CUBIC INTERACTION VERTEX OF HIGHER-SPIN FIELDS WITH EXTERNAL CONSTANT ELECTROMAGNETIC FIELD

I.L. Buchbinder

TSPU

- Construction of interaction vertex of massive and massless bosonic higher spin fields with external constant electromagnetic field in linear approximation in external field
- Aspects of causality

Based on a recent work in collaboration with T.V. Snegirev and Yu.M Zinoviev, arXiv:1204.2341[hep-th].

Contents

Motivations

- Construction of interacting Lagrangians, which describe coupling of the higher-spin fields to each other or to low-spin fields or to external fields is one of the central lines of modern development in higher-spin field theory.
- Naive switching on the interaction (e.g. minimal) does not work for higher spin fields and yields the problems of inconsistency.
- In general, two types of interaction problems are considered in field theory, interactions among the dynamical fields and couplings of dynamical fields to external background. In conventional field theory, these problems are closely related. However, in higher spin theory, where the generic interaction Lagrangians are not established so far, these two types of interactions can be studied as independent problems.
- Cubic vertex for dynamical fields was studied many authors beginning with pioneer works by Bengtsson-Bengtsson-Brink, Berends-Bugrers-van Dam, and Fradkin-Vasiliev.

Higher spin fields coupling to external fields is not so well elaborated.
Basic results:

- Velo-Zwanziger problem (acausal propagation of spin 3/2 and spin 2 fields in constant external electromagnetic fields).
- Derivation of consistent Lagrangian for massive spin-2 in constant external electromagnetic field from string theory (Argyres and Nappi) and recent extension of this result for arbitrary integer-spin field (Porrati, Rahman and Sagnotti). Field interpretation?
- Examples of cubic interactions of higher spin fields with electromagnetic field for some partial cases (Zinoviev).
- Examples of higher spin couplings to external electromagnetic field (Porrati and Rahman).

General problem of constructing the massive and massless, bosonic and fermionic higher spin interaction vertices with external electromagnetic field (not obligatory constant)

Aspects of Lagrangian formulation for free higher spin fields

Bosonic field with mass m and spin $s=n$, $\phi_{\mu_1\mu_2\ldots\mu_n}(x)$ is defined by Dirac-Fierz-Pauli constraints:

$$\phi_{\mu_1\mu_2\ldots\mu_n}=\phi_{(\mu_1\mu_2\ldots\mu_n)}$$
$$(\partial^2+m^2)\phi_{\mu_1\mu_2\ldots\mu_n}=0$$
$$\partial^{\mu_1}\phi_{\mu_1\mu_2\ldots\mu_n}=0$$
$$\phi^{\mu_1}{}_{\mu_1\mu_3\ldots\mu_n}=0$$

Lagrangian construction for free higher spin fields (Singh and Hagen for massive case, Fronsdal for massless case):

- True Lagrangian depends not only on basic field with spin s but also on non-propagating fields with spin less s (auxiliary fields).
- Eliminating the auxiliary fields from the equations of motion yields correct Dirac-Fierz-Pauli constraints for basic field.
- Massive bosonic spin s field.
  Lagrangian is given in terms of totally symmetric traceless tensor fields with spins $s, s-2, \ldots, 1, 0$.
- Massless bosonic spin s field.
  Lagrangian is given in terms totally symmetric traceless tensor fields with spins $s, s-2$.
  It can be imbedded into a single double traceless spin s field.

Electromagnetic potential A_μ enters into Lagrangian either through covariant derivative or through the electromagnetic field strength $F_{\mu\nu}$ directly.
The approach to the vertex construction is based on two points.

- Gauge invariance.
  Lagrangian $\mathcal{L}$ is constructed to be invariant under the gauge transformation δ, i.e. the vanishing of variation $\delta\mathcal{L} = 0$ (up to the total divergence).
- Perturbative consideration.
  Lagrangian in constructed as a sum of terms, which are linear, quadratic and so on in external field strength F

  $$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + ...$$

  where $\mathcal{L}_0$ is the free Lagrangian of dynamical fields, $\mathcal{L}_1$ is quadratic in dynamical fields and linear in strength F and so on.
  The gauge transformations are also written as the series

  $$\delta = \delta_0 + \delta_1 + ...$$

  where δ_0 are gauge transformations of free theory, δ_1 are linear in strength F one and so on.

Aim: construction in explicit form of the first correction to Lagrangian $\mathcal{L}_1$ and first correction to gauge transformation δ_1. Both these corrections are linear in strength F. The Lagrangian $\mathcal{L}_1$, being quadratic in dynamical fields and linear in external field, defines the cubic coupling of higher spin fields to external electromagnetic field.

Gauge variation of action:

$$\delta\mathcal{L} = (\delta_0 + \delta_1)(\mathcal{L}_0 + \mathcal{L}_1) = \delta_0\mathcal{L}_0 + \delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 + \delta_1\mathcal{L}_1 = 0$$

Finding the $\mathcal{L}_1$:

- Most general expressions for gauge transformations δ_1 and Lagrangian $\mathcal{L}_1$ on the base of Lorentz symmetry and gauge invariance up to the numerical coefficients.
- Getting the equations for the coefficients and their solution.
- Stuekelberg fields in massive case (Zinoviev, BRST approach).

Remark: the higher spin fields are real and numerated by index $i = 1, 2$ (fundamental representation of $SO(2)$ group).

Remark: in arbitrary external electromagnetic field a number of derivatives in vertex will increase with value of spin. Specific features of constant external field is that it is sufficient to use only two derivatives for dynamical fields

- Massless charged field of arbitrary integer spin s is described by doublet of totally symmetric real tensor rank-s fields $\Phi_{\mu_1\mu_2\ldots\mu_s}{}^{i}$, $i=1,2$, satisfying the double traceless condition $\Phi^{\alpha\beta}{}_{\alpha\beta\mu_1\ldots\mu_{s-4}}{}^{i}=0$
- Notations

$$\Phi_{\mu_1\mu_2\ldots\mu_s}{}^{i}=\Phi_s{}^{i}, \qquad \partial^{\mu_1}\Phi_{\mu_1 s-1}{}^{i}=(\partial\Phi)_{s-1}{}^{i}, \qquad g^{\mu_1\mu_2}\Phi_{\mu_1\mu_2 s-2}{}^{i}=\tilde{\Phi}_{s-2}{}^{i}$$

- Fronsdal Lagrangian for free theory

$$\mathcal{L}_0=$$

$$(-1)^s\frac{1}{2}[\partial^\mu\Phi^{s\ i}\partial_\mu\Phi_s{}^{i}-s(\partial\Phi)^{s-1,\ i}(\partial\Phi)_{s-1}{}^{i}+s(s-1)(\partial\Phi)^{\mu_1 s-2,\ i}\partial_{\mu_1}\tilde{\Phi}_{s-2}{}^{i}-$$
$$-\frac{s(s-1)}{2}\partial^\mu\tilde{\Phi}^{s-2,\ i}\partial_\mu\tilde{\Phi}_{s-2}{}^{i}-\frac{s(s-1)(s-2)}{4}(\partial\tilde{\Phi})^{s-3,\ i}(\partial\tilde{\Phi})_{s-3}{}^{i}]$$

- Gauge transformations

$$\delta_0\Phi_s{}^{i}=\partial_{(\mu_1}\xi_{s-1)}{}^{i}, \quad \tilde{\xi}_{s-3}{}^{i}=0$$

$\xi_{s-1}{}^{i}$ is symmetric traceless rank-$(s-1)$ tensor field (the tilde means a trace).

- First order gauge invariance condition

$$\delta_0 \Phi_s{}^i \left(\frac{\delta S_1}{\delta \Phi_s{}^i} \right) + \delta_1 \Phi_s{}^i \left(\frac{\delta S_0}{\delta \Phi_s{}^i} \right) = 0$$

- Most general anzatz for first correction to free Lagrangian

$$\begin{gathered}
\mathcal{L}_1 = (-1)^s \varepsilon^{ij} F^{\alpha\beta} [a_1 \partial^\mu \Phi_\alpha{}^{s-1,\, i} \partial_\mu \Phi_{\beta s-1}{}^j + \\
+ a_2 (\partial\Phi)_\alpha{}^{s-2,\, i} (\partial\Phi)_{\beta s-2}{}^j + \\
+ a_3 \partial_\alpha \Phi_\beta{}^{s-1,\, i} (\partial\Phi)_{s-1}{}^j + a_4 (\partial\Phi)_\alpha{}^{s-2,\, i} \partial_\beta \tilde{\Phi}_{s-2}{}^j + \\
+ a_5 (\partial\Phi)_\alpha{}^{\mu_1 s-3,\, i} \partial_{\mu_1} \tilde{\Phi}_{\beta s-3}{}^j + a_6 \partial^\mu \tilde{\Phi}_\alpha{}^{s-3,\, i} \partial_\mu \tilde{\Phi}_{\beta s-3}{}^j + \\
+ a_7 \partial_\alpha \tilde{\Phi}_\beta{}^{s-3,\, i} (\partial\tilde{\Phi})_{s-3}{}^j + a_8 (\partial\tilde{\Phi})_\alpha{}^{s-4} (\partial\tilde{\Phi})_{\beta s-4}{}^j]
\end{gathered}$$

- Most general anzatz for first correction to free gauge transformations after some (F-dependent) redefinition of fields and parameters

$$\delta_1 {\Phi_s}^i = \gamma \varepsilon^{ij} g_{(\mu_1\mu_2} F^{\alpha\beta} \partial_\alpha {\xi_{\beta s-2)}}^j$$

- One arbitrary real parameter γ
- First order gauge invariance condition yields equations for arbitrary parameters
- Solutions to equations

$$a_3 = 2a_1 = \frac{1}{2}\gamma s(d+2s-6)$$

$$a_4 = -2a_2 = \frac{1}{2}\gamma s(s-1)(d+2s-6)$$

$$a_4 = -2a_6 = 2a_7 = \frac{1}{4}\gamma s(s-1)(s-2)(d+2s-6)$$

$$a_8 = -\frac{1}{16}\gamma s(s-1)(s-2)(s-3)(d+2s-6)$$

- Single arbitrary real parameter γ of inverse mass square dimension

Lagrangian and gauge transformations are written as follows:

$$\mathcal{L} = \mathcal{L}_{00} + \mathcal{L}_{01} + \mathcal{L}_{02} + \mathcal{L}_{10} + \mathcal{L}_{11} + ...$$

$$\delta = \delta_{00} + \delta_{01} + \delta_{10} + \delta_{11} + ...$$

- First index in gauge transformations means a power of fields (including the electromagnetic potential A_μ in strength $F_{\mu\nu}$. Second index in gauge transformations means a total number of derivatives (including the derivatives of electromagnetic potential A_μ in strength $F_{\mu\nu}$).
  $\delta_{kn} \sim \partial^n \Phi^k \xi$
- First index in Lagrangian means a power of fields higher quadratic (including field A_μ in strength $F_{\mu\nu}$). Second index in Lagrangian means a total number of derivatives.
  $\mathcal{L}_{kn} \sim \partial^n \Phi^{k+2}$

Illustration of general procedure on an example of spin-2 field.

- Set of fields $\Phi^a = \{h_{\mu\nu},\ b_\mu,\ \varphi\}$
  Basic field $h_{\mu\nu}$, auxiliary Stueckelberg fields b_μ and φ.
- Free Lagrangian

$$\mathcal{L}_0 = \mathcal{L}_{00} + \mathcal{L}_{01} + \mathcal{L}_{02}$$

$$\mathcal{L}_{02} = \frac{1}{2}\partial^\alpha h^{\mu\nu}\partial_\alpha h_{\mu\nu} - (\partial h)^\mu(\partial h)_\mu + (\partial h)^\mu\partial_\mu h - \frac{1}{2}\partial^\mu h\partial_\mu h -$$

$$-\frac{1}{2}\partial^\mu b^\nu\partial_\mu b_\nu + \frac{1}{2}(\partial b)(\partial b) + \frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi$$

$$\mathcal{L}_{01} = m[\alpha_1 h^{\mu\nu}\partial_\mu b_\nu - \alpha_1 h(\partial b) + \alpha_0 b^\mu\partial_\mu\varphi]$$

$$\mathcal{L}_{00} = m^2[-\frac{1}{2}h^{\mu\nu}h_{\mu\nu} + \frac{1}{2}hh + \frac{1}{2}\alpha_1\alpha_0 h\varphi + \frac{d}{2(d-2)}\varphi^2]$$

- Free gauge transformations

$$\delta_0 = \delta_{00} + \delta_{01}$$

$$(\delta_{01} + \delta_{00})h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)} + m\frac{\alpha_1}{d-2}g_{\mu\nu}\xi$$

$$(\delta_{01} + \delta_{00})b_\mu = \partial_\mu\xi + m\alpha_1\xi_\mu$$

$$\delta_{00}\varphi = -m\alpha_0\xi$$

$$(\alpha_1)^2 = 2, \quad (\alpha_0)^2 = 2\frac{d-1}{d-2}$$

- Free gauge invariance

$$\delta_0\mathcal{L}_0 = 0$$

- Set of fields $\Phi^a = \{h_{\mu\nu}{}^i,\ b_\mu{}^i,\ \varphi^i\},\ i = 1, 2$
- Minimal interaction

$$\partial_\mu \to D^{ij}_\mu = \delta^{ij}\partial_\mu + e_0\varepsilon^{ij}A_\mu, \qquad \varepsilon^{ij} = -\varepsilon^{ji}, \qquad \varepsilon^{12} = 1,$$

- Violation of gauge invariance because of non-zero commutator $[D^{ik}_\mu, D^{kj}_\nu] = e_0\varepsilon^{ij}F_{\mu\nu}$. Non-invariant part in linear approximation

$$\bar\delta_0\bar{\mathcal{L}}_0 = (\delta_{00}\bar{\mathcal{L}}_{02} + \bar\delta_{01}\bar{\mathcal{L}}_{01}) + \bar\delta_{01}\bar{\mathcal{L}}_{02}$$

$$(\delta_{00}\bar{\mathcal{L}}_{02} + \bar\delta_{01}\bar{\mathcal{L}}_{01}) = me_0\varepsilon^{ij}\xi^i_\mu[-\alpha_1 F^{\alpha\mu}b^j_\alpha]$$

$$\bar\delta_{01}\bar{\mathcal{L}}_{02} = e_0\varepsilon^{ij}\xi^i_\mu[-4F^{\alpha\beta}\partial_\alpha h^{\mu,\,j}_\beta - 2F^{\alpha\mu}(\partial h)^j_\alpha + 3F^{\alpha\mu}\partial_\alpha h^j] + e_0\varepsilon^{ij}\xi^i[2F^{\alpha\beta}\partial_\alpha b^j_\beta]$$

Cancelation of terms violating gauge invariance. Non-minimal interaction?

- Non-minimal correction to Lagrangian, surviving in massless limit $e_0 \to 0,\ m^2 \to 0,\ \frac{e_0}{m^2} = const$

$$\mathcal{L}_1 = \mathcal{L}_{13}$$

$$\mathcal{L}_{13} = \frac{e_0}{m^2}\varepsilon^{ij}F^{\alpha\beta}[a_1\partial^\mu h_\alpha^{\nu,\,i}\partial_\mu h_{\nu\beta}^j + a_2(\partial h)_\alpha^i(\partial h)_\beta^j + a_3\partial_\alpha h_\beta^{\mu,\,i}(\partial h)_\mu^j +$$

$$+a_4(\partial h)_\alpha^i\partial_\beta h^j + b_1\partial^\mu b_\alpha^i\partial_\mu b_\beta^j + b_2\partial_\alpha b_\beta^i(\partial b)^j]$$

- Non-minimal correction to gauge transformations

$$\delta_1 = \delta_{12}$$

$$\delta_{12}h_{\mu\nu}^i = \gamma_2\frac{1}{m^2}\varepsilon^{ij}g_{\mu\nu}F^{\alpha\beta}\partial_\alpha\xi_\beta^j$$

- Unknown real coefficients $\gamma_2,\ a_1, ..., a_4, b_1, b_2$

- Gauge invariance. Part of conditions are literally coincides with the analogous conditions in massless theory. The coefficients $a_1, ..., a_4$ are determined. All other are not fixed.
- Gauge invariance is still violated. More corrections to gauge transformations and Lagrangian?
- Additional corrections to Lagrangian

$$\mathcal{L}_{12} = \frac{1}{m}\varepsilon^{ij}F^{\alpha\beta}[c_1\partial_\alpha h_\beta^{\mu,\,i}b_\mu^j + c_2(\partial h)_\alpha^i b_\beta^j + c_3\partial_\alpha h^i b_\beta^j + c_4\partial_\alpha b_\beta^i\varphi^j]$$

$$\mathcal{L}_{11} = \varepsilon^{ij}F^{\alpha\beta}[d_1 h_\alpha^{\mu,\,i}h_{\mu\beta}^j + d_2 b_\alpha^i b_\beta^j]$$

- Additional correction to gauge transformations

$$\delta_{11}b_\mu^i = \delta_1\frac{1}{m}\varepsilon^{ij}F_\mu{}^\alpha\xi_\alpha^j$$

Massive spin-2 theory. Fixation of the coefficients

- New arbitrary coefficients
- Use of conditions of gauge invariance in first order in $F_{\mu\nu}$. System of algebraic equations for the coefficients.
- All coefficients are determined in terms of two arbitrary parameters γ_2 and b_1
- The same procedure works perfectly for any massive integer spin field. System of equations for the coefficients is extremely complicated in general case. Solution is found in terms of two arbitrary real parameters

Velo-Zwanziger problem. Higher spin field equations of motion contain some number of algebraic and differential constraints. Causality of equations of motion can be clarified only after all constraints are taken into account

Statement: After eliminating all constraints the equations of motion for spin-2 field coupled to constant electromagnetic background in linear order in strength $F_{\mu\nu}$ contain the higher (second) derivatives only in form of d'Alambertian what guarantees a causal propagation.

- 1st step. Fixation of gauge transformation and elimination of the auxiliary Stueckelberg fields b_μ and φ. The Lagrangians take the form

$$\mathcal{L}_0 = \frac{1}{2} D^\alpha h^{\mu\nu} D_\alpha h_{\mu\nu} - (Dh)^\mu (Dh)_\mu - (DDh)h - \frac{1}{2} D^\mu h D_\mu h -$$

$$-\frac{1}{2} m^2 h^{\mu\nu} h_{\mu\nu} + \frac{1}{2} m^2 hh$$

$$\mathcal{L}_1 = \frac{1}{m^2} \varepsilon^{ij} F^{\alpha\beta} [a_1 \partial^\mu h_\alpha^{\nu,\,i} \partial_\mu h_{\nu\beta}^j + a_2 (\partial h)_\alpha^i (\partial h)_\beta^j + a_3 \partial_\alpha h_\beta^{\mu,\,i} (\partial h)_\mu^j +$$

$$+ a_4 (\partial h)_\alpha^i \partial_\beta h^j + m^2 d_1 h_\alpha^{\mu,\,i} h_{\mu\beta}^j]$$

- 2nd step. Algebraic constraint (the terms quadratic in F are omitted)

$$\left(D^{ik}_\mu D^{kj}_\nu - \frac{m^2}{d-2}\delta^{ij} g_{\mu\nu} + 2\frac{\delta_1}{m^2\alpha_1}\varepsilon^{ij} F^\alpha{}_\mu \partial_\alpha \partial_\nu\right)\left(\frac{\delta \mathcal{S}}{\delta h_{\mu\nu}{}^j}\right) =$$

$$= -m^4 \frac{d-1}{d-2} h^i = 0$$

It yields $h^i = 0$

- Differential constraint

$$\left(g_{\mu\alpha} D^{ij}_\nu + \frac{\gamma_2}{2m^2}\varepsilon^{ij} g_{\mu\nu} F^\sigma{}_\alpha \partial_\sigma\right)\left(\frac{\delta \mathcal{S}}{\delta h_{\mu\nu}{}^j}\right) = 0.$$

Using $h^i = 0$, one gets

$$-m^2 (Dh)_\alpha{}^i + \varepsilon^{ij}[(2e_0 + d_1)F^{\sigma\rho}\partial_\sigma h_{\alpha\rho}{}^j + (1-d_1)F^\sigma{}_\alpha (\partial h)_\sigma{}^j] = 0$$

It yields $(Dh) \sim F$, $(DDh) \sim F^2$

- 3rd step. Equation of motion in first order in F

$$-(D^2 h_{\mu\nu})^i - m^2 h_{\mu\nu}{}^i - \frac{1}{m^2}\varepsilon^{ij} F^\alpha{}_{(\mu}(a_1\partial^2 h_{\nu)\alpha}{}^j - m^2 d_1 h_{\nu)\alpha}{}^j) +$$
$$+\varepsilon^{ij}(2e_0 + d_1 + \frac{a_3}{2}) F^{\alpha\beta}\partial_\alpha\partial_{(\mu}h_{\nu)\beta}{}^j = 0$$

- 4rth step. Second derivatives included both into d'Alambertian and into $\varepsilon^{ij}(2e_0 + d_1 + \frac{a_3}{2}) F^{\alpha\beta}\partial_\alpha\partial_{(\mu}h_{\nu)\beta}{}^j$.
  Theory is formulated in terms of two free parameters. One can prove that there exists a value of the parameter b_2 such that $2e_0 + d_1 + \frac{a_3}{2} = 0$.
- After eliminating the constraints and fixing one of the free parameters, the second derivatives enter into equations of motion only in form of d'Alambertian.
  One can prove that the only free parameter is γ_2. This parameter determines the non-minimal correction to gauge transformations.
- The same mechanism works for massive spin-3 field as well. Analysis is more complicated since not all auxiliary Stueckelberg fields can be gauged away and we should consider more constraints. Nevertheless, after eliminating the constraints and fixing one of the free parameters, derivatives enter into Lagrangian only in form of d'Alambertian.

Summary

- Cubic vertex of interaction of massive and massless integer spin fields with constant external electromagnetic field in d-dimensional flat space is completely constructed.
- Construction is based on gauge invariance. Stueckelberg auxiliary fields are used in case of massive fields.
- Cubic vertex is a deformation of free Lagrangian by the terms linear in electromagnetic strength. Such a deformation violates free gauge invariance and to restore the gauge invariance we should deform the free gauge transformations by the terms linear in strength.
- Causal propagation of massive spin-2 and spin-3 fields is proved in first order in strength. Analysis for arbitrary spin field is analogous to spin-2 and spin-3 cases but technically it is more complicated.

Further development

- Fermionic higher spin fields
- Higher powers of external strengths
- Non-constant electromagnetic background
- Higher spin fields in three dimensions
- Higher spin fields in external gravitational fields besides dS or AdS backgrounds
- Quantum effective action

THANK YOU VERY MUCH!