Some solutions in galileon theory

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OUTLINE

- Introduction and motivation
- Spherical symmetry
- Variation of the Newton constant in a class of theories
- Conclusion

motivation

Nicolis et al'09

Motivated by Dvali-Gabadadze-Porrati model of gravity.
Scalar field theory with the properties:

- it directly couples to matter, $L \supset \pi T$
  - additional (fifth) force, like all scalar-tensor models, modified cosmological dynamics, Dark energy, inflation etc
- Vainshtein mechanism (like in massive gravity or DGP),
  - the scalar is screened in local measurements
- galilean symmetry, invariance under $\pi(x) \to \pi(x) + b_\mu x^\mu + c$
  - motivated by DGP, the cosmological solution of the form,

$$\pi = C + B_\mu x^\mu + A_{\mu\nu} x^\mu x^\nu + O(x^3 H^3)$$

  - loop correction do not produce other terms;
- up to second order derivatives in equations of motion,
  - no Ostrogradski ghosts

galileon lagrangians

Flat space-time (curvature and the backreaction of the galileon onto the metric are neglected)

Nicolis et al'09

$$\mathcal{L}_\pi = \sum_{i=1}^{i=5} c_i \mathcal{L}_i, \quad \mathcal{L}_i \sim \pi^i$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi, \\
\mathcal{L}_2 &= -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi, \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\pi)^2\,\Box\pi, \\
\mathcal{L}_4 &= -\frac{1}{4}(\Box\pi)^2\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\Box\pi\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots \\
\mathcal{L}_5 &= -\frac{1}{5}(\Box\pi)^3\partial_\mu\pi\partial^\mu\pi + \frac{3}{5}(\Box\pi)^2\partial_\mu\pi\partial_\nu\pi\partial^\mu\partial^\nu\pi + \ldots
\end{aligned}$$

equations of motion of galileon

Equations of motion (in flat space-time)

Nicolis et al'09

$$
\begin{aligned}
\mathcal{E}_1 &= 1 \\
\mathcal{E}_2 &= \Box\pi \\
\mathcal{E}_3 &= (\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2 \\
\mathcal{E}_4 &= (\Box\pi)^3 - 3\,\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3 \\
\mathcal{E}_5 &= (\Box\pi)^4 - 6(\Box\pi)^2(\partial_\mu\partial_\nu\pi)^2 + 8\,\Box\pi(\partial_\mu\partial_\nu\pi)^3 + 3\left[(\partial_\mu\partial_\nu\pi)^2\right]^2 - 6(\partial_\mu\partial_\nu\pi)^4
\end{aligned}
$$

However, the perturbations of metric were neglected.
(metric couples to galileon nonminimally, through the higher derivatives
=> higher-order derivatives may appear in EOM for metric!)

covariant galileon

Indeed, naive covariantization, $\partial_\mu \to D_\mu$, leads to higher-order derivative EOMs for $\mathcal{L}_4, \mathcal{L}_5$ terms.

Deffayet et al'09

Can be cured by adding non-minimal scalar-metric coupling to "flat" Galileon

$$\mathcal{L}_\pi = \sum_{i=1}^{i=5} a_i \mathcal{L}_i,$$

$$\mathcal{L}_1 = \pi, \quad \mathcal{L}_2 = \partial_\mu \pi \partial^\mu \pi, \quad \mathcal{L}_3 = (\partial\pi)^2 \Box\pi$$

$$\mathcal{L}_4 = -\left(\pi_{;\alpha}\pi^{;\alpha}\right)\left[2\left(\Box\pi\right)^2 - 2\left(\pi_{;\mu\nu}\pi^{;\mu\nu}\right) - \frac{1}{2}\pi_{;\mu}\pi^{;\mu}R\right],$$

$$\mathcal{L}_5 = \left(\pi_{;\lambda}\,\pi^{;\lambda}\right)\left[\left(\Box\pi\right)^3 - 3\left(\Box\pi\right)\left(\pi_{;\mu\nu}\,\pi^{;\mu\nu}\right) + 2\left(\pi_{;\mu}{}^{\nu}\,\pi_{;\nu}{}^{\rho}\,\pi_{;\rho}{}^{\mu}\right) - 6\left(\pi_{;\mu}\,\pi^{;\mu\nu}\,G_{\nu\rho}\,\pi^{;\rho}\right)\right].$$

covariant galileon (II)

In a curved space time the galilean symmetry is lost

$$\cancel{\pi(x) \to \pi(x) + b_\mu x^\mu + c}$$

One may consider more general class of theories:

1. No Ostrogradski ghosts
2. Vainshtein mechanism

generalization of galileon

$$\mathcal{L}^{(1)} = V(\varphi, X)$$

$$\mathcal{L}^{(2)} = K(\varphi, X)$$

$$\mathcal{L}^{(3)} = G^{(3)}(\varphi, X)\,\Box\varphi$$

$$\mathcal{L}^{(4)} = G^{(4)}_{,X}(\varphi, X)\left[(\Box\varphi)^2 - (\nabla\nabla\varphi)^2\right] + R\,G^{(4)}(\varphi, X)$$

$$\mathcal{L}^{(5)} = G^{(5)}_{,X}(\varphi, X)\left[(\Box\varphi)^3 - 3\Box\varphi\,(\nabla\nabla\varphi)^2 + 2\,(\nabla\nabla\varphi)^3\right] - 6G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi\,G^{(5)}(\varphi, X)$$

Deffayet et al'11

$V, K, G^{(3)}, G^{(4)}, G^{(5)}$ are arbitrary functions of the fields and its the canonical kinetic term X

Second-order derivative EOMs:
"generalized" Galileons
or
Horndeski theory

Horndeski'74

the Vainshtein mechanism

For the Vainshtein mechanism to work it is (generically) sufficient to have a non-linear kinetic term ("k-mouflage" gravity),

$$S = M_P^2 \int d^4x \sqrt{-g} \left[(1+\varphi) \frac{R}{2} + K(\varphi, \partial\varphi, \partial^2\varphi, \ldots) \right] + S_{\mathrm{matter}}$$

EB, Deffayet, Ziour et al'09

$$\partial^2 h + \partial^2 \varphi = M_{\mathrm{P}}^{-2} T$$

$$\partial^2 h + \mathcal{E}_\varphi = 0$$

$$\Longrightarrow \quad \partial^2 \varphi + \mathcal{E}_\varphi = M_{\mathrm{P}}^{-2} T$$

$$\partial^2 \varphi = M_{\mathrm{P}}^{-2} T$$

$$h \neq h_{\mathrm{GR}}$$

$$\partial^2 \varphi \ll \mathcal{E}_\varphi = M_{\mathrm{P}}^{-2} T$$

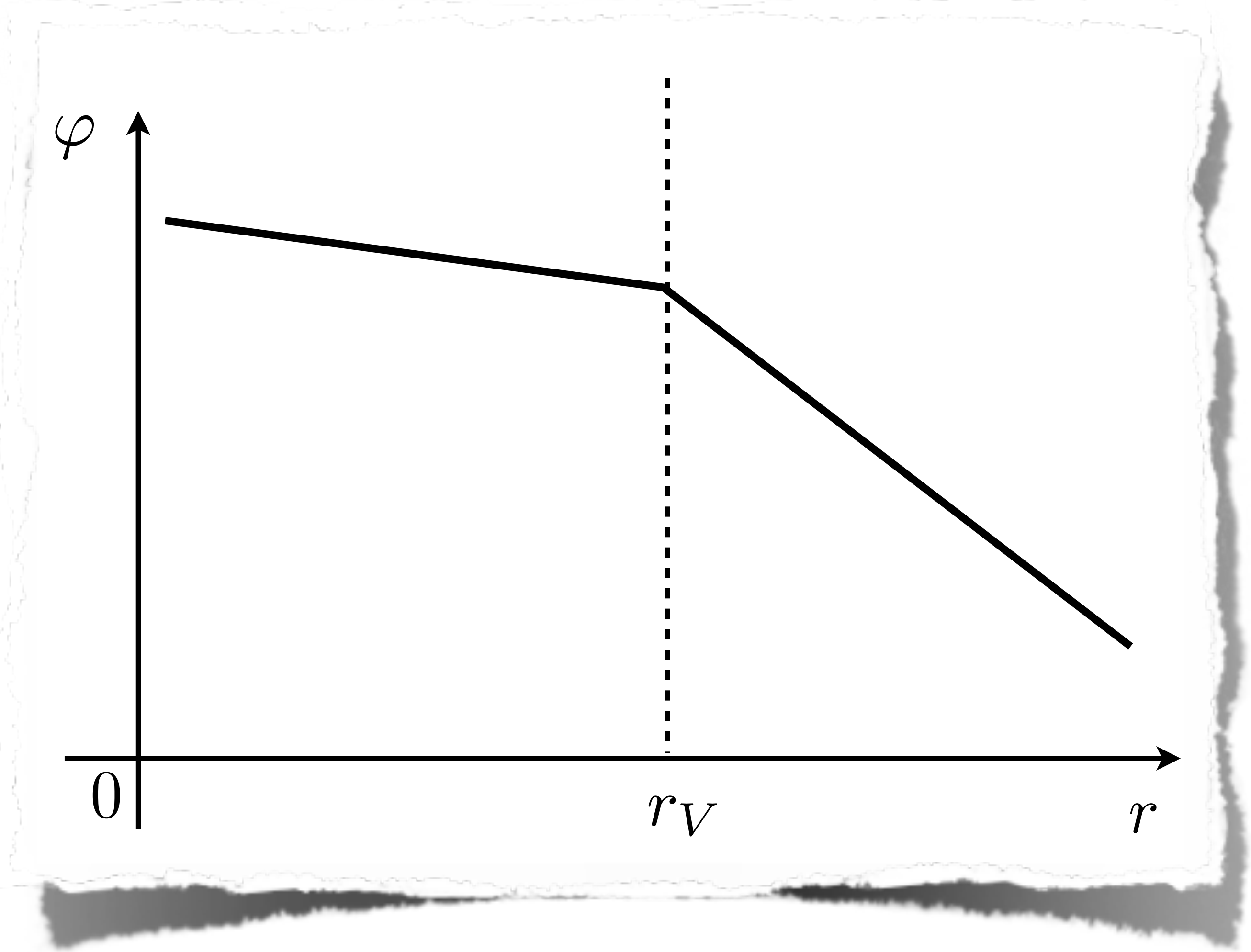
$$h \approx h_{\mathrm{GR}}$$

spherical symmetry

general case (I)

$$\partial^2\varphi + \mathcal{E}_\varphi = \frac{T}{M_{\mathrm{P}}^2}$$

$$\mathcal{E}_\varphi \sim M^{\#}\varphi^{\#}\partial\varphi^{\#} \xrightarrow{\partial \to 1/r} \mathcal{E}_\varphi \sim M^{2-l}r^{-l}\varphi^{n}$$



l+n>3

$$\varphi \sim \frac{r_S}{r},\ r > r_V$$

$$\varphi \sim (r_S M)^{\frac{1}{n}}\,(rM)^{\frac{l-3}{n}}\ ,\ r < r_V$$

$$r_V \sim M^{-1}\,(Mr_S)^{\frac{n-1}{l+n-3}}$$

general case (II)

EOM is identically zero on flat space time,

but non-zero on curved background

$$\mathcal{E}_\varphi \sim r_S^{\#} M^{\#} \varphi^{\#} \partial \varphi^{\#} \xrightarrow{\partial \to 1/r} \mathcal{E}_\varphi \sim r_S^{-s} M^{2-l-s} r^{-l} \varphi^n$$

$$\varphi \sim \frac{r_S}{r}, \; r > r_V$$

$$\varphi \sim (r_S M)^{\frac{s+1}{n}} (rM)^{\frac{l-3}{n}}, \; r < r_V$$

$$r_V \sim M^{-1} (M r_S)^{\frac{n-s-1}{n+l-3}}$$

some examples

decoupling limit of DGP

$$\frac{\mathcal{L}}{M_{\rm P}^2} = \frac{1}{M^2}\left(\partial\varphi\right)^2 \Box\varphi$$

l=4, n=2, s=0

$$\mathcal{E}_\varphi = \frac{1}{M^2}\left(\frac{4\varphi'\varphi''}{r} - \frac{2\varphi'^2}{r^2}\right) \sim M^{-2}r^{-4}\varphi^2$$

$$r_V \sim \left(\frac{r_S}{M^2}\right)^{1/3} \qquad \varphi' \sim M\left(\frac{r_S}{r}\right)^{1/2}$$

---

pure k-essence

$$\frac{\mathcal{L}}{M_{\rm P}^2} = \frac{1}{M^2}\left(\partial\varphi\right)^4$$

l=4, n=3, s=0

$$\mathcal{E}_\varphi \propto \frac{1}{M^2}\varphi'^2\varphi'' \sim M^{-2}r^{-4}\varphi^3$$

$$r_V \sim \left(\frac{r_S}{M}\right)^{1/2} \qquad \varphi' \sim \left(\frac{M^2 r_S}{r^2}\right)^{1/3}$$

time-varying Newton’s constant

ingredients

action

$$S = \frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}\left(R+\mathcal{L}_{\rm s}+\mathcal{L}_{\rm NL}\right)+S_m\left[\tilde{g}_{\mu\nu},\psi_m\right],$$

$$\mathcal{L}_{\rm s} = -\left(\partial\varphi\right)^2$$

$\mathcal{L}_{\rm NL}$ — nonlinear self-interaction of φ

$$\tilde{g}_{\mu\nu} = \mathcal{A}^2(\varphi)g_{\mu\nu}$$

$\mathcal{L}_{\rm NL}$: - Restores GR thanks to the Vainshtein mechanism,
- shift-symmetric, $\varphi \to \varphi + \mathrm{const.}$

eoms

$$M_{\mathrm{P}}^2 G_{\mu\nu} = T_{\mu\nu}^{(\mathrm{st})} + T_{\mu\nu}^{(\mathrm{NL})} + T_{\mu\nu}^{(\mathrm{m})}$$

$$\nabla_\mu \left(\nabla^\mu \varphi + J_{\mathrm{NL}}^\mu\right) = -\alpha(\varphi) M_{\mathrm{P}}^{-2} T^{(\mathrm{m})}$$

$$\alpha(\varphi) \equiv d \ln\left(\mathcal{A}\right) / d\varphi$$

$$J_{\mathrm{NL}}^\mu \equiv -\delta \mathcal{L}_{\mathrm{NL}} / \delta \varphi_{,\mu}$$

we can choose at present $\mathcal{A}(\varphi) = 1$

$T^{(\mathrm{m})} = \mathcal{A}^4(\varphi) \tilde{T}^{(\mathrm{m})}$ but we neglect variation of the scalar field in the r.h.s of the field equation

homogeneous evolution

$$\ddot{\varphi}_{\rm cosm} + 3H\dot{\varphi}_{\rm cosm} - \nabla_0\left(J^0_{\rm NL}\right) = \alpha(\varphi) M_{\rm P}^{-2} T^{(\rm m)}$$

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+
scalar is not in cosmological Vainshtein regime

$\Rightarrow$ $|\dot{\varphi}_{\rm cosm}| \sim \alpha H$ + decaying solution

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$\rho_\varphi \gg \rho_{\rm m}$

i.e. self-accelerated scenario

$\Rightarrow$ $|\dot{\varphi}_{\rm cosm}| \sim H_0$

local effects (I)

$$\varphi'(r) + J^r_{\mathrm{NL}}\left[\varphi'(r)\right] = \frac{r_s}{2r^2}$$

$$\Downarrow$$

$$\varphi_{\mathrm{static}}(r) = \int dr\, \varphi'_{\mathrm{static}}(r) + \mathcal{C}$$

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$$\varphi_{\mathrm{approx}}(t, r) = \int dr\, \varphi'_{\mathrm{static}}(r) + \dot{\varphi}_{\mathrm{cosm}}(t_0)t$$

small corrections due to additional small constant in the equation and due to the curvature

local effects (II)

$$\boxed{\nabla_\mu \left(\nabla^\mu \varphi + J^\mu_{\rm NL}\right) = -\alpha(\varphi) M_{\rm P}^{-2} T^{(\rm m)}}$$

$$\varphi_{\rm cosm}(t) = \varphi_{\rm cosm}(t_0) + \dot{\varphi}_{\rm cosm}(t_0)\, t.$$

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$$\Downarrow$$

second order ODE on $\varphi(r)$

$$\varphi(r=\infty) = 0$$
$$\varphi'(r=0) = 0$$

$\dot\varphi(t,r)$ is set by the cosmological evolution even inside the regions where the Vainshtein screening operates.

variation of the Newton constant

variation of the scalar field => variation of the Newton constant

Einstein frame -> Jordan frame

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In general two effects:
exchange of the scalar degree of freedom and
rescaling of the coordinates due to the conformal transformation of the metric.

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$$\left|\dot{G}/G\right| \approx 2\alpha\dot{\varphi}_{\rm cosm}(t)$$

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$|\dot{G}/G| \sim \alpha^2 H_0$ matter dominated era

$|\dot{G}/G| \sim \alpha H_0$ scalar field domination

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Experiment (Lunar Laser Ranging): $|\dot{G}/G| < 0.02 H_0$

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violate one of the assumptions:

1. break shift symmetry, for example add a mass term

$$m^2\varphi^2 \to \dot{\varphi}_{min} \sim \alpha H\dot{H}/m^2$$

2. Particular disformal coupling, e.g. TeVeS: no evolution of Newton's constant

a note about brane models

DGP model
equivalent to a scalar-tensor theory of the Galileon type in UV
(in particular in the decoupling limit),

BUT

not fully described by such a theory at cosmological scales
(IR limit).
No variation of the Newton constant is found in DGP! Lue&Starkman'03

conclusion

- shift-symmetric non-minimally coupled theories are tightly constrained (ruled out?)