nabla_\mu\,\Lambda_{\mu,\,\nu} + \nabla_\nu\,\lambda_{\mu\mu} - \frac{1}{2}\,\nabla_\mu\,\lambda_{\mu\,\nu} + \frac{\beta}{L}\,[\nabla_\nu,\nabla\mu]\,\xi_\mu \\ \delta\,\cancel{\varphi^{(1)}_{\mu\,\mu}} = \nabla_\mu\,\xi_\mu + \frac{\alpha}{L}\,\lambda_{\mu\,\mu} \end{cases}$$

up to GpG leaving it invariant

Gauge-per-gauge $\longrightarrow$ $$\delta\,\Lambda_{\mu,\,\nu} = \nabla_\nu\,\theta_\mu - \nabla_\mu\,\theta_\nu\,, \quad \delta\,\cancel{\lambda_{\mu\,\mu}} = -\,2\,\nabla_\mu\,\theta_\mu\,, \quad \delta\,\cancel{\xi_\mu} = \frac{b}{L}\theta_\mu$$

via ``gauge-per-gauge'' fixing

Lagrangian $\longrightarrow$ $$\mathcal{L} = \frac{1}{2}\,\varphi^{(0)}\left(M - \frac{m_0}{L^2}\right)\varphi^{(0)}$$

Constraints $\longrightarrow$ $$\begin{cases} \nabla^{\,\alpha}\,(\Lambda_{\,\alpha,\,\mu} - \frac{1}{2}\,\cancel{\lambda_{\alpha\,\mu}}) + \frac{k}{L}\,\cancel{\xi_\mu} = 0 \\ \nabla^{\,\alpha}\,\cancel{\xi_\alpha} = 0 \end{cases}$$

thus completing our argument

§ III

Comments & Conclusions

Comments & Conclusions

Symmetric tensors: _in the traceful case $\rightarrow$ reducible spectrum: spin s, s-2, . . .

_via a suitable field redefinition the Lagrangian *diagonalises*:

$$\varphi = \phi_s + O_{s-2}\,\phi_{s-2} + O_{s-4}\,\phi_{s-4} \cdots + O_{s-2k}\,\phi_{s-2k} + \cdots$$

s.t.

$$\mathcal{L} = \frac{1}{2}\,\varphi\,M\,\varphi = \frac{1}{2}\sum_{k=0}^{[\frac{s}{2}]} \hat{b}_{\,k,s,D}\,\phi_{s-2k}\,M\,\phi_{s-2k}$$

where $\phi'_{s-2k} \equiv 0$ ➡ *block-diagonal single-particle Lagrangians*

Comments & Conclusions

Symmetric tensors: _in the traceful case $\rightarrow$ reducible spectrum: spin s, s-2, . . .

_via a suitable field redefinition the Lagrangian *diagonalises*:

$$\varphi \,=\, \phi_s \,+\, O_{s-2}\,\phi_{s-2} \,+\, O_{s-4}\,\phi_{s-4} \cdots + O_{s-2k}\,\phi_{s-2k} \,+ \cdots$$

s.t.

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\,M\,\varphi \,=\, \frac{1}{2}\sum_{k=0}^{[\frac{s}{2}]}\hat{b}_{\,k,s,D}\,\phi_{s-2k}\,M\,\phi_{s-2k}$$

where $\phi'_{s-2k} \,\equiv\, 0$ ➞ *block-diagonal single-particle Lagrangians*

However, the redefinition is bound to be *non-local;* e.g.:

$$O_{s-2} \,=\, \eta \,-\, 2\,\frac{\partial^{\,2}}{\Box}\,\cdots$$

Comments & Conclusions

Symmetric tensors: _in the traceful case $\rightarrow$ reducible spectrum: spin s, s-2, . . .

_via a suitable field redefinition the Lagrangian *diagonalises*:

$$\varphi = \phi_s + O_{s-2}\,\phi_{s-2} + O_{s-4}\,\phi_{s-4} \cdots + O_{s-2k}\,\phi_{s-2k} + \cdots$$

s.t.

$$\mathcal{L} = \frac{1}{2}\,\varphi\, M\,\varphi = \frac{1}{2}\sum_{k=0}^{[\frac{s}{2}]} \hat{b}_{\,k,s,D}\,\phi_{s-2k}\, M\,\phi_{s-2k}$$

where $\phi'_{s-2k} \equiv 0$ ➙ *block-diagonal single-particle Lagrangians*

However, the redefinition is bound to be *non-local;* e.g.:

$$O_{s-2} = \eta - 2\,\frac{\partial^{\,2}}{\Box}\,\cdots$$

➽ What happens in vertices?

Comments & Conclusions

Constrained gauge parameter $\leftrightarrow$ partial gauge-fixing

Comments & Conclusions

Constrained gauge parameter $\leftrightarrow$ partial gauge-fixing

Fronsdal/Labastida: Minimal extensions

$$\mathcal{F} \quad \longrightarrow \quad \mathcal{A} = \mathcal{F} - 3\,\partial^{3}\,\alpha$$

Comments & Conclusions

Constrained gauge parameter $\leftrightarrow$ partial gauge-fixing

Fronsdal/Labastida: Minimal extensions

$$\mathcal{F} \quad \longrightarrow \quad \mathcal{A} = \mathcal{F} - 3\,\partial^{3}\,\alpha$$

Transverse-invariant, symmetric tensors: tensionless limit of free Open String Field theory action

$$\mathcal{L} = \frac{1}{2}\,\varphi\,M\,\varphi + 2\,\varphi\,\partial^{2}\,\mathcal{D} - 2\binom{s}{2}\,\mathcal{D}\hat{M}\,\mathcal{D}$$

Comments & Conclusions

Constrained gauge parameter $\leftrightarrow$ partial gauge-fixing

Fronsdal/Labastida: Minimal extensions

$$\mathcal{F} \quad \longrightarrow \quad \mathcal{A} = \mathcal{F} - 3\,\partial^{3}\,\alpha$$

Transverse-invariant, symmetric tensors: tensionless limit of free Open String Field theory action

$$\mathcal{L} = \frac{1}{2}\,\varphi\,M\,\varphi + 2\,\varphi\,\partial^{2}\,\mathcal{D} - 2\binom{s}{2}\,\mathcal{D}\hat{M}\,\mathcal{D}$$

unconstrained counterpart of mixed, AdS Lagrangian?

Closeby directions

➙ *Reducible description of mixed-symmetry fields on (A)dS*

➙ *Fermions*

➙ *Massive & partially massless reps.*

➙ *(cubic) vertices & their relation to triplet interactions*