Some new results about massive gravity

(and the « Vainshtein mechanism »)

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(APC, CNRS Paris)

Astroparticules
et Cosmologie

1. Introduction

C.D., T. Jacobson 2012
Babichev, C.D., Ziour, 2009, 2010

2. « Non linear massive gravity » and the « Vainshtein mechanism »

Blas, C.D., Garriga, 2005
C.D., Dvali, Gabadadze, Vainshtein 2002

3. Generic properties of bimetric space-times and some applications

Ginzburg Conference on Physics
Moscow, May 29th 2012

1.1. Introduction: Why « massive gravity » ?

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dark energy ?

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Dark matter
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Dark matter
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➡ One obviously needs a very light graviton
(of Compton length of order of the size of the Universe)

1.2. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

Pauli-Fierz action: second order action for a massive spin two

$$\int d^4x \underbrace{\sqrt{g} R_g} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta} \right)$$

second order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$

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Only Ghost-free (quadratic) action for a
massive spin two   Pauli, Fierz 1939

(NB: $h_{\mu\nu}$ is TT: **5 degrees of freedom)**

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vDVZ discontinuity
(van Dam, Veltman;
Zakharov; Iwasaki 1970)

The propagators read

propagator for $m=0$ $$D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{\boxed{2}p^2} + \mathcal{O}(p)$$

propagator for $m\neq0$ $$D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{\boxed{3}(p^2-m^2)} + \mathcal{O}(p)$$

2. Non linear Pauli-Fierz theory and the « Vainshtein Mechanism »

Can be defined by an action of the form

Isham, Salam, Strathdee, 1971

$$S = \int d^4x \sqrt{-g} \ \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f, g],$$

Einstein-Hilbert action
for the *g* metric

Matter action
(coupled to metric *g*)

Interaction term coupling
the metric *g* and the non
dynamical metric *f*

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The interaction term $S_{int}[f,g],$ is chosen such that

- It is invariant under diffeomorphisms
- It has flat space-time as a vacuum
- When expanded around a flat metric ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $f_{\mu\nu} = \eta_{\mu\nu}$)
  It gives the Pauli-Fierz mass term

Leads to the e.o.m. $M_P^2 G_{\mu\nu} = \left(T_{\mu\nu} + T^g_{\mu\nu}(f,g)\right)$

Matter energy-momentum tensor

Effective energy-momentum tensor (*f,g* dependent)

- Some working examples

$$S^{(2)}_{int} = -\frac{1}{8}m^2 M_P^2 \int d^4x \sqrt{-f}\, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau}\right)$$

(Boulware Deser)

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(Arkani-Hamed, Georgi, Schwartz)

with $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

- Infinite number of models with similar properties
- Have been investigated in different contexts
  - « f-g, strong, gravity » Isham, Salam, Strathdee 1971
  - « bigravity » Damour, Kogan 2003
  - « Higgs for gravity » t’Hooft 2007, Chamseddine, Mukhanov 2010

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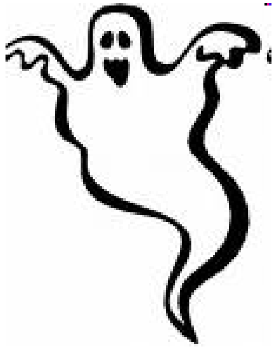
Generically: a 6th ghost-like degree of freedom propagates (Boulware-Deser 1972)



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  - de Rham, Gabadadze,Tolley 2010, 2011

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Look for static spherically symmetric solutions with the ansatz (not the most general one)

$$\left\{\begin{array}{rcl} g_{AB}dx^A dx^B & = & -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB}dx^A dx^B & = & -dt^2 + dr^2 + r^2 d\Omega^2 \end{array}\right.$$

Gauge transformation

$$\left\{\begin{array}{rcl} g_{\mu\nu}dx^\mu dx^\nu & = & -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \\ f_{\mu\nu}dx^\mu dx^\nu & = & -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 \end{array}\right.$$

Which can easily be compared to Schwarzschild

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Then look for an expansion in
G_N (or in $R_S \propto G_N M$) of the would-be solution

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

(For $R \ll m^{-1}$)

$$\nu(R) = -\frac{R_S}{R}(1 \quad + \ldots$$

$$\lambda(R) = +\frac{1}{2} \quad \frac{R_S}{R}(1 \quad + \ldots$$

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Wrong light bending!
(vDVZ discontinuity)

This coefficient equals +1
in Schwarzschild solution

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$$\nu(R) = -\frac{R_S}{R}(1 + \mathcal{O}(1)\epsilon + \ldots$$

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with $\epsilon = \frac{R_S}{m^4 R^5}$

Vainshtein 1972
In « some kind »
[Damour et al. 2003]
of non linear PF

Introduces a new length scale R_v in the problem below which the perturbation theory diverges!

For the sun: bigger than solar system!

with $R_v = (R_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein 1972

There exists an other perturbative expansion at smaller distances, defined around (ordinary) Schwarzschild and reading:

$$\left.\begin{aligned} \nu(R) &= -\tfrac{R_S}{R}\left\{1+\mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\} \\ \lambda(R) &= +\tfrac{R_S}{R}\left\{1+\mathcal{O}\left(R^{5/2}/R_v^{5/2}\right)\right\} \end{aligned}\right\} \quad \text{with} \quad R_v^{-5/2} = m^2 R_S^{-1/2}$$

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- This goes smoothly toward Schwarzschild as *m* goes to zero
- This leads to corrections to Schwarzschild which are non analytic in the Newton constant

To summarize: 2 regimes

$$\nu(R) = -\frac{R_S}{R}\left(1+\mathcal{O}(1)\epsilon+\cdots\right) \quad \text{with} \quad \epsilon = \frac{R_S}{m^4R^5}$$

Valid for $R \gg R_V$ with $R_V = \left(R_S m^{-4}\right)^{1/5}$

Standard perturbation theory around flat space

Crucial question: can one join the two regimes in a single existing non singular (asymptotically flat) solution? **(Boulware Deser 72)**

Expansion around Schwarzschild solution

$$\nu(R) = -\frac{R_S}{R}\left(1+\mathcal{O}\left(R^{5/2}/R_V^{5/2}\right)\right)$$

Valid for $R \ll R_V$

This was investigated (by numerical integration) by Damour, Kogan and Papazoglou 2003

No non-singular solution found matching the two behaviours (always singularities appearing at finite radius) and hence failure of the « Vainshtein mechanism »

(see also Jun, Kang 1986)

We (Babichev, C.D., Ziour) reinvestigated this issue using more sophisticated methods and found solutions featuring the Vainshtein recovery

(with the Arkani-Hamed, Georgi, Schwartz potential and a source)

To obtain our solutions, we used the « Decoupling Limit », and various (asymptotic) expansions, and we first…

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« shooted »

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Then « relaxed »

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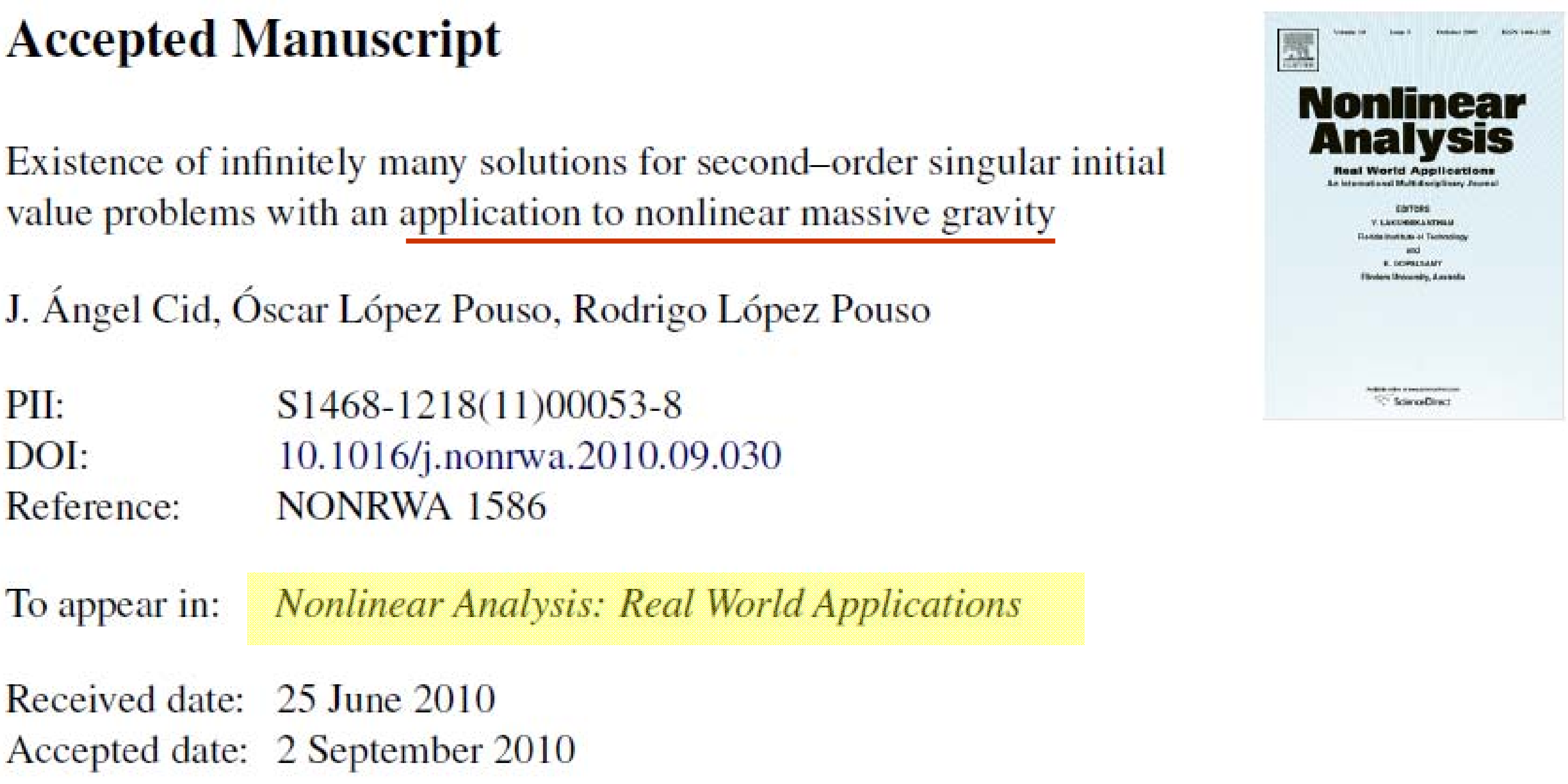
One crucial issue: existence of infinitely many solutions at infinity (in the decoupling limit: we have two different mathematical proofs of that)



Accepted Manuscript

Existence of infinitely many solutions for second–order singular initial value problems with an application to nonlinear massive gravity

J. Ángel Cid, Óscar López Pouso, Rodrigo López Pouso

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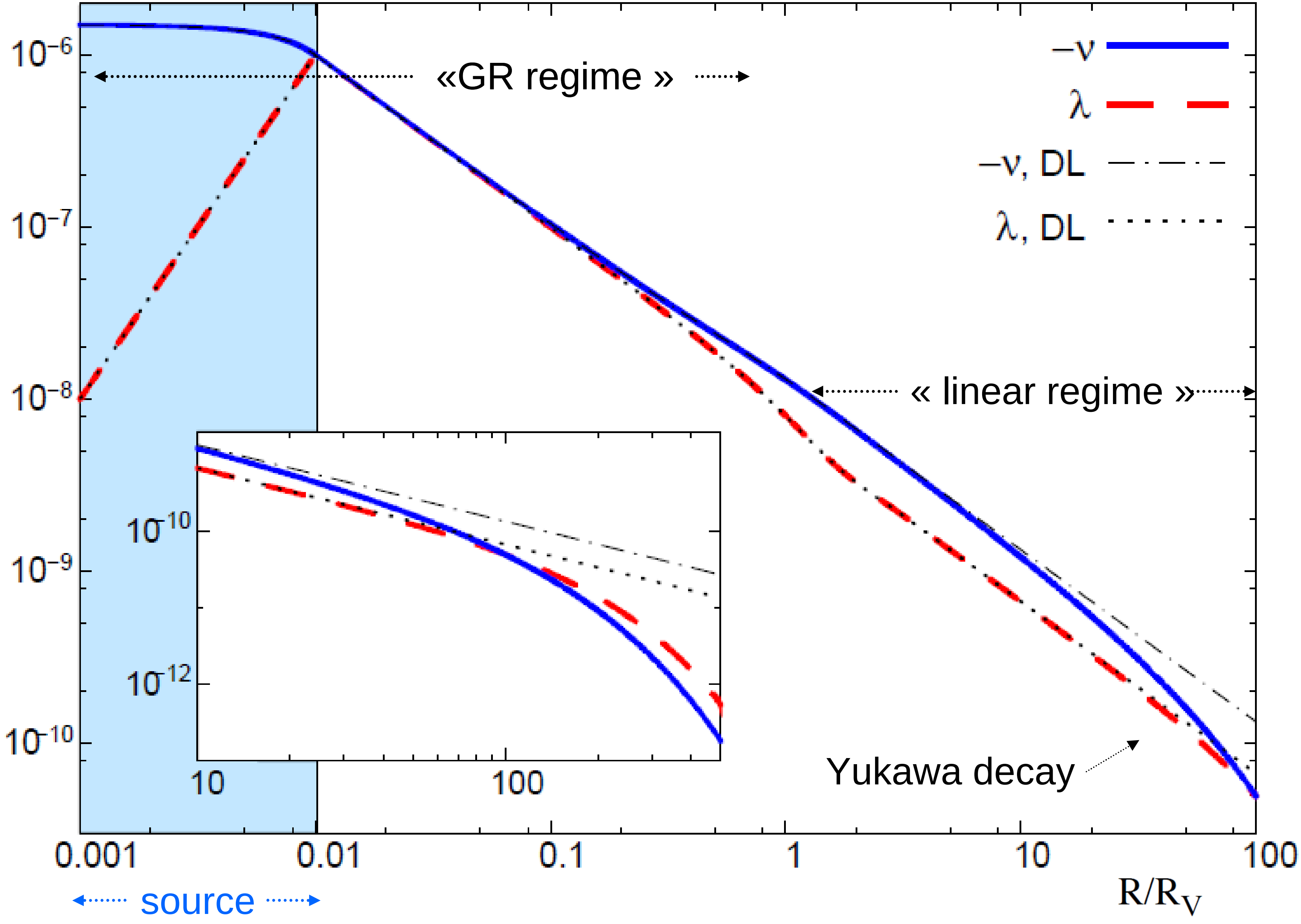
DOI: 10.1016/j.nonrwa.2010.09.030
Reference: NONRWA 1586

To appear in: *Nonlinear Analysis: Real World Applications*

Received date: 25 June 2010
Accepted date: 2 September 2010

Numerical solutions (of the full non linear system)



«GR regime »

« linear regime »

Yukawa decay

source

So the Vainshtein’s mechanism does really work even in sick theories (NB: our numerical results were confirmed by M. Volkov) !

Solutions were obtained for very low density objects. We did not find numerically what is happening for dense objects (and BHs).

3. Generic properties of horizon structure (and some consequences)

3.1 Formal results

C.D.,T.Jacobson, CQG 2012

Consider a theory with two metrics, $g_{\mu\nu}$ and $f_{\mu\nu}$

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Proposition 1: Suppose the Killing vector ∂_t is null at $r = r_H$ with respect to $g_{\mu\nu}$. Then if both metrics are diagonal and describe smooth geometries at r_H, ∂_t must also be null with respect to $f_{\mu\nu}$ at $r = r_H$.

i.e. both metric must have the same horizon

First proof (1a)

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- When both metrics are static and spherically symmetric, they can be put in the form (in a common coordinate system)

$$\begin{aligned} f_{\mu\nu}dx^\mu dx^\nu &= -J(r)dt^2 + K(r)dr^2 + r^2 d\Omega^2 \\ g_{\mu\nu}dx^\mu dx^\nu &= -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + D(r)d\Omega^2 \end{aligned}$$

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It must be regular at the horizon $r=r_H$ if both metrics are regular there

But $A(r_H)=0$, and J/A, K/C and r^2/D have the same sign, so cannot cancel

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➡ One must have $J(r_H) = 0$
(and hence the killing horizon of g is also one for f)

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- If a space-time has a Killing horizon, then, under rather general assumptions, it has a « virtual » bifurcation surface.

- More precisely:

if a space-time is static (with « t » reflection symmetry) or stationary axisymmetric with « t-ϕ » reflection symmetry, and if the surface gravity of the horizon is non zero and constant

then

There is an extension of a neighborhood of the horizon to one with a bifurcate Killing horizon

(i.e. a Killing horizon which contains a bifurcation surface)

(NB: this applies to any space-time without assuming anything concerning the field equations)

Moreover (Racz-Wald 1996)

Any Killing invariant tensor field sharing the t or the t-ϕ reflection symmetry of the metric

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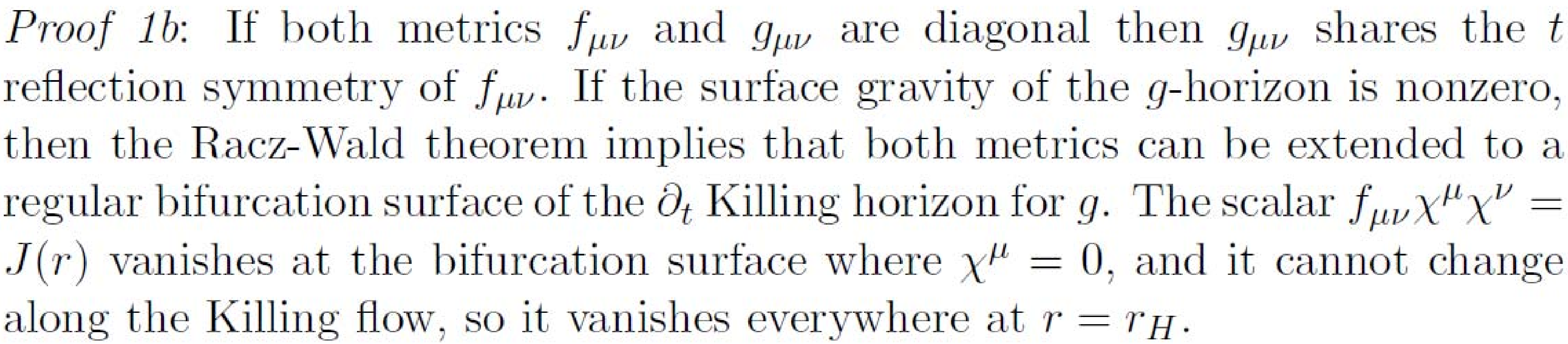

Proof 1b: If both metrics $f_{\mu\nu}$ and $g_{\mu\nu}$ are diagonal then $g_{\mu\nu}$ shares the t reflection symmetry of $f_{\mu\nu}$. If the surface gravity of the g-horizon is nonzero, then the Racz-Wald theorem implies that both metrics can be extended to a regular bifurcation surface of the ∂_t Killing horizon for g. The scalar $f_{\mu\nu}\chi^\mu\chi^\nu = J(r)$ vanishes at the bifurcation surface where $\chi^\mu = 0$, and it cannot change along the Killing flow, so it vanishes everywhere at $r = r_H$.

(where χ is the killing vector)

NB: This extends to the stationary-axisymmetric case

This does not preclude the existence of two geometries one with a Killing horizon and one without….

But only implies that the non-horizon geometry cannot possess the *t*-ϕ (or t in the previous case) reflection symmetry

E.g.: the existence of a non zero B in the g metric can allow both geometries to be regular at the horizon.

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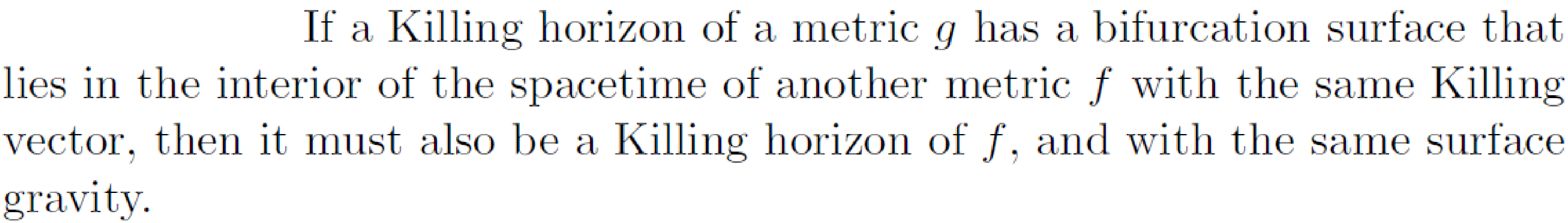
When this is the case (i.e. when the Killing horizon is not a Killing horizon for the other metric)

The bifurcation surface of the g spacetime cannot lie in the interior of the f space-time

Conversely, when the horizon coincide, they must have the same surface gravity (see. e.g. M. Volkov arXiv:1202.6682)

This can be put together as



If a Killing horizon of a metric g has a bifurcation surface that lies in the interior of the spacetime of another metric f with the same Killing vector, then it must also be a Killing horizon of f, and with the same surface gravity.

3.2 Some Consequences

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Indeed, in the standard way of looking at Vainshtein mechanism of « massive gravity » one has two (commonly) diagonal metric

« Massive metric »

Flat metric

$$\begin{aligned} g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \end{aligned}$$

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In any theory where the Vainshtein mechanism is working for recovering a solution close to the Schwarschild Black Hole, the *g* metric must have a (spherical) Killing horizon at $r=r_H$ … this must also be a killing horizon for *f*

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In any theory where the Vainshtein mechanism is working for recovering a solution close to the Schwarschild Black Hole, the *g* metric must have a (spherical) Killing horizon at $r=r_H$ … this must also be a killing horizon for *f*

Impossible:
Minkowski ST has no spherical Killing horizons (but only planar)

3.2 Some Consequences

3.2.1. (Standard) Vainshtein mechanism does not work for black holes

Indeed, in the standard way of looking at Vainshtein mechanism of « massive gravity » one has two (commonly) diagonal metric

« Massive metric »

Flat metric

$$\begin{aligned} g_{AB} dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB} dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \end{aligned}$$

In any theory where the Vainshtein mechanism is working for recovering a solution close to the Schwarschild Black Hole, the *g* metric must have a (spherical) Killing horizon at $r=r_H$ … this must also be a killing horizon for *f*

Impossible:
Minkowski ST has no spherical Killing horizons (but only planar)

NB: this applies also to the new massive gravity of
de Rham, Gabadadze, Tolley (and in particular to
solutions of Nieuwenhuizen; Gruzinov, Mirbabayi)

3.2.2. Causal structure of « type I » static spherically symmetric solutions of non linear massive gravity

$$
\begin{aligned}
f_{\mu\nu}dx^\mu dx^\nu &= -J(r)dt^2 + K(r)dr^2 + r^2 d\Omega^2 \\
g_{\mu\nu}dx^\mu dx^\nu &= -A(r)dt^2 + 2B(r)dtdr + C(r)dr^2 + D(r)d\Omega^2
\end{aligned}
$$

« Type I » solutions: those with $B \neq 0$

Salam, Strathdee 1977
Isham, Storey 1978

(as opposed to « type II » solutions, with B = 0, such as the ones discussed so far when addressing the Vainshtein mechanism - (cf. « λ, μ, ν ansatz ») previous part of this talk)

Some Type I solutions are known analytically and simple

(Salam, Strathdee 1977, Isham, Storey, 1978;
see also Berezhiani, Comelli, Nesti, Pilo, 2008)

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

$$f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3\beta}(1-p)dt^2 - 2Ddtdr - Adr^2 - 2/3r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

With
$$\begin{cases} A = \frac{2}{3\beta}(1-q)^{-2}\left(p + \boxed{\beta} - q - \beta q\right) \\ D^2 = \left(\frac{2}{3\beta}\right)^2(1-q)^{-2}(p-q)(p+\beta-1-\beta q) \end{cases}$$

Integration constant

and
$$\begin{cases} p = \frac{2M_f}{r} + \frac{2\Lambda_f}{9}r^2 \\ q = \frac{2M_g}{r} + \frac{\Lambda_g}{3}r^2 \end{cases}$$

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Integration constant

and
$$\begin{cases} p = \frac{2M_f}{r} + \frac{2\Lambda_f}{9}r^2 \\ q = \frac{2M_g}{r} + \frac{\Lambda_g}{3}r^2 \end{cases}$$

Both metric are of Schwarzschild-(A)dS form (no sign of vDVZ or massive gravity!)

Namely, the change of variable $d\tilde{t} = \frac{1}{\sqrt{\beta}}\left\{dt \mp \frac{\sqrt{(p-q)(p+\beta-1-\beta q)}}{(1-q)(1-p)}dr\right\}$

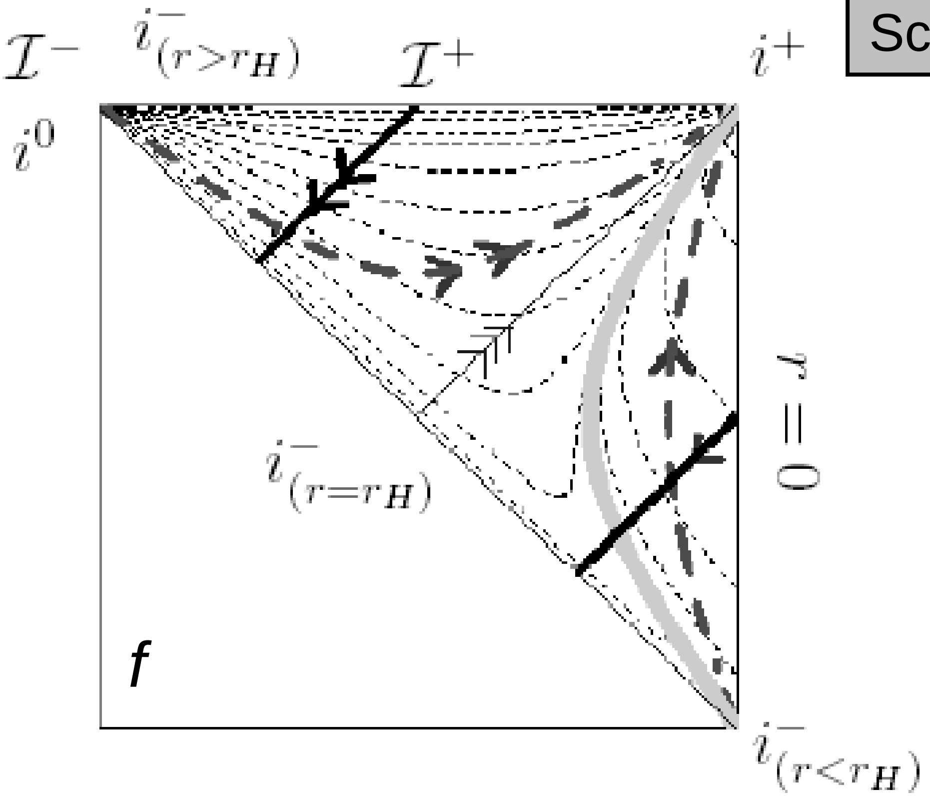
Put the metric $f_{\mu\nu}$ in the usual static form of S(A)dS:

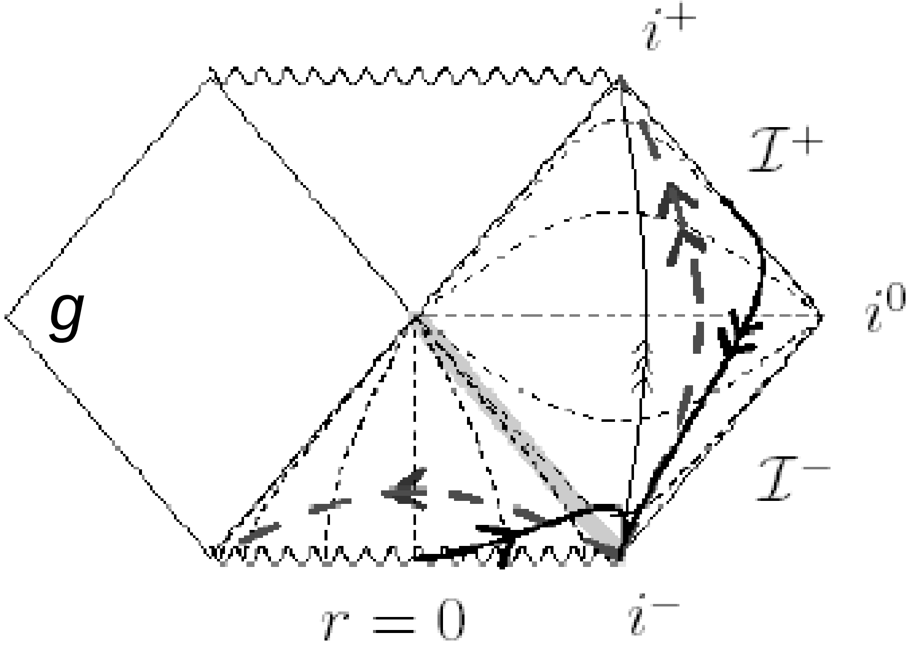
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = \frac{2}{3}\left\{(1-p)d\tilde{t}^2 - (1-p)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right\}$$

Causal Structure

Blas, C.D., Garriga 2005

E.g. de Sitter (r_H) with Schwarzschild (r_s) with $r_s < r_H$





Causal Structure

Blas, C.D., Garriga 2005

E.g. de Sitter (r_H) with
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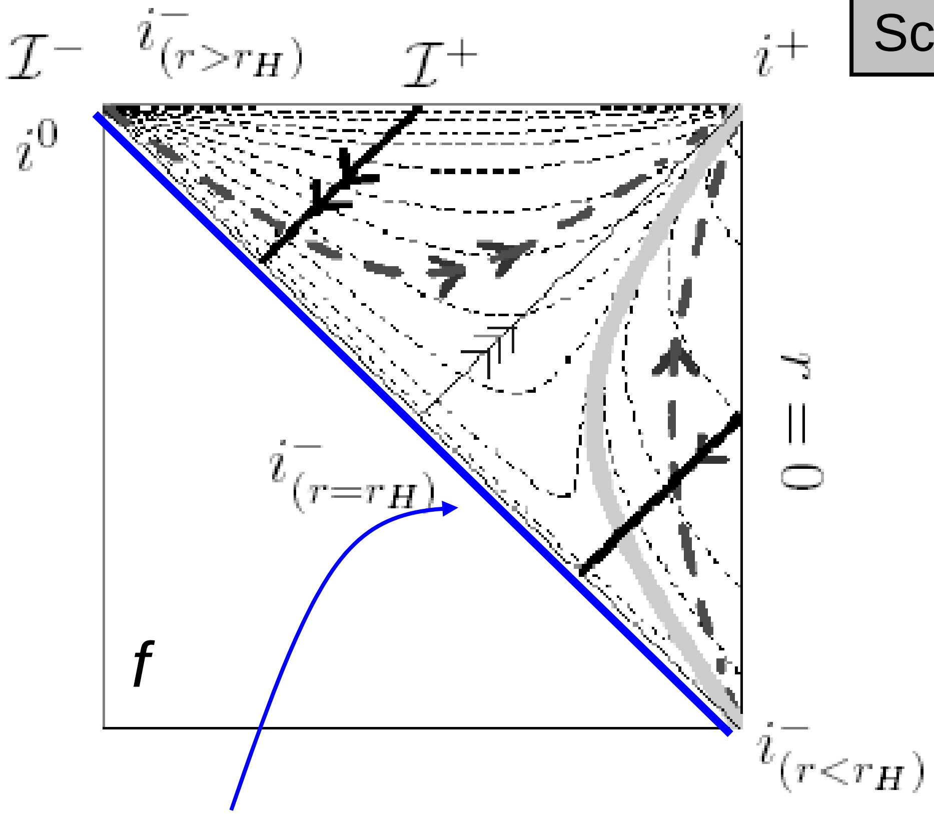
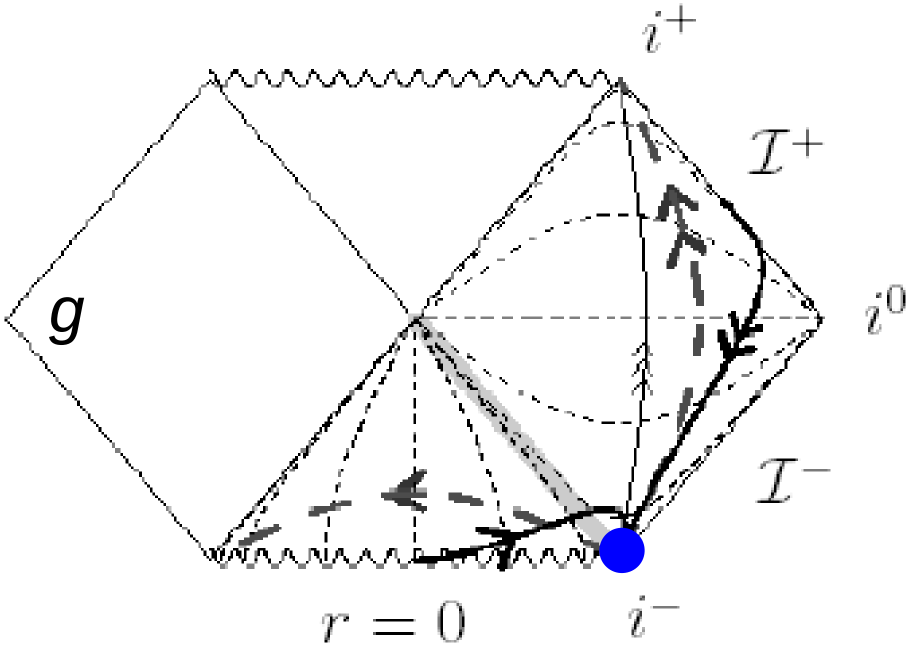


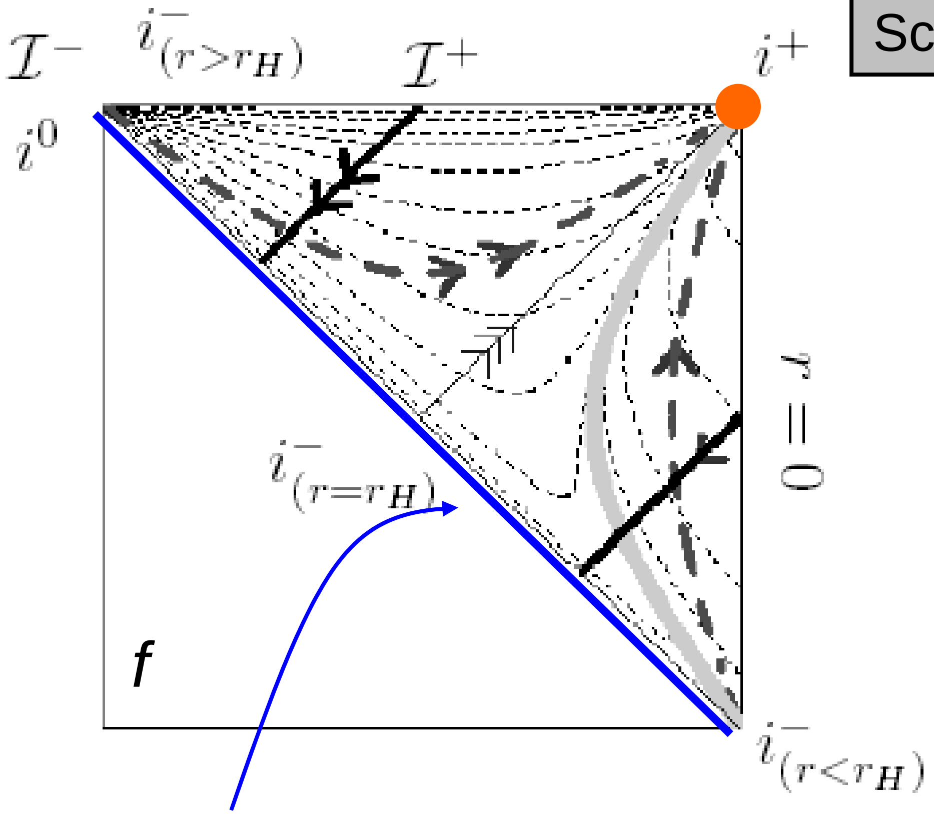


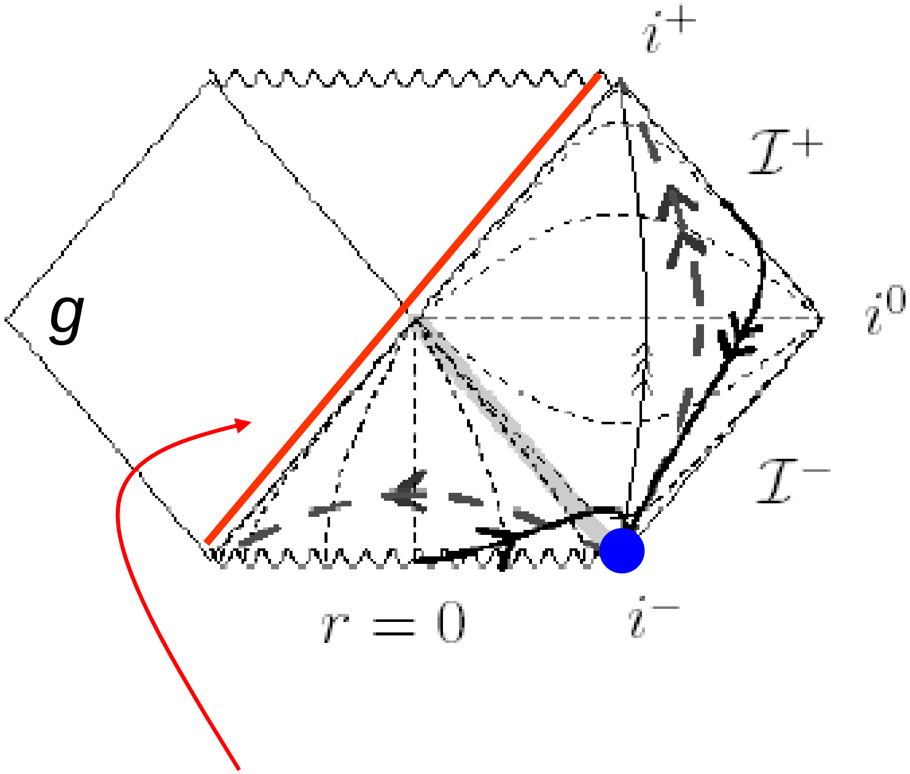
Part of the dS horizon
mapped into the past
timelike infinity of $r=r_H$
2-sphere of Schwarzschild

Causal Structure

Blas, C.D., Garriga 2005

E.g. de Sitter (r_H) with Schwarzschild (r_s) with $r_s < r_H$





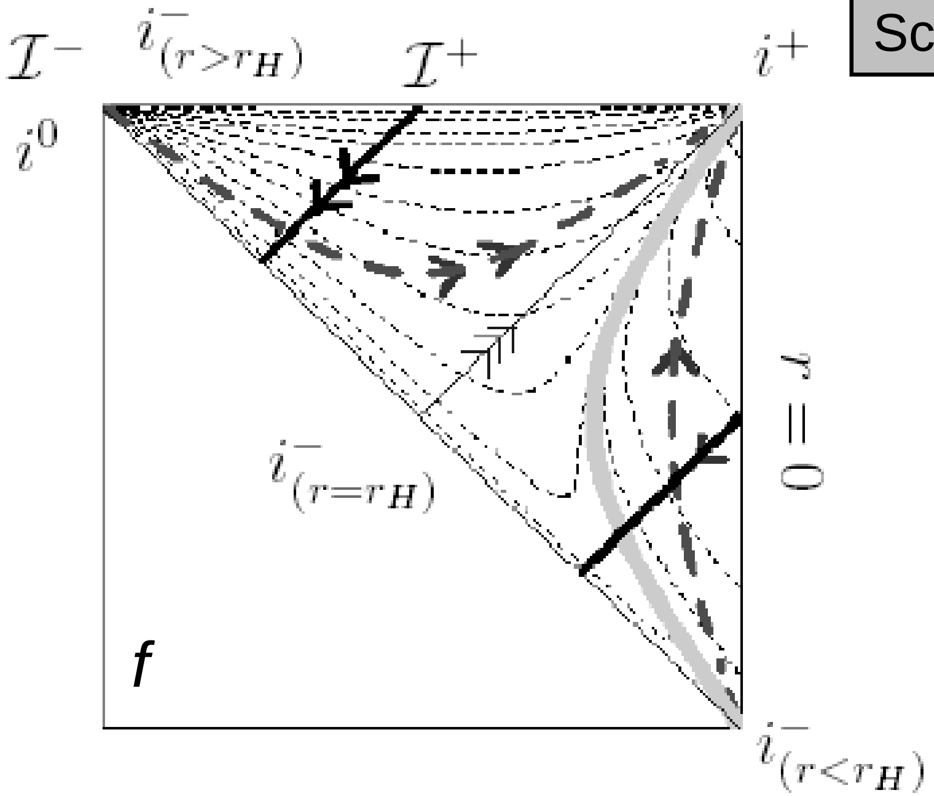
Part of the dS horizon mapped into the past timelike infinity of $r=r_H$ 2-sphere of Schwarzschild

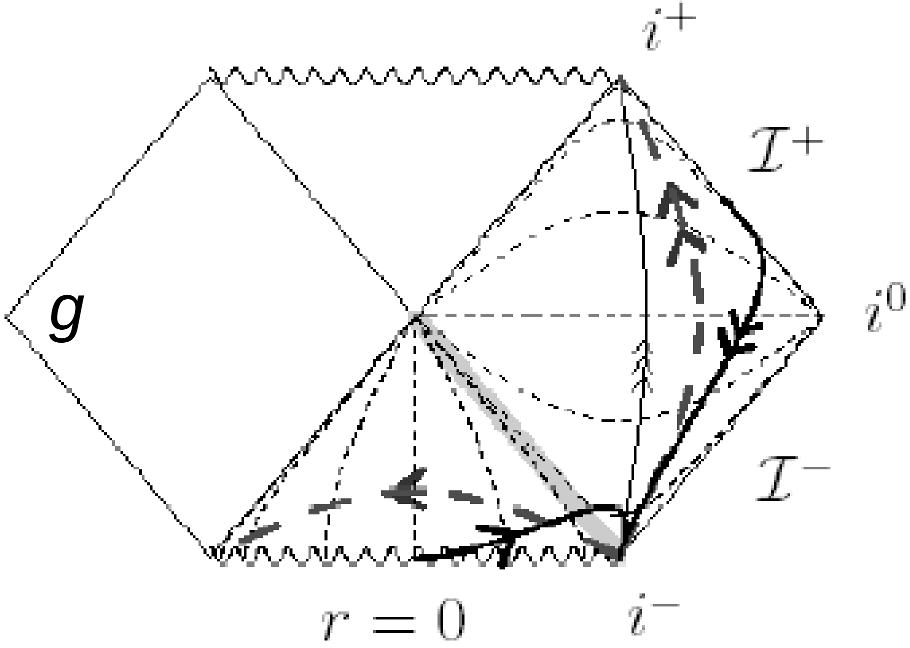
Part of the Schwarzshild horizon mapped into the future timelike infinity of $r=r_s$ 2-sphere of de Sitter

Causal Structure

Blas, C.D., Garriga 2005

E.g. de Sitter (r_H) with Schwarzschild (r_s) with $r_s < r_H$





Bifurcation sphere of one space-time does not lie in the interior of the other …

Conclusions (of the second part)

There exist interesting global constraints on putting together two metrics on a same manifold

- One simple consequence: failure of the usual Vainshtein mechanism to recover Black holes (but there exist non diagonal solutions crossing the horizon)
- Consequence for superluminal issues ?
- One simple question: What is the ending point of spherical collapse ?

Close to the horizon, the situation (with a working Vainshtein mechanism) would be similar to the following simple example:

1. Consider 4D schwarzschild ST in static coordinates

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

2. On this space-time (for $r>r_H=2M$) define a new metric as

$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

⇒ There is no extension of this construction in a neighborhood of the horizon where both metric are non singular

⇒ (or) any « bi-diagonal Vainshtein recovery » of Schwarschild must stop at the horizon (or before)

3.2.2. A contrasting example is

1. Consider 4D schwarzschild ST in Eddington-Finkelstein coordinates

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(1-\frac{2M}{r}\right)dv^2 + 2dvdr + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

2. On this space-time define a new metric as

$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -dv^2 + 2dvdr + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

This metric is flat, and extend beyond the future horizon of the Schwarzschild ST

In coordinate system where the Schwarzschild metric takes the usual diagonal form, the *f* metric is not diagonal

3.2.2. A contrasting example is

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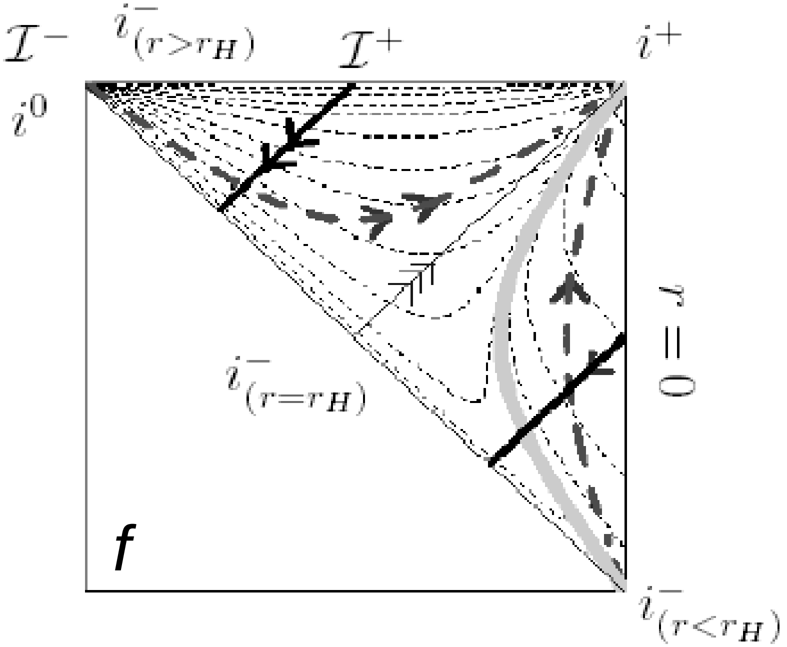
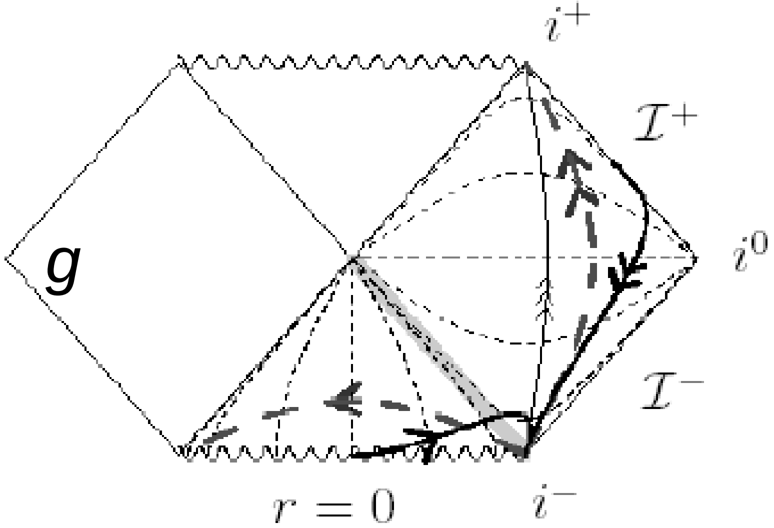
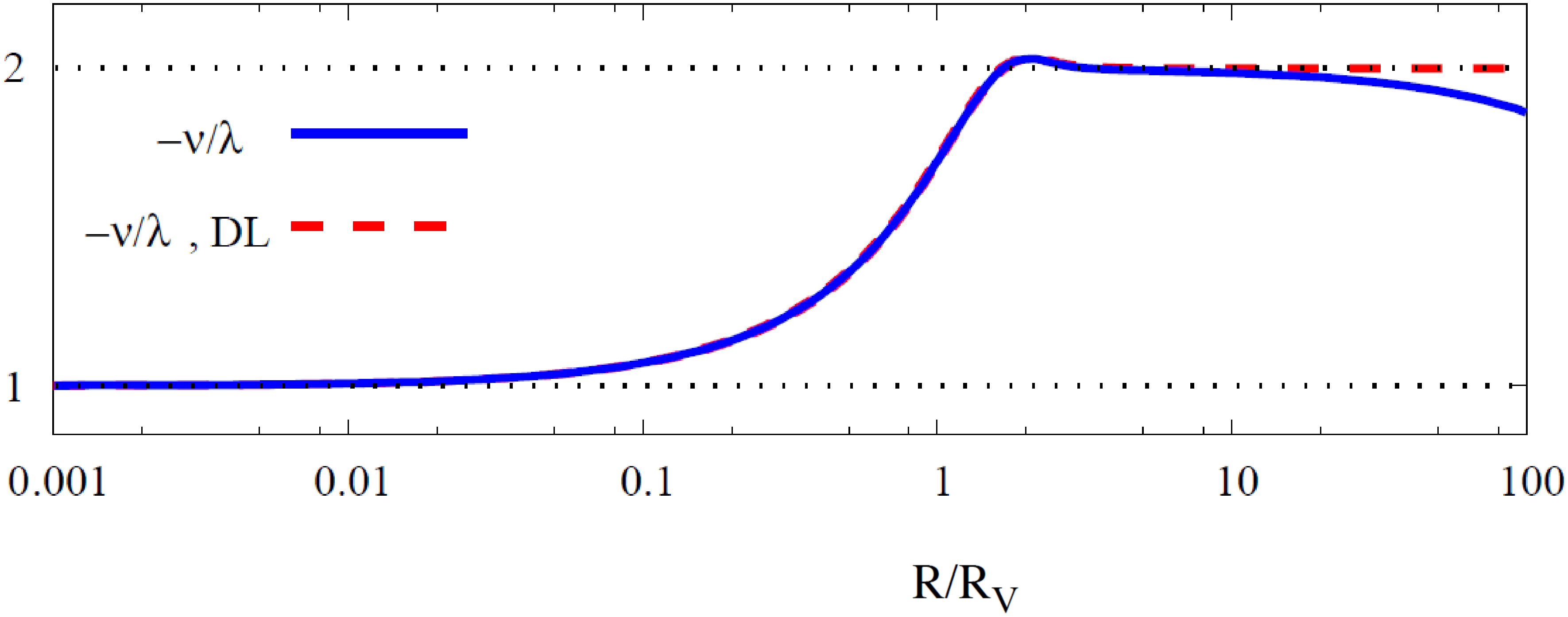
$\sim$

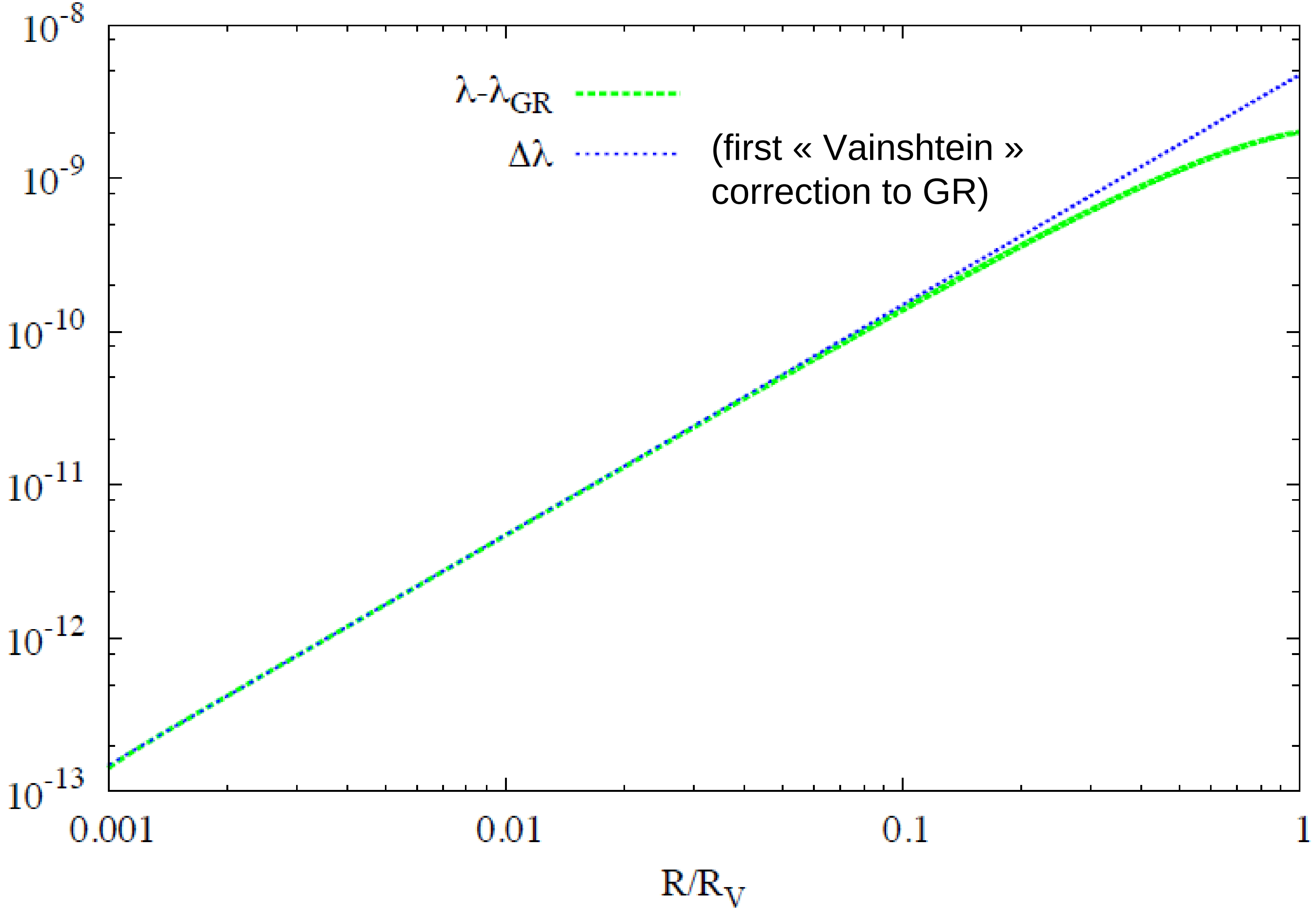




The vDVZ discontinuity gets erased for distances smaller than R_V as expected



Corrections to GR in the $R \ll R_V$ regime



(first « Vainshtein » correction to GR)