Heavy Quark Potentials via Gauge/String Duality

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Outline

- Motivation & Background
  - Cornell Model
  - strings in 4d
- Think Different

  a new approach using strings in 5d

  - overview of the approach
  - first example: heavy quark potential
  - second example: baryonic potential
  - third example: some hybrid potentials
- Conclusion and Future Work

Phenomenological Models and Strings in 4d

✦ Cornell model

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2} + C$$

Eichten et al

three free parameters are adjusted to fit the charmonium spectrum

$$\kappa \approx 0.48\,, \qquad a \approx 2.34\ \mathrm{GeV}^{-1}\,, \qquad C = -0.25\ \mathrm{GeV}$$

✦ effective string models in 4d

$$E_n = \sigma r + C + \frac{\pi}{r}\left(n - \frac{d-2}{24}\right) + \underset{\text{1/r corrections}}{\blacksquare}\,, \qquad E_0 = V$$

Luescher-Weisz

It is a series in powers of 1/r.

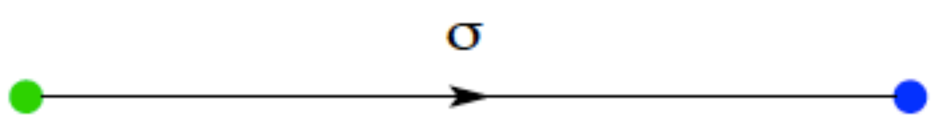
$$E_n = \sigma r\left(1 + \frac{2\pi}{\sigma r^2}(n - \frac{d-2}{24})\right)^{\frac{1}{2}} + C$$

Arvis

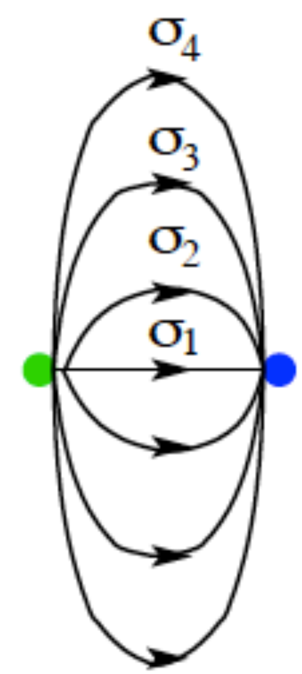
For more discussion, see $http://online.itp.ucsb.edu/online/novelnum12/teper/$

Think Different

✦ Stringy reason for the 5th "dimension"



Faraday picture of fluxes
(also from the Cornell potential)



$$\sigma_n l_n = const$$

Left: a string-like flux tube of tension σ. Right: a "fat string" as a collection of thin strings of different tensions $\{\sigma_n\}$.

- Continuos spectrum. σ can be promoted to a new spacetime coordinate.

  Polyakov: Liouville field as the 5th dimension.

- Discrete spectrum $\longrightarrow$ generalized Veneziano models. Andreev-Siegel

✦ A **big** question to ask:

quantum fluctuations in 4d $\approx$ geometry modification or $\{\sigma_n\}$ in 5d

The Model: soft wall metric model

5-dimensional Euclidean background metric (in string frame)

$$ds^2 = \frac{R^2}{z^2} h(z)\Big((dx^i)^2 + dz^2\Big)\,,\;\; i = 1, \dots, 4\,,\;\; h(z) = \exp\{cz^2\}$$

It is a one-parameter deformation of AdS.
The same number of free parameters as in the Cornell model.

✦ History

Hirn and Satz (2005): z^4-deformation $\quad h(z) = \exp\{kz^4\}$

Son et al (2006): soft wall dilaton model $\quad h(z) = 1\,,\;\; \phi = cz^2$

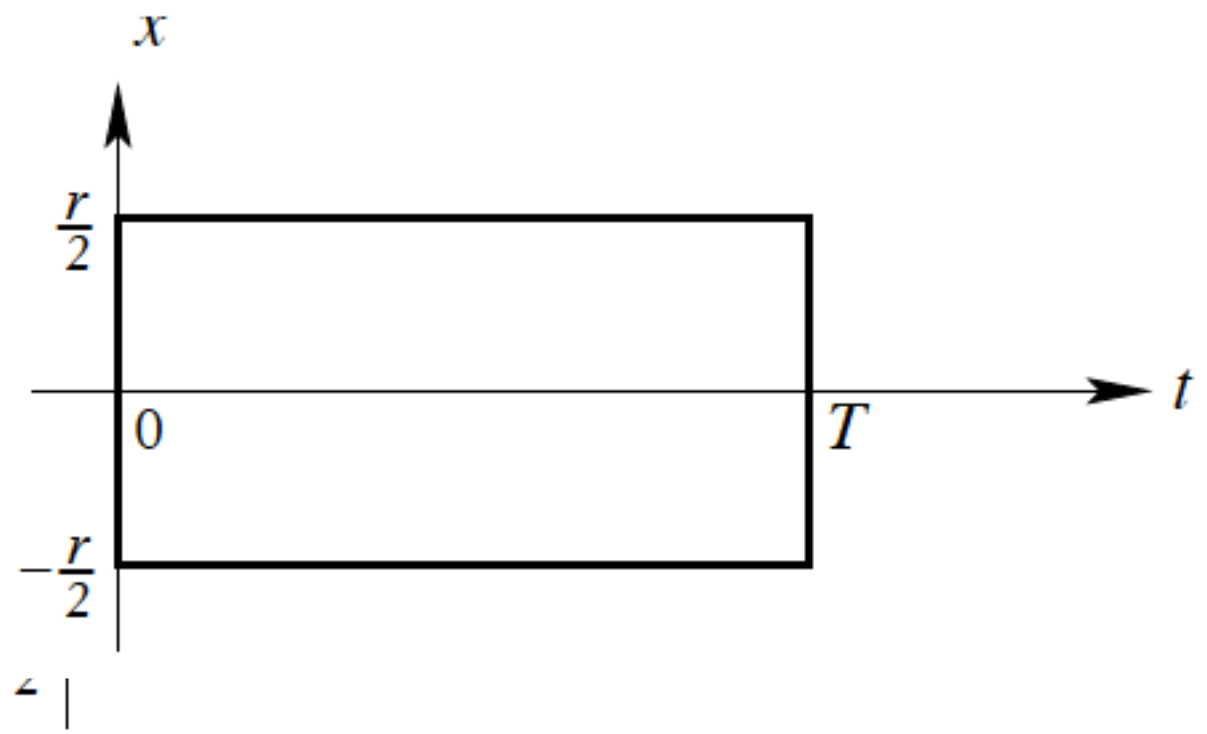
Metsaev (2000): Regge like spectrum of KK modes $\quad m^2 = cn\,,\;\; n = 1, \dots$

✦ Phenomenology

For the ρ-meson radial excitations $\quad c \approx 0.9\,\mathrm{GeV}^2 \quad$ Andreev

Example I: heavy quark potential

Take a rectangular Wilson loop



then use the proposal

$$\langle W(C)\rangle \sim \exp\{-S_{NG}\}$$

Rey-Yee-Maldacena

The potential is written in parametric form

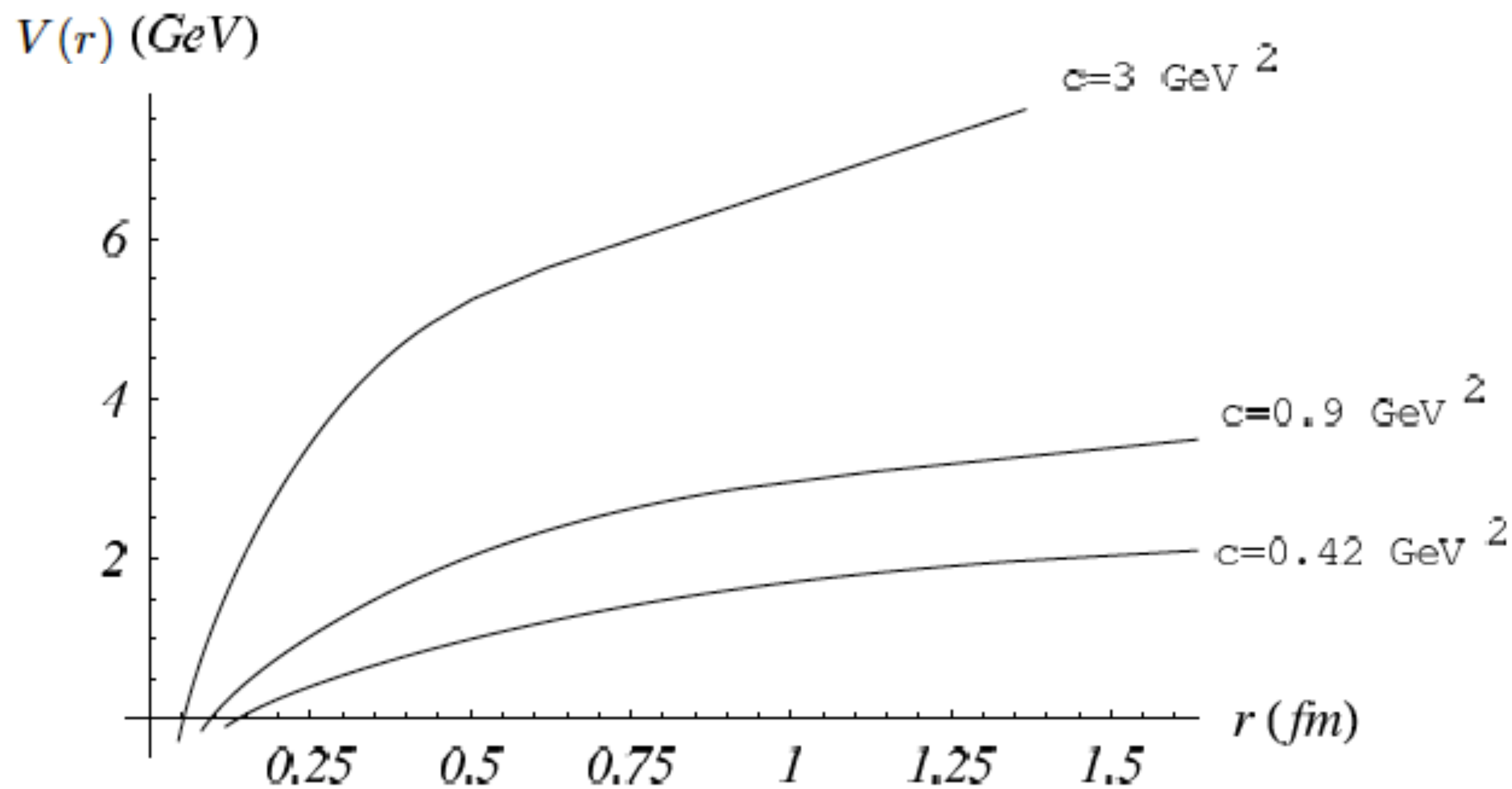
$$r = 2\sqrt{\frac{\lambda}{c}}\int_0^1 dv\, v^2 \exp\{\frac{1}{2}\lambda(1-v^2)\}\Big(1-v^4\exp\{\lambda(1-v^2)\}\Big)^{-\frac{1}{2}}$$

$$V = \frac{g}{\pi}\sqrt{\frac{c}{\lambda}}\left(-1+\int_0^1 dv\, v^{-2}\left[\exp\{\tfrac{1}{2}\lambda v^2\}\Big(1-v^4\exp\{\lambda(1-v^2)\}\Big)^{-\frac{1}{2}}-1\right]\right)+C$$

with λ a parameter and $g = \dfrac{R^2}{\alpha'}$

Andreev-Zakharov

Analysis of the potential



We can investigate the properties of V at long and short distances analytically.

$$V(r) = \sigma r + C + \ldots\,, \qquad \sigma = \frac{ge}{4\pi} c$$

$$V(r) = -\frac{\alpha}{r} + C + \sigma_0 r + \ldots\,, \qquad \sigma_0 = \frac{\Gamma^4(1/4)}{8\pi^2 e}\sigma \approx 0.81\sigma$$

It makes it different from the Cornell model.

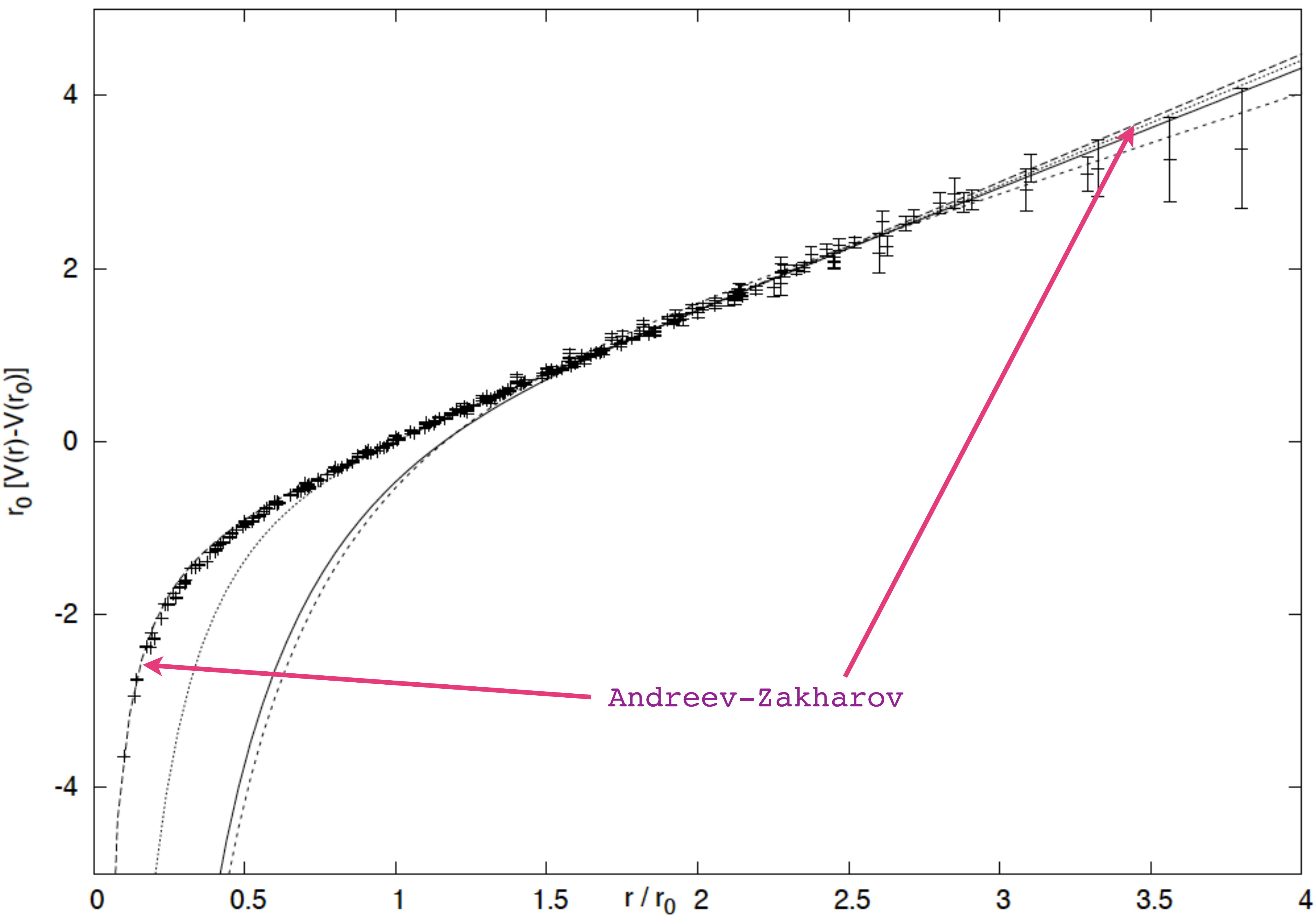
Fixing the parameters

There are three free parameters: c, $g = \dfrac{R^2}{\alpha'}$, and C

Options:

- the Cornell model
- the lattice data
- the heavy meson spectrum

5d string models vs lattice (pure SU(3) glue)



White: solving the inverse problem

Heavy meson spectrum from V

Flavor	Level	$J=0$			$J=1$		
		Particle	Th. mass	Exp. mass [6]	Particle	Th. mass	Exp. mass [6]
$c\bar{q}$	$1S$	D	1.862	1.867	D^*	2.027	2.008
	$2S$		3.393			2.598	2.622
	$3S$		2.837			2.987	
$c\bar{s}$	$1S$	D_s	1.973	1.968	D_s^*	2.111	2.112
	$2S$		2.524			2.670	
	$3S$		2.958			3.064	
$c\bar{c}$	$1S$	η_c	2.990	2.980	J/ψ	3.125	3.097
	$2S$		3.591	3.637		3.655	3.686
	$3S$		3.994			4.047	4.039
$b\bar{q}$	$1S$	B	5.198	5.279	B^*	5.288	5.325
	$2S$		5.757			5.819	
	$3S$		6.176			6.220	
$b\bar{s}$	$1S$	B_s	5.301	5.366	B_s^*	5.364	5.412
	$2S$		5.856			5.896	
	$3S$		6.266			6.296	
$b\bar{c}$	$1S$	B_c	6.310	6.286	B_c^*	6.338	6.420
	$2S$		6.869			6.879	
	$3S$		7.221			7.228	
$b\bar{b}$	$1S$	η_b	9.387	9.389	Υ	9.405	9.460
	$2S$		10.036			10.040	10.023
	$3S$		10.369			10.371	10.355
	$4S$		10.619			10.620	10.579

Gianuzzi: solving the Salpiter equation

Surprises from example I

- c is of order $0.9\,\mathrm{GeV}^2$

Consistency with the soft wall model estimate from the ρ-meson Regge trajectory.

- g is of order 1

If $g = \frac{R^2}{\alpha'} = \sqrt{\lambda}$, it seems likely that in the case of interest supergravity based phenomenology is not reliable.
The gauge theory is neither strongly nor weakly coupled.
Is this a reason why such a warped geometry has not been seen in SUGRA?

- Are corrections to $V(\alpha', 1/N)$ small?

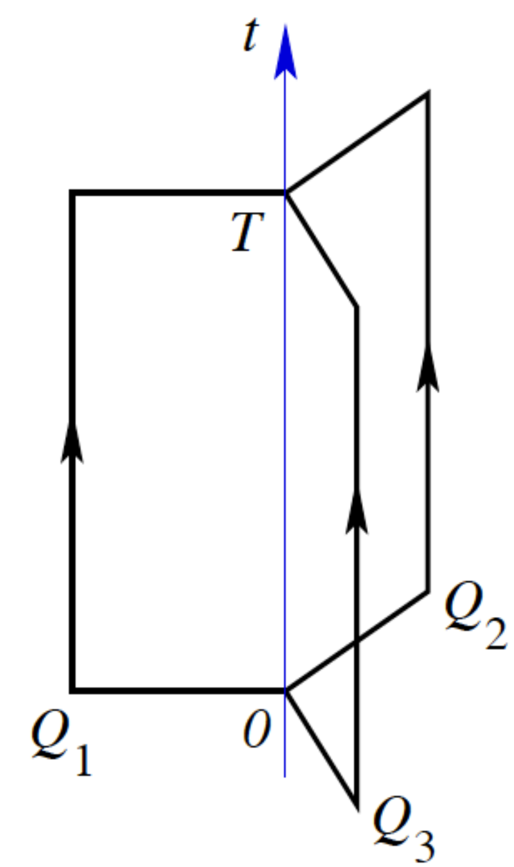
Do we really need to calculate the corrections?
Or leave it as a mean string theory approximation.

Example II: baryonic potential

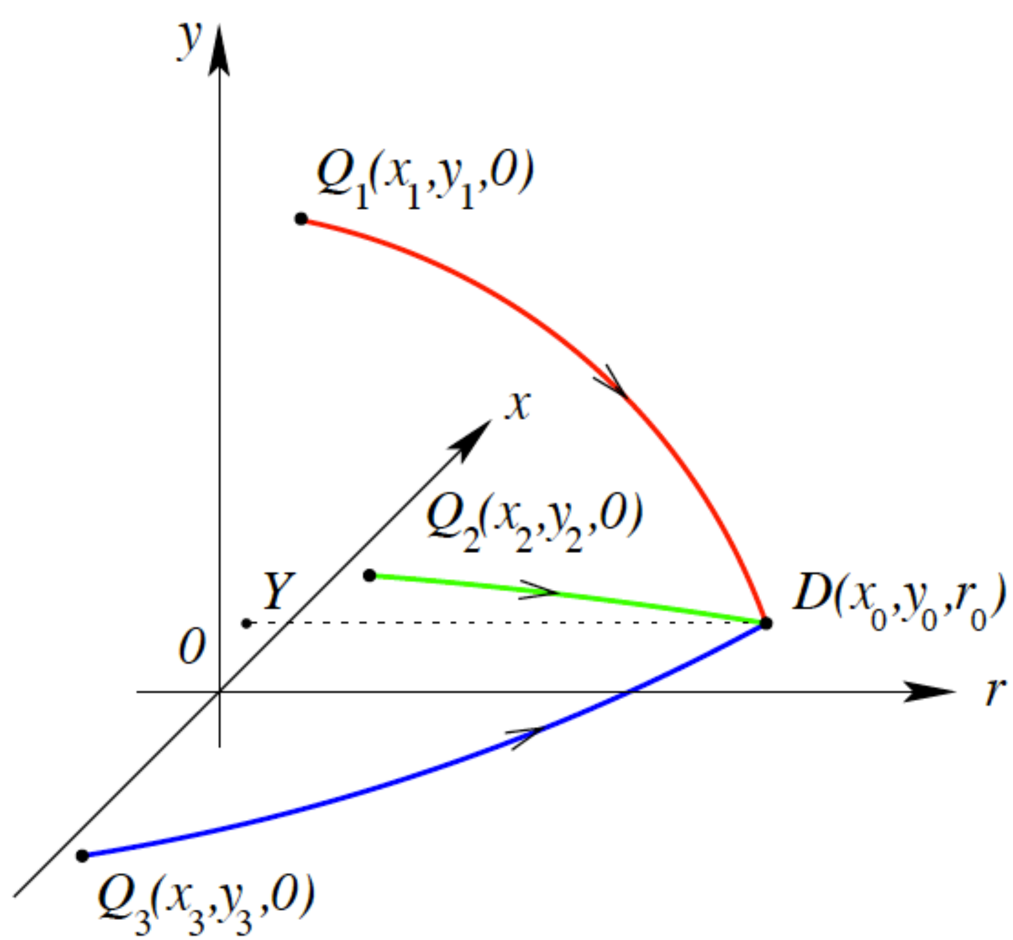
Take a baryonic Wilson loop for SU(3)

then use the proposal $$\langle W(C)\rangle \sim \exp\{-S_b\}$$

with $$S_b = \sum_{i=1}^{3} S_{NG}(i) + S_{bv}$$



A 5-dimensional view



Analysis of the potential

In general, the analysis is complicated: 7 equations and 3 parameters.

An illustrative example - the most symmetric configuration of the quarks.

Also take the baryonic vertex as a point like particle in 5d: $S_{bv} = mR\sqrt{h(r_0)}T$

The potential is given by

$$r = \sqrt{\frac{\lambda}{c}\rho}\int_0^1 dv\, v^2 \exp\{\lambda(1-v^2)\}\Big(1-\rho\, v^4 \exp\{2\lambda(1-v^2)\}\Big)^{-1/2}$$

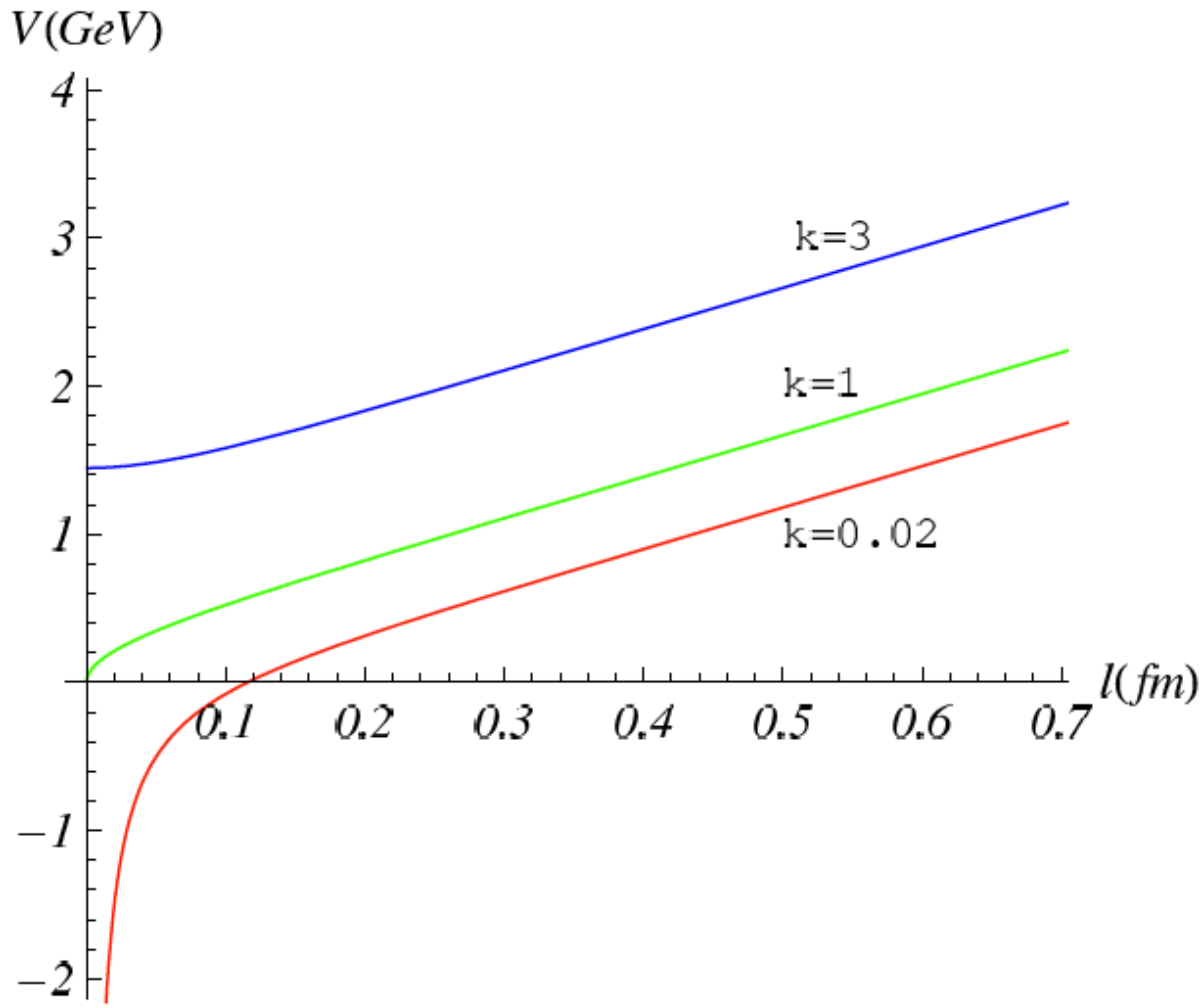
$$V = C + 3g\sqrt{\frac{c}{\lambda}}\bigg[\kappa \exp\{\lambda/2\} + \int_0^1 \frac{dv}{v^2}\bigg(\exp\{\lambda v^2\}\Big(1-\rho\, v^4 \exp\{2\lambda(1-v^2)\}\Big)^{-1/2} - 1 - v^2\bigg)\bigg]$$

with $\kappa = \frac{1}{3}\frac{mR}{g}$ and $\rho = 1-\kappa^2(1-\lambda)^2\exp\{-\lambda\}$

Andreev

Now, # parameters 3+1: c, g, C , and κ

Fixing the parameter



At short distances the form of the potential depends on the value of κ .

One possibility to fix it is to assume $$\alpha_{3q} = \frac{1}{2}\alpha_{q\bar{q}}$$

It gives $\kappa \approx 0.02$

Additional remarks to example II

- For a generic quark configuration:

  universality of the string tension $\sigma_{q\bar{q}} = \sigma_{3q}$

  the Y-law at large quark separations

- generalization to SU(N) is easy

- generalization to spatial baryonic loops

  (no lattice simulations yet?)

  universality of the spatial string tension $\sigma^{(s)}_{q\bar{q}} = \sigma^{(s)}_{3q}$ also at finite T

  the Y-law for the pseudopotential at large quark separations

Example III: hybrid potentials

✦ Hybrid mesons

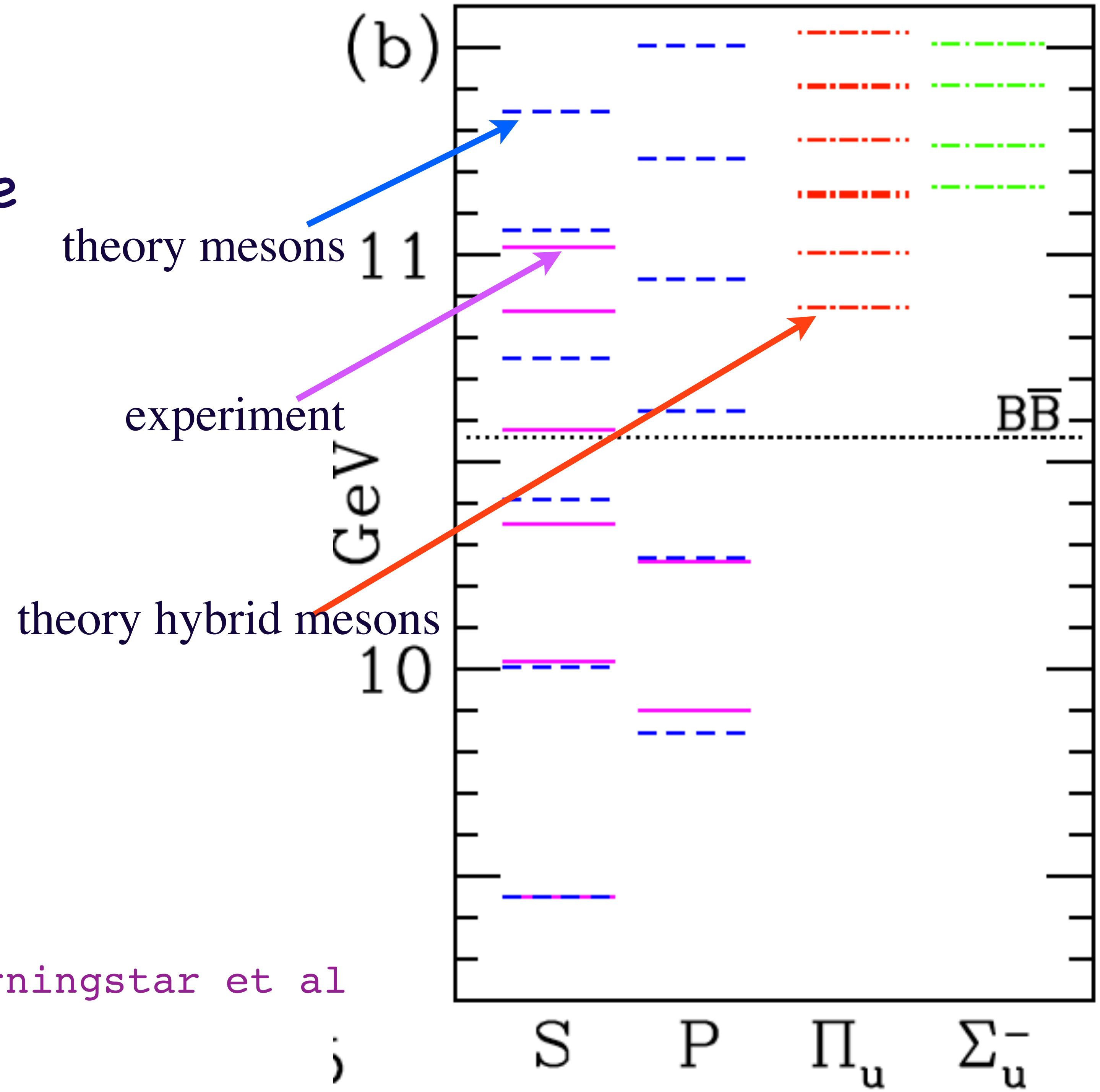
excited states of the flux tube

Isgur-Paton

to get hybrid meson spectrum

$$V_0 \rightarrow V_n$$

in the Schroedinger equation



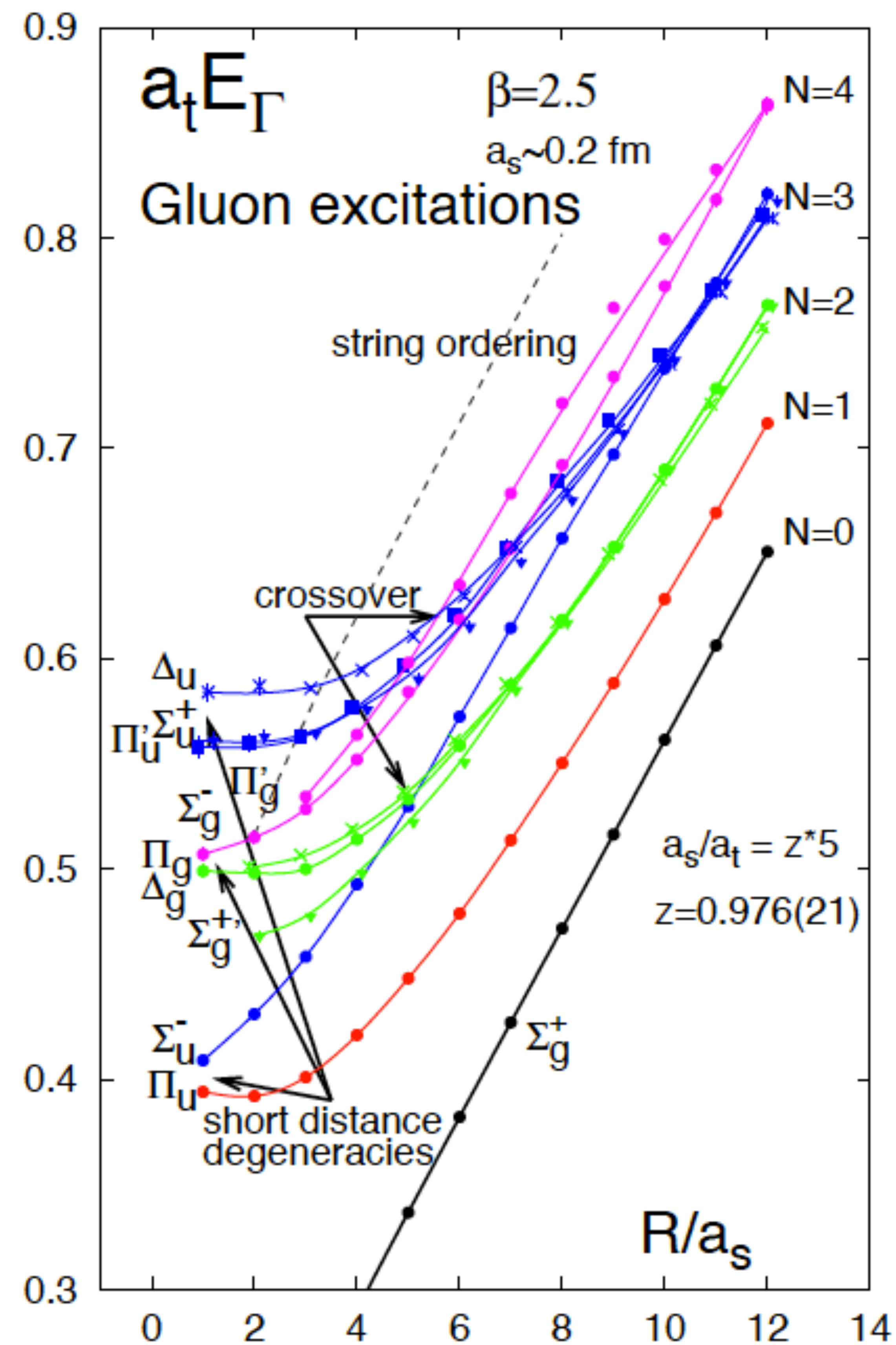
Morningstar et al

Excited states of the gluon flux tube

classification via reps. of D_{4h}

here $\Sigma_g^+ \equiv V$

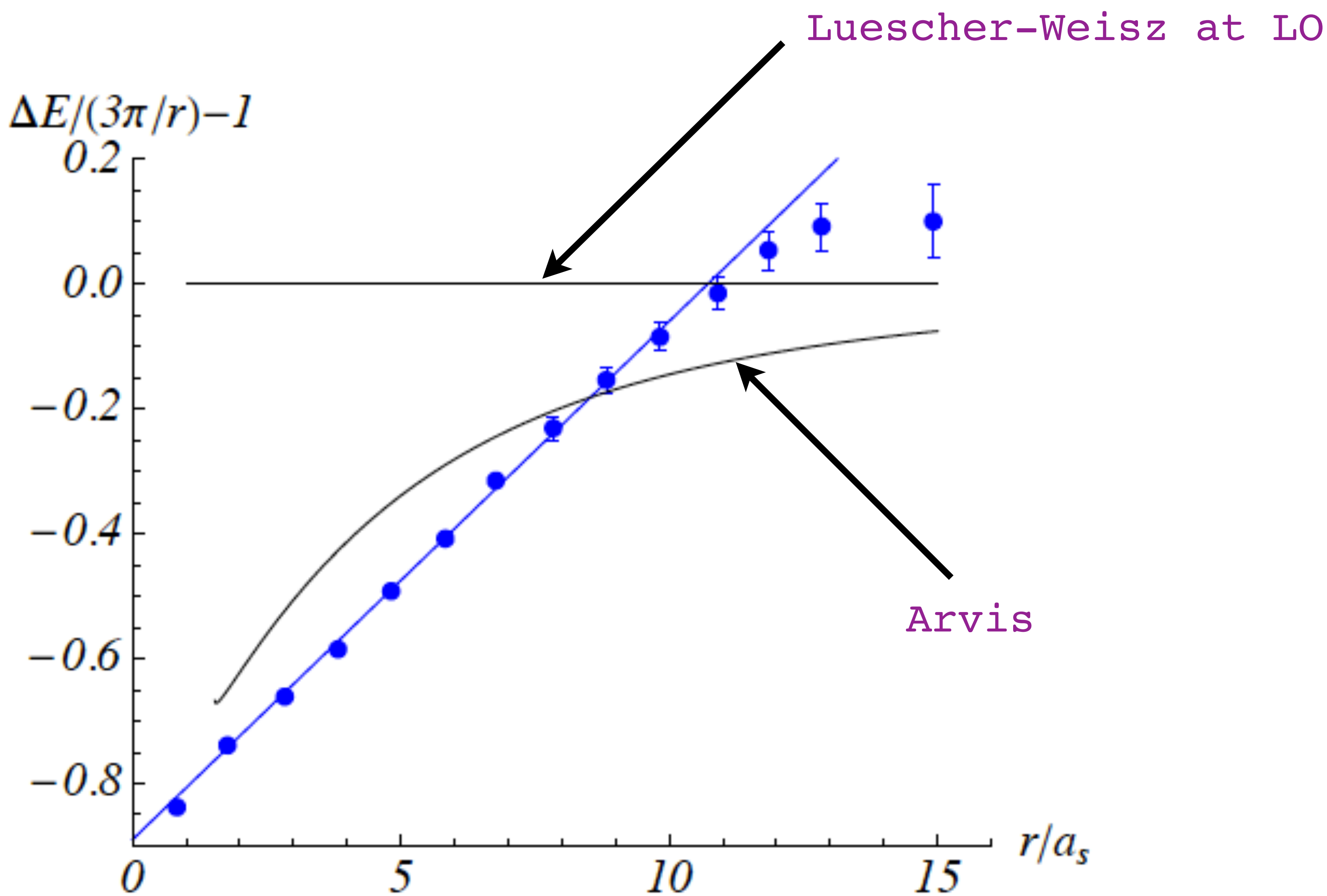
if Λ is a projection of angular momentum onto the quark-antiquark axis, then $\Sigma' s$ have $\Lambda = 0$, etc



Morningstar et al

Where do 4d strings fail?

The energy gap between Σ_u^- and Σ_g^+



5d model for the Σ's

assume that flux excitations are due to a little loop/baryon-antibaryon vertices

so take $S_\Sigma = \sum_{i=1}^{2} S_{NG}(i) + S_{bb}$

then treat the loop as a point-like defect with $S_{bb} = mR\mathcal{V}(r_0)T$

(Warning: obviously, this approximation fails at short distances)

The potential is given by

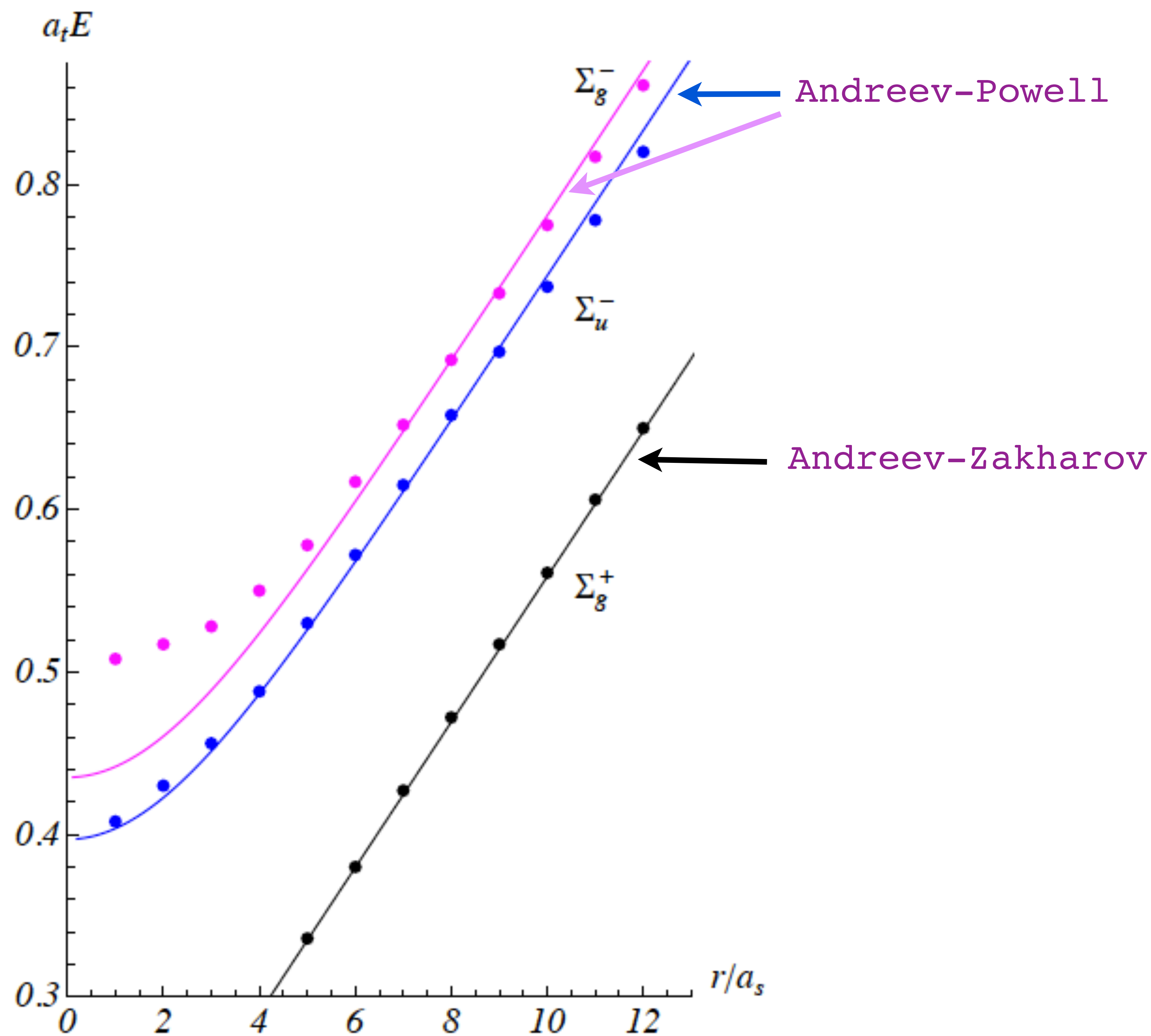
$$r = 2\sqrt{\frac{\lambda}{c}\bar{\rho}}\int_0^1 dv\, v^2 \exp\{\lambda(1-v^2)\}\Bigl(1-\bar{\rho}\, v^4\exp\{2\lambda(1-v^2)\}\Bigr)^{-1/2}$$

$$V_\Sigma = C + 2g\sqrt{\frac{c}{\lambda}}\Biggl[\kappa\,\exp\{-2\lambda\} + \int_0^1 \frac{dv}{v^2}\biggl(\exp\{\lambda v^2\}\Bigl(1-\bar{\rho}\, v^4\exp\{2\lambda(1-v^2)\}\Bigr)^{-1/2} - 1 - v^2\biggr)\Biggr]$$

with $\kappa = \frac{1}{2}\frac{mR}{g}$ and $\bar{\rho} = 1-\kappa^2(1+4\lambda)^2\exp\{-6\lambda\}$ Andreev-Powell

Now, # parameters 3+1: c, g, C, and κ

And this is how it works



Additional remarks to example III

✦ a flux loop is tricky

baryon vertex is nothing but a D5-brane in 10d Witten

baryon/antibaryon vertices -> $D_5\bar{D}_5$ bound state in 10d

(Warning: stability at short separations)

the form of S_{bb} can depend on a warp factor of the internal space

✦ Unlike V, there is no Coulomb term

note that Luescher-Weisz have it

Approximation breaks down

"In my opinion, string theory in general may be too ambitious. We know too little about string dynamics to attack the fundamental questions of the right vacua, hierarchies, to choose between anthropic and misanthropic principles etc. The lack of control from the experiment makes going astray almost inevitable. I hope that gauge/string duality somewhat improves the situation. There we do have some control, both from experiment and from numerical simulations. Perhaps it will help to restore the mental health of string theory."

A.M. Polyakov