Elastic scattering of hadrons

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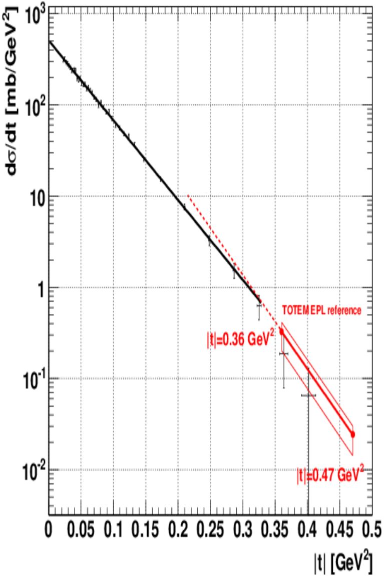
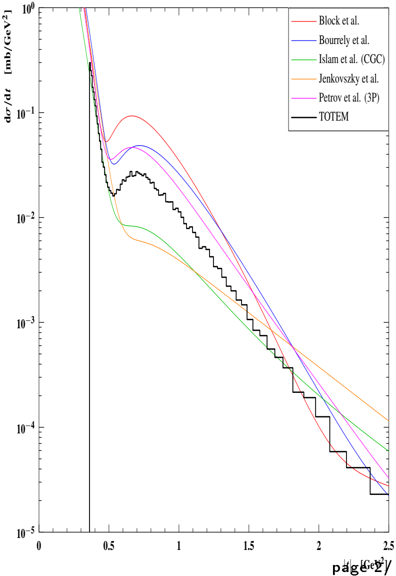
THREE REGIONS: the diffraction cone, the Orear regime, the hard parton scattering





FIVE characteristics: $\sigma_t(s)$, $\sigma_{el}(s)$, $\frac{d\sigma}{dt}(s,t)$, $\rho(s,t)$, $B(s,t)$

NOTE: s-dependence of σ_t, σ_{el} and (s,t)-dependence of $\frac{d\sigma}{dt}$, ρ, B.

$$\sigma_t(s) = \frac{\mathrm{Im}A(p,\theta=0)}{s}$$

$$\sigma_{el}(s) = \int_{t_{min}}^{0} dt \frac{d\sigma}{dt}(s,t)$$

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{16\pi s^2}|A|^2 = \frac{1}{16\pi s^2}(\mathrm{Im}A(s,t))^2(1+\rho^2(s,t))$$

$$\rho(p,\theta) = \frac{\mathrm{Re}A(p,\theta)}{\mathrm{Im}A(p,\theta)}$$

The diffraction cone $[s \approx 4p^2;\ t = -2p^2(1-\cos\theta) \approx -p^2\theta^2]$

$$\frac{d\sigma}{dt} / \left(\frac{d\sigma}{dt}\right)_{t=0} = e^{Bt} \approx e^{-Bp^2\theta^2}$$

The amplitude in the diffraction cone (Gaussian, imaginary)

$$A(p,\theta) \approx is\sigma_t e^{Bt/2} \approx 4ip^2\sigma_t e^{-Bp^2\theta^2/2} \qquad (\rho^2(s,0) < 0.02)$$

OUR GUESSES ABOUT ASYMPTOTICS

$$\sigma_t(s) \leq \frac{\pi}{2m_\pi^2} \ln^2(s/s_0)$$

THE BLACK DISK: $\sigma_t = 2\pi R^2$; $R = R_0 \ln s$; $\frac{\sigma_{el}}{\sigma_t} = \frac{\sigma_{in}}{\sigma_t} = \frac{1}{2}$

$B(s) = \frac{R^2}{4}$; $\rho_0 \equiv \rho(s, t=0) = \frac{\pi}{\ln s}$ None observed in experiment!

THE GRAY DISKS: two parameters - radius+opacity

Gray and Gaussian disks ($X = \sigma_{el}/\sigma_t$; $Z = 4\pi B/\sigma_t$; $\alpha \leq 1$)

Model	$1 - e^{-\Omega}$	σ_t	B	X	Z	XZ	X/Z
Gray	$\alpha\theta(R-b)$	$2\pi\alpha R^2$	$R^2/4$	$\alpha/2$	$1/2\alpha$	$1/4$	α^2
Gauss	$\alpha e^{-b^2/R^2}$	$2\pi\alpha R^2$	$R^2/2$	$\alpha/4$	$1/\alpha$	$1/4$	$\alpha^2/4$

The energy behavior

$\sqrt{s}$, GeV	2.70	4.74	6.27	7.62	13.8	62.5	546	1800	7000
X	0.42	0.27	0.24	0.22	0.18	0.18	0.21	0.23	0.25
Z	0.64	1.09	1.26	1.34	1.45	1.50	1.20	1.08	1.00
XZ	0.27	0.29	0.30	0.30	0.26	0.25	0.26	0.25	0.25
X/Z	0.66	0.25	0.21	0.17	0.16	0.12	0.18	0.21	0.25

THEORETICAL APPROACHES

1. Geometrical picture and eikonal

The impact parameter (**b**) representation

$$A(s,t=-q^2)=\frac{2s}{i}\int d^2be^{i\mathbf{qb}}(e^{2i\delta(s,\mathbf{b})}-1)=2is\int d^2be^{i\mathbf{qb}}(1-e^{-\Omega(s,\mathbf{b})})$$

Two or three regions of the internal hadron structure.
Heisenberg relation: large b (external regions) - small $|t|$, small b (internal regions) - large $|t|$. 15-25 parameters!
E.g., the diffraction profile ("Fermi") function is

$$\Gamma(s,b)=1-\Omega(s,b)=g(s)\left[\frac{1}{1+e^{(b-r)/a}}+\frac{1}{1+e^{(-b+r)/a}}-1\right]$$

and special shapes for internal regions. UNITARIZATION! but...
Geometrical scaling.

2. Electromagnetic analogies

The droplet model and electromagnetic form factors:

$$F(t)\propto G^2(t)(a^2+t)/(a^2-t);\quad G(t)=(1-t/m_1^2)^{-1}(1-t/m_2^2)^{-1}.$$

3. Reggeon exchanges

$$\Omega(s,\mathbf{b}) = S(s)F(\mathbf{b}^2) + (\mathrm{non-leading\ \ terms})$$

$S(s)$ is crossing symmetric and reproduces Pomeron trajectory

$$S(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}$$

$F(\mathbf{b}^2)$ is the Bessel transform of Pomeron and Reggeon vertices $F(t)$ with electromagnetic or exponential form factors.

$$A_P(s,t) = i\frac{a_P s}{b_P s_0}[r_1^2(s)e^{r_1^2(s)(\alpha_P-1)} - \epsilon_P r_2^2(s)e^{r_2^2(s)(\alpha_P-1)}], \qquad (1)$$

where $r_1^2(s) = b_P + L - i\pi/2,\ r_2^2(s) = L - i\pi/2,\ L = \ln(s/s_0)$.

$$A_R(s,t) = a_R e^{-i\pi\alpha_R(t)/2} e^{b_R t}(s/s_0)^{\alpha_R(t)} \qquad (2)$$

with $\alpha_P(t) = \alpha_0 - \gamma \ln(1 + \beta\sqrt{t_0 - t})$ - non-linear;
$\alpha_R(t) = a_R + b_R t$ - linear trajectories.

4. QCD-inspired approaches

Gluons and quarks as active partons. Similar form factors.
Most approaches are rather successful in fits of the diffraction cone slope $B(s)$, $\sigma_t(s)$, $\sigma_{el}(s)$ in a wide interval of energies.

All models fail! **OCCAM RAZOR!**

The unitarity condition

$$\mathrm{Im}A(p,\theta) = I_2(p,\theta) + F(p,\theta) =$$
$$\frac{1}{32\pi^2}\int\int d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 A(p,\theta_1)A^*(p,\theta_2)}{\sqrt{[\cos\theta - \cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2)-\cos\theta]}} + F(p,\theta$$

The region of integration

$$|\theta_1 - \theta_2| \le \theta, \qquad \theta \le \theta_1 + \theta_2 \le 2\pi - \theta$$

I_2 – two-particle intermediate states (σ_{el}), F – inelastic ones (overlap function $\to \sigma_{inel}$). For angles θ outside the diffraction cone one amplitude in I_2 is at small angles and another at large ones. Thus, the **linear** integral equation outside the diffraction cone

$$\mathrm{Im}A(p,\theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi B}}\int_{-\infty}^{+\infty} d\theta_1 f_\rho e^{-Bp^2(\theta-\theta_1)^2/2}\mathrm{Im}A(p,\theta_1) + F(p,\theta).$$

$f_\rho = 1 + \rho_0\rho(\theta_1)$.

Analytical solution if $F(p,\theta) \ll \mathrm{Im}A(p,\theta)$ and $f_\rho \approx$const outside the diffraction cone!

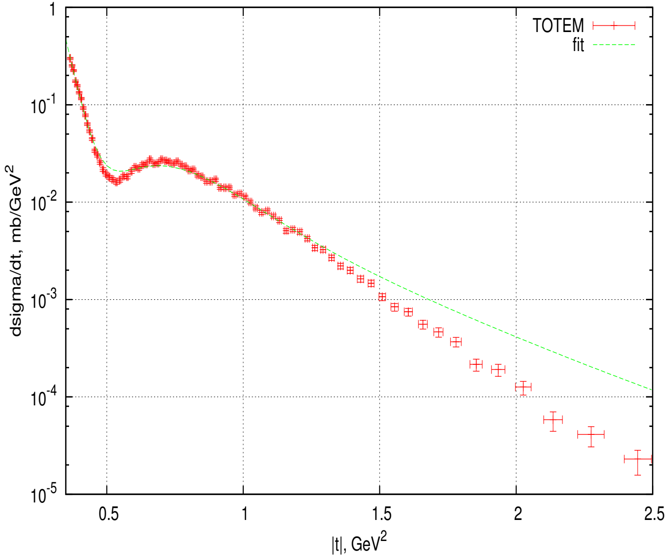
The elastic differential cross section **outside** the diffraction cone contains the exponentially decreasing with θ (or $\sqrt{|t|}$) term (Orear regime!) with imposed on it damped oscillations:

$$\ln\left(\frac{d\sigma}{Cdt}\right) \approx -2\sqrt{2B|t|\ln(Z/f_\rho)} + D\exp[-\sqrt{2\pi B|t|}]\cos(\sqrt{2\pi B|t|} - \phi)$$

The experimentally measured diffraction cone slope B and total cross section σ_t determine mainly the shape of the differential cross section in the Orear region of transition from the diffraction peak to large angle parton scattering. The value of $Z = 4\pi B/\sigma_t$ is so close to 1 at 7 TeV that the fit is very sensitive to f_ρ. Thus, it becomes possible for the first time to estimate the ratio ρ outside the diffraction cone from fits of experimental data. NEW!
At the LHC, its average value is negative and equal to -2!
i.e., $f_\rho = 1 + 0.14\bar{\rho} \approx 0.72$ to fit the slope in Orear region!

Do we approach the black disk limit $Z \to 0.5$?
To fit Orear slope, the decrease of Z must be compensated by the decrease of $f_\rho = 1 + \rho_0\rho$ but $\rho_0 \propto \ln^{-1} s$ asymptotically! Is it possible that ρ in Orear region increases in modulus being negative?

Fit at 7 TeV (dip+Orear in $0.3 < |t| < 1.5$ GeV^2)



In the Orear region, the overlap function $F(p, \theta)$ was neglected and $f_\rho = 1 + \rho_0 \rho(t)$ was approximated by a constant!

The proof of the assumption about the small overlap function

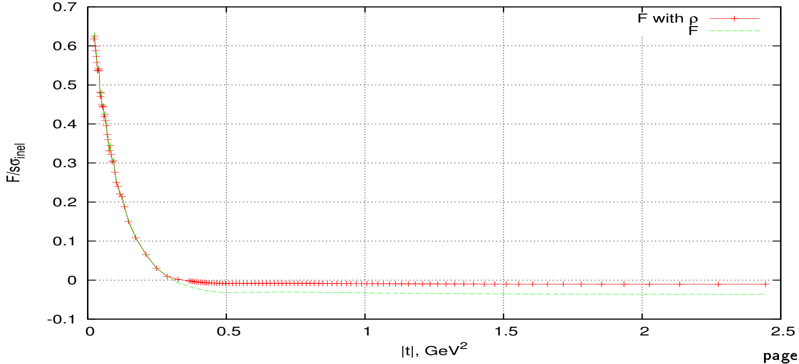
$F(p,\theta)$ computed from experimental data is negligible outside cone:

$$F(p,\theta) = \; 16p^2\left(\pi\frac{d\sigma}{dt}/(1+\rho^2)\right)^{1/2} -$$

$$\frac{8p^4 f_\rho}{\pi}\int_{-1}^{1} dz_2 \int_{z_1^-}^{z_1^+} dz_1 \left[\frac{d\sigma}{dt_1}\cdot\frac{d\sigma}{dt_2}\right]^{1/2} K^{-1/2}(z,z_1,z_2),$$

$z_i = \cos\theta_i;\quad K(z,z_1,z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2,$

$z_1^{\pm} = zz_2 \pm [(1-z^2)(1-z_2^2)]^{1/2}$



The real part outside the diffraction cone

At $t = 0$, it is known from Coulomb-nuclear interference experimentally (at lower than LHC energies) and from dispersion relations theoretically. ρ_0 at LHC may be about 0.13 - 0.14.

No experimental results for $\rho(t)$ are available.

Our estimate from the fit at 7 TeV is the first attempt with f_ρ =const, i.e., with $\rho(t)$ replaced by some average value.

However, $\rho(t)$ can be calculated if the imaginary part is known:

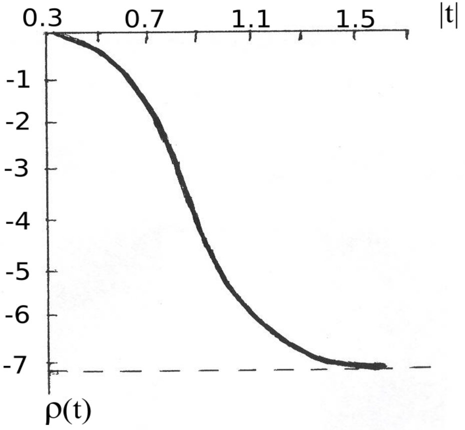
$$\rho(t) = \rho_0 \left[1 + \frac{t(d\mathrm{Im}A(t)/dt)}{\mathrm{Im}A(t)} \right]$$

Then the equation for $\rho(t)$ follows from the unitarity condition

$$\frac{dv}{dx} = -\frac{v}{x} - \frac{2}{x^2} \left(\frac{Ze^{-v^2} - 1}{\rho_0^2} - 1 \right)$$

$x = \sqrt{2B|t|}$, $v = \sqrt{\ln(Z/f_\rho)}$,
$\rho(t) = (Ze^{-v^2} - 1)/\rho_0$, where v is the solution of the equation.

The behavior of $\rho(t)$ in the Orear region



Asymptotics at $|t| \to \infty$ $\rho \to -1/\rho_0$.

Then $f_\rho \to 0$ and $\ln(Z/f_\rho) \to \infty$!

The slope **steepens** with $|t|$ - see Fig. with the fit

Prediction: some changes are expected in this region of $|t|$!

The black disk limit requires $f_\rho < 0.5$, if some slope survives asymptotically in the Orear region, and then $\bar{\rho}(t) < -\frac{\ln s}{2\pi}$.

Experimentally observed $|t|^{-8}$-regime in pp-scattering.

The dimensional counting

$d\sigma/dt|_{AB\to CD} \propto s^{-n+2}f(t/s)$ at large s and t and fixed ratio s/t, n is the total number of fields in A, B, C, D which carry a finite fraction of the momentum. Assuming quark constituents, the $s\to\infty$, fixed-t/s prediction for pp-scattering is $d\sigma/dt \propto s^{-10}$.
For n partons participating in a single hard scattering

$$A_1(s,t) \propto \left(\frac{s_0}{s}\right)^{\frac{n}{2}-2} f_1(s/t)$$

There exists the formula for m hard scatterings.

The coherent scattering

1. Coherent exchange by three gluons between three pairs of quarks
The propagators of three gluons and their couplings give rise to $\alpha_S^6|t|^{-6}$-dependence and two powers in the denominator are added by kinematical factors.
2. Multi-Pomeron exchange with one large-p_T Pomeron.

Conclusions

- Models describe the diffraction peak but fail outside it.
- At intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations.
- The experimental data on elastic *pp* differential cross section at low and high ($\sqrt{s}$=7 TeV) energies have been fitted in this region with well described position of the dip and Orear slope.
- The fit allows for the first time at 7 TeV to estimate the ratio of real to imaginary parts of the elastic scattering amplitude ρ far from forward direction t=0. It happened to be about -2.
- This value of ρ is explained by the unitarity condition.
- The overlap function is small and negative in the Orear region. That confirms the assumption used in solving the unitarity equation. Important corollary: the phases of inelastic amplitudes are crucial in any model of inelastic processes.